A Study of Cognitive Orbits Based on Man-machine Interactions

Cheng Rengui[1,*], Liu Changyong[2] and Meng Shimin[1,3]

[1]School of Mathematics and Computer of Wuyi University,Wuyishan China; [2]School of Modern Information Service of Wuyi University, Wuyishan, China; [3]UlinkM.com INC., Wuyishan, China

Abstract: Representation-Computation is one of the topics consistently focused by researchers in the field of cognitive science. The representation of cognitive structures in the advanced cognition is an important part of research. How to observe and empirically validate the cognitive structures is meaningful. Based on man-machine interaction environments, a novel "Brain cognitive body - coupling Situations - information Manifolds (BSM)" cognition observation platform has been created, which allows users to observe and analyze cognitive processes and structures, selecting tree-based networks and other networks to represent low-dimensional cognitive topological manifolds, and to re-structure the cognitive structures and processes. Based on cognitive manifolds and cognitive dynamics, a cognitive process can be abstracted as a dynamical orbit in a low-dimensional cognitive manifold. The behaviors of the cognitive orbit are observed, related data are collected and the process is restructured in a low-dimensional cognitive manifold. The cognition is also analyzed by introducing mathematical methods, such as calculus and topology.

Keywords: Cognitive orbits, cognitive structure, man-machine interaction, topological manifold, visualization.

INTRODUCTION

The cognitive structure is a type of important representation of advanced cognitive activities (*e.g.* learning and thinking) and cognitive psychology. Piaget introduced category theory, morphisms and group theory to study cognitive structures [1]. However, from the perspective of empirical research, the cognitive structures are difficult to be observed due to lack of digitalization and visualization. Before understanding what cognitive structures are, first the cognitive structures should be observed, their data should be collected and these should be re-structured under the framework of mathematics in order to obtain more information about the fine cognitive structures and their dynamic features.

From the point of view of empirical research, the cognitive structure research can be explored at three layers, *i.e.* observation, detection and prediction. Undoubtedly, the observation (how can the cognitive structures in brain be observed?) is the most important issue in the empirical research of cognitive structures. Since 2011, based on advanced views of situated cognition, distributed cognition, embodied cognition and extended cognition, it has been found that cognitions could take place in a brain-situation-coupling space, and due to which "Cognitive Coupling Observation Principle (CCOP)" has been proposed [2-4]. Similar to studies in other disciplines, computer techniques are introduced to build man-machine interaction environments based on CCOP and create a novel "Brain cognitive body - coupling Situations - information Manifolds (BSM)" cognition observation platform.

*Address correspondence to this author at the School of Mathematics and Computer of Wuyi University, Wuyishan, 354300, China;
Tel/Fax: 086 0599-5136186; E-mail: crg@ wuyiu.edu.cn

The BSM observation consists of the following three points of views. 1) In category theory, the brain cognitive body, coupling situations and information manifolds are three types of abstractions that can represent cognitive structures or complete cognitive computation. 2) There are cognitive morphisms in BSM. The cognitive objects and relations in brain can be mapped to man-machine situations by coupling morphisms, and then mapped to information manifolds for digitalization. 3) The cognitive natural isomorphism and natural transformation can be applied to the BSM platform [5]. These principles ensure the objectiveness and covariance of cognitive objects and relations in SM digital systems and the ones in brain.

After completely understanding the objectiveness and covariance of BSM observation, further a mathematical system of cognitive structure observation is built based on topological manifold theories, and following propositions are made: 1) cognitions occur in high-dimensional cognitive manifolds; 2) coupling man-machine interaction environments are the local measurement of high-dimensional cognitive manifolds; 3) cognitive symbol sequences are collected in frame-based vectorized man-machine interaction situations. 4) local cognitive structures are mapped to low-dimensional cognitive metric spaces for dimensionality reduction.

As Fig. (**1**) shows, the high-dimensional cognitive situations are coupled to low-dimensional manifolds through coupling situations. The man-machine situations can be the cognitive situations which appear on computer screens; the low-dimensional manifolds are the representations of data in tables or graphs collected during the cognitive processes.

With the help of BSM, more accurate cognitive information can be collected and a digital environment can be

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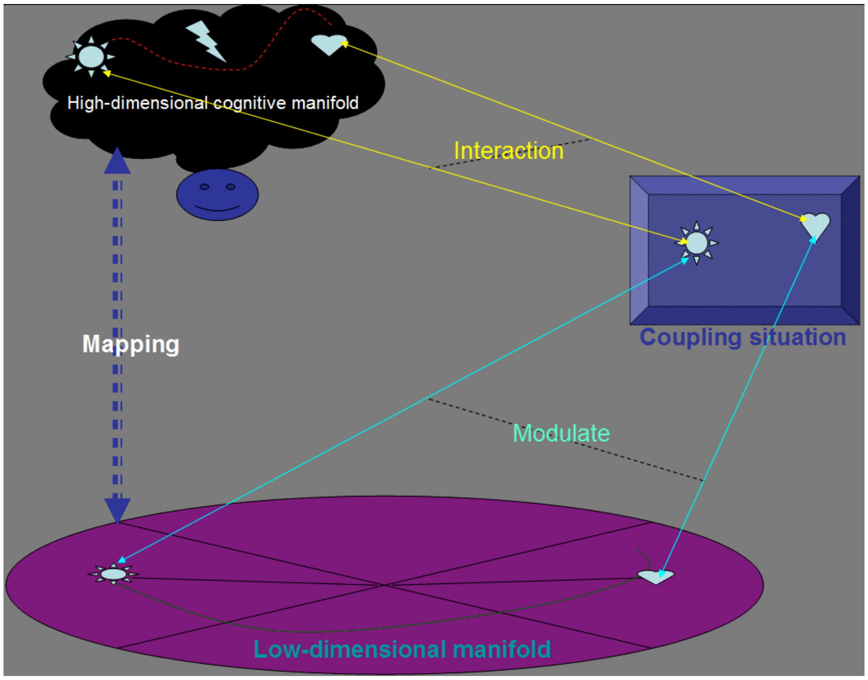

Fig. (1). Category-theory-based cognition observation in BSM. High-dimensional cognitive objects are induced by the coupling situations. The cognitive information is collected through man-machine interactions. Modulated by computers, the coupling situations can perform more collection interactions for the controllability of the cognition observation.

built that can be adjusted for cognitive dynamics. Moreover, accurate laboratories for cognitive dynamics research can also be provided [6, 7]. For example, like "cloud chamber" and "accelerator" in high energy physics, cognitive digital manifolds can be created to explore cognitive secrets.

In this paper, parts of the progress in cognitive structure observation are demonstrated, the structures of low-dimensional cognition spaces, the mathematical description and the basis of dynamical orbits are analyzed, and the foundation for cognitive structure detection and cognitive dynamic orbit prediction is provided. Advanced cognitive and thinking processes are selected as objects being observed in BSM, as the content of the advanced cognitive processes is important and is easily collected in the man-machine interaction observation.

LAYERED STRUCTURES OF BSM-BASED COGNITIVE LOW-DIMENSIONAL MANIFOLDS

Since a large amount of cognitive information is collected by BSM, these data need to be restructured. The basis of the re-structuring is to select a background manifold or space. In other words, it is to select a low-dimensional cognitive manifold that a high-dimensional cognitive manifold is mapped to. Thus, investigating the structures of low-dimensional manifolds is important.

In the field of psychology, the goal of studying the structures of low-dimensional cognitive manifolds is to solve the issues of how the cognitive structures can be represented and what manifolds the cognitive structures can be represented in. These cognitive representation issues are the core of cognitive psychology research.

Manifolds the Foundation of Cognitive Dynamical Systems

BSM is a novel platform that differs from traditional cognitive psychology research platforms. In traditional psychology research, cognitive data is collected by face-to-face interviews and questionnaires. Although computers and BCI devices are being used after the emergence of computers, but traditional cognitive psychology researchers have never applied cognitive coupling methods to data collection by inducing cognition to out-of-brain situations. Therefore, the traditional psychology researchers mainly focus on statistics and data processing rather than building cognitive models based on mathematics or cognitive dynamical systems.

Cognitive dynamics is a main trend of the development of current cognitive science and a promising path to higher achievement in the field of cognitive science and psychology as shown in Fig. (**2**). The essence of dynamics research is to provide manifolds and representations in spaces for objects, and introduce mathematical methods for empirical research.

Like Newton's mechanics [8], the studied space can co-exist with Euclidean geometry, which can be measured in a Cartesian coordinate system, Galilean transformation; Einstein's general relativity, the studied space is a curved manifold that can co-exist with Riemannian geometry, in which only local spaces can be measured in rectangular coordinate systems. The metric and tensor have become important concepts in Riemannian manifolds.

Cognitive dynamics could not be studied without mathematics. Due to high complexity of cognition, the cognitive issues cannot be solved by simple mathematical tools. Like Piaget who used mathematical theories to understand cognitive structures, we should pay more attention to mathematical theories, so that these mathematical theories and methods proposed in the 20th century or even the latest methods can be confidently introduced in our research.

Manifold is always a research focus, irrespective of being Riemannian manifold of general relativity or contemporary advanced physics and mathematics. Shiing-Shen Chern [9], a leader in differential geometry, predicted that the mathematical research would be dominated by manifold studies in future. Traditional real and complex spaces are only local situations of manifolds.

In mathematics, it is assumed that complex cognitive spaces are manifolds. The content in a local space of a manifold can be observed and measured; measuring a local space of a manifold is a type of manifold analysis, which divides a manifold into a number of small measurable components. The manifold can also be analyzed by using particular methods in order to obtain the characteristics of an entire space or a wide range of the space.

Based on cognitive coupling, cognition is coupled to out-of-brain man-machine interaction situations by BSM through coupling situations. The cognitive content in brain can be indirectly perceived by observing man-machine cognitive coupling situations. Cognitive coupling observation shows consistency with a point of view in advanced cognitive philosophy, which states that cognition occurs in the coupling interaction of the brain and situations.

In the paragraphs above, the mathematical background of BSM-based cognitive observation has been preliminarily demonstrated. In the next step, how to re-structure the data collected by BSM is described, which is similar to astronomy, re-structuring the states of astronomical objects by using mathematical theories and tools after data collection from telescopes.

Layered Structures of BSM-based Cognitive Low-Dimensional Manifolds

The simplest type of manifolds is the topological manifold. A local space of a topological manifold can be mapped to a Euclidean space, which can also be considered as building a Cartesian coordinate system in a low-dimensional manifold for measurement. However, the cognition is a complex process. In many cases, a local space of a cognitive manifold cannot be easily mapped to a Euclidean space. Therefore, a better low-dimensional manifold needs to be selected for representing the content in a local space of a high-dimensional manifold.

In mathematics, a manifold can be approximated by a graph, or the manifold and the corresponding graph can be considered as an isomorphism to some extent. A graph G=(V, E) can be abstracted by a set of vertices V and a set of edges E. Given a sample set X from a d-dimensional manifold, a one-to-one mapping between the sample data and the vertices in G is created. The similarity of a pair of the sample data is defined as an edge in G. So, a graph is built by the sample set. Graphs and manifolds share a number of features, the most important one of which is that both of them can be embedded into a Euclidean space. One corresponding graph of a manifold is a topological object. Its topological properties can be represented by edge weights. For example, a complex cognitive network can be used to describe a low-dimensional manifold of a high-dimensional cognitive manifold, in which a node represents a cognitive object and an edge represents a logic relation between two cognitive points [10].

The cognitive topological structures can also be considered from the perspective of the cognition occurrence. Cognition takes place in the coupling process of brain and situations. What type of topological structures do we perceive most in contemporary cognitive situations? Apparently, the textbooks are the basis of learning and are organized in tree structures. Therefore, from the perspective of cognition occurrence, the tree structure is the most widely used cognitive topological structure in the internalization of education environments.

Tree structure is an important non-linear structure, in which nodes have multiple branches and are organized in multiple layers. The tree structure is similar to trees in the real world. There are a number of examples of tree structures in the objective world. For instance, genealogy and the organization of institutions can by represented by trees. In the field of computer science, the tree structure is widely applied in compilers, presenting the grammar structures of source codes; in database systems, the tree structures can be used to organize information; in algorithm analysis, the execution of programs can be described by trees.

Trees and networks are similar to some extent. A tree structure can be easily transformed to a network structure, but it is difficult to transform a network structure back to a tree structure, because no loop exists in a tree. In this research, an attribute "link" is introduced to every node of a tree, which indicates a connection from the current node to other nodes. The connection is called the "virtual link". With the "virtual link", all topological networks can be described by trees, which makes networks equivalent to the trees with "virtual links". For the convenience of presentation, these are known as "tree-based network" for short in the paper.

The tree-based networks can be unlimitedly extended. Child nodes in a particular node in the tree-based network can be generated on demand. In BSM, if a finer-grained observation on a cognitive object is required, a child node of the cognitive object currently being observed is generated.

In the previous paragraphs, manifolds have been analyzed, tree-based networks have been selected as the structures of low-dimensional cognitive manifolds, and summarized as uniform structures in digital environments.

In general, objects and elements in a high-dimensional manifold are mapped to a low-dimensional topological manifold and represented by tree-based networks through coupling situations on the BSM-based cognitive observation platform. The process is called cognitive structure imaging. Similar to films in traditional photography, the tree-based networks are the cognitive "films" on the BSM observation platform.

On the BSM platform, first an observed object obj is determined, and then a COM observation frame on obj is built in coupling situations. When any operation is performed on obj by users, cognitive symbol sequences are collected and

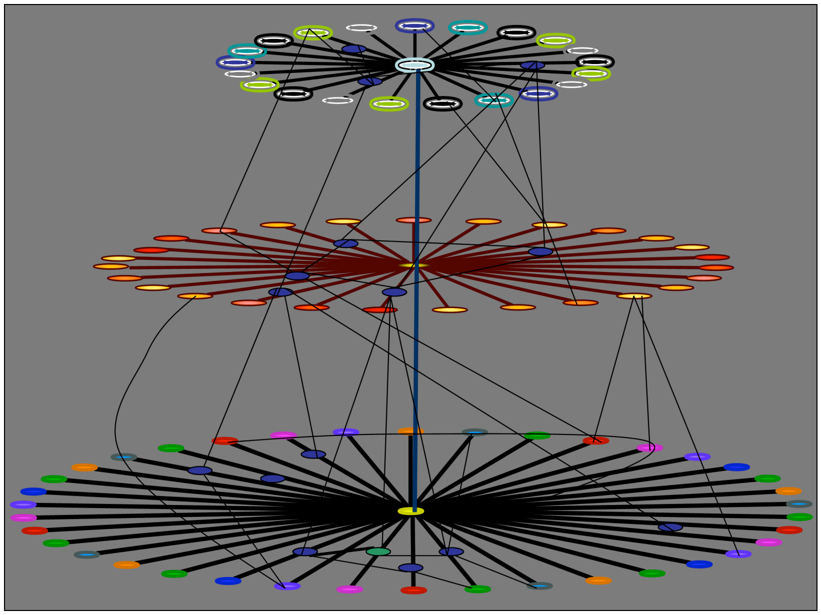

Fig. (2). A layered model of a cognitive structure. (A tree-based network of order one is displayed for easy understanding).

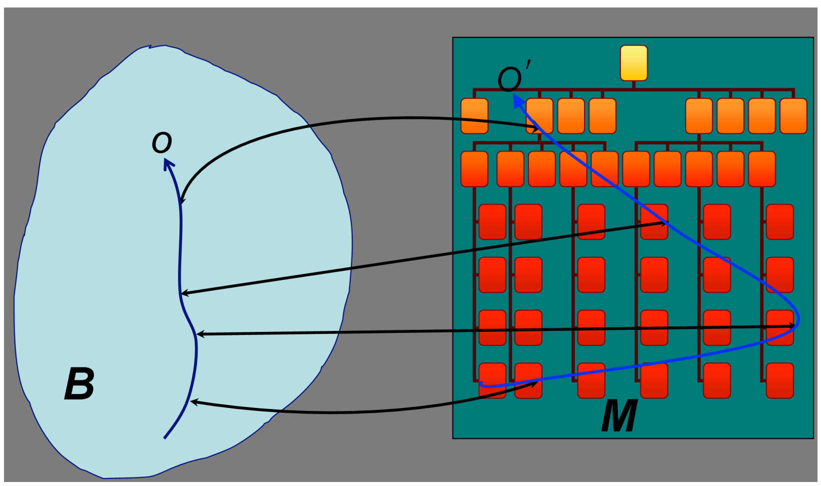

Fig. (3). The projection of a cognitive orbit.

the observation information is imaged to "films" (tree-based networks).

Based on the extensibility of the tree-based networks and the feasibility of the observation on the BSM platform, a large amount of content and structure in high-dimensional cognitive manifolds can be mapped to information manifolds for cognitive structure digitalization, which is difficult to apply to traditional cognitive psychology and pedagogy.

Analysis of BSM-based Low-dimensional Manifolds

On the BSM-based cognition observation platform, the process of man-machine interactions is designed and users' cognitive data are collected in the man-machine coupling situations. These cognitive data are re-structured in tree-based topological spaces, as shown in Fig. (**3**). The BSM observation consists of the following steps: 1) design a cognitive orbit O; 2) observe O and collect its observation data on the BSM platform; and 3) obtain a cognitive orbit O' by re-structuring O in a tree-based space.

1) **Cognitive orbit design**: cognitive orbits are designed based on the experience in psychology and pedagogy. For example, the content and process of learning and observation points; an assumption of the design is that the cognition occurs in Euclidean spaces or tree-based network spaces.

2) **Cognitive symbol collection**: the man-machine interactions and the cognitive data collection are designed based on the content of the cognition. For example, keyboard inputs, the position of the mouse, cognitive results, operation times, *etc*.

3) **Cognitive process re-structuring**: based on the description of cognitive orbits in tree-based spaces, the cognitive data is restructured in low-dimensional cognitive manifolds.

Key parts of the observation process are how to design the content of low-dimensional cognitive manifolds, and what cognitive data is collected during the observation. During the process, more attention should be paid to key data collection. Moreover, an appropriate design of low-

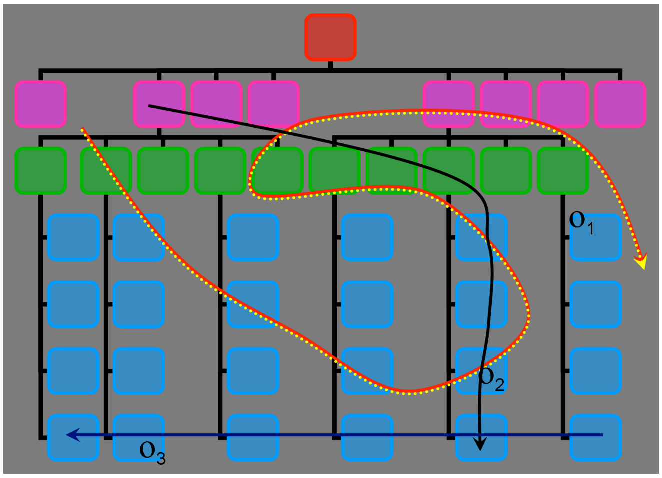

Fig. (4). A cognitive orbit in a tree-based network space.

dimensional cognitive spaces should be selected by using scientific methods.

Fig. (**3**) displays an observation result from the process described above. More mathematical operations can be further performed on the low-dimensional cognitive manifold. For example, it is assumed that there is a metric between each pair of points in the tree-based network space. 1) The time t is measured in a uniform manner; the time is a variable in the integration; 2) if K is a function of t, *i.e.* K=f(t), and the addition operation can be applied to K at different points, thus, 3) the following integration of K on the orbit o from the time t_a to t_b can be obtained.

$$\boldsymbol{E}=\int_{t_a}^{t_b} f(t)dt$$

With powerful mathematical tools, such as integration, more cognitive dynamics researches can be effectively conducted.

Based on low-dimensional cognitive manifolds, more designs of cognitive orbits can be achieved, such as the orbit prediction based on mathematical methods or experience in the cognitive process, and the orbits being observed in real observation. The predicted and observed cognitive orbits can be compared on the BSM-based platform, in order to conduct cognition analysis and research more effectively.

In Fig. (**4**), it can be assumed that there are multiple potential orbits that can be designed for mastering a particular cognitive skill, and the orbits can be visualized by curves in tree-based spaces.

A CASE STUDY OF COGNITIVE ORBITS

Analysis of BSM-based Cognitive Mechanics

A mental calculation learning system is designed, in which the mental calculation questions are displayed on the left, and a digital keyboard is shown on the right. Like rain, the questions fall from top to bottom. Before the questions reach the ground, users need to input their answers through clicking the "submit" button on the keyboard. If an incorrect answer is submitted, all questions keep on falling; otherwise, the corresponding question disappears on the screen. The remaining time and training score are displayed at the top right corner of the screen [11].

According to the specification of Newton systems, a simple cognitive mechanical model is created. Let M be the cognitive value (similar with mass in physics) of a cognitive object. (1) Let C denote the score of each test, which can be considered as velocity; (2) T denotes the sequence of the tests, which can be considered as a time variable; (3) A describes the acceleration of the improvement in the tests (like acceleration in physics); (1) the force of cognition can be calculated as F=M*A; (5)thus, a cognitive orbit model for mental calculation has been established, which is similar to

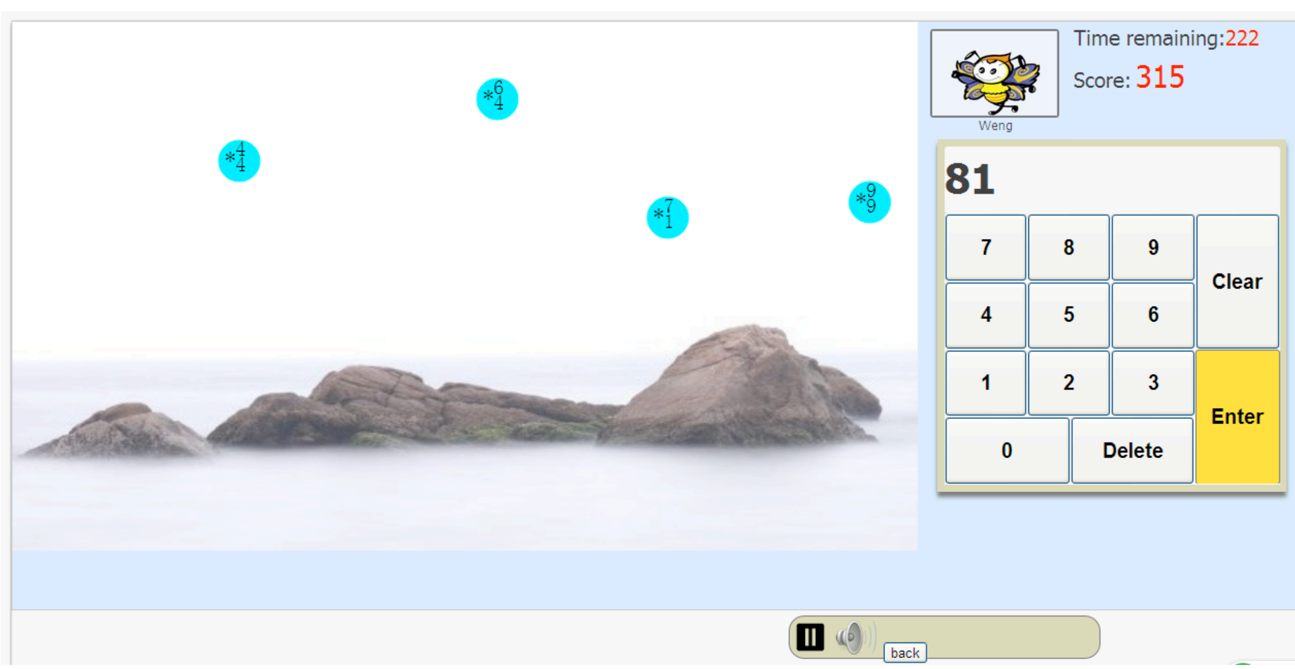

Fig. (5). A man-machine interface for collecting mental calculation training data.

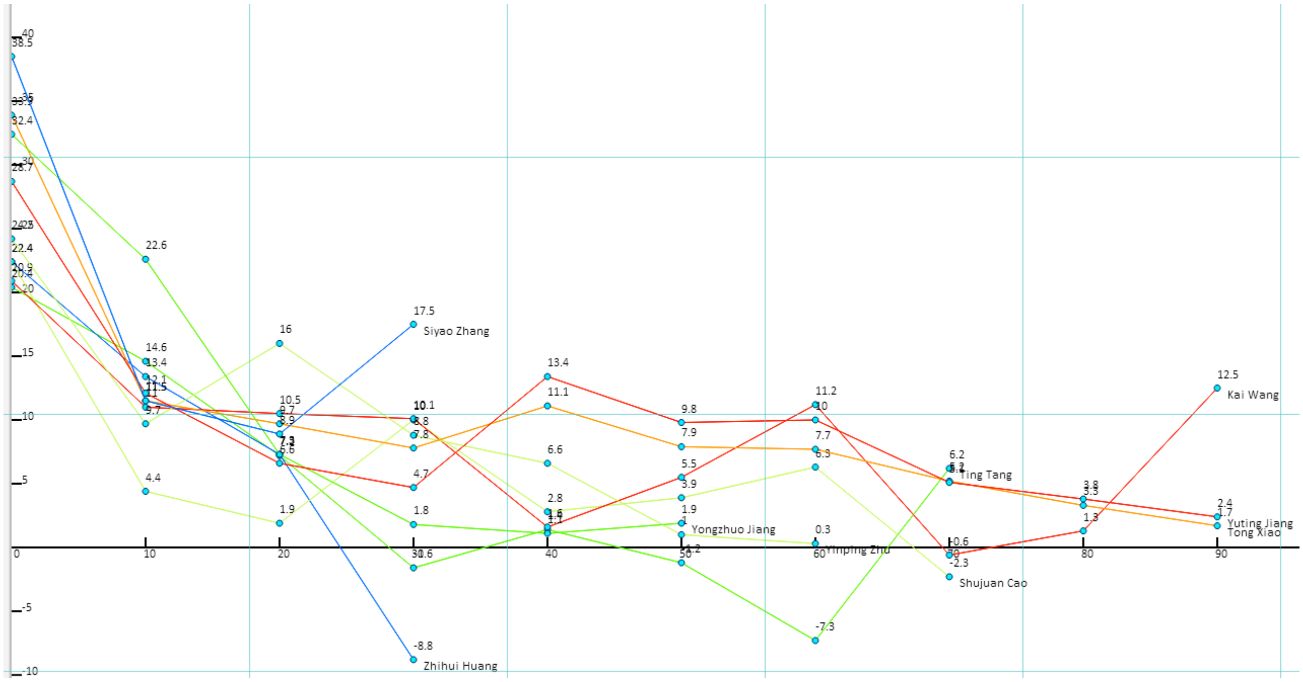

Fig. (6). Acceleration of mental calculation training.

the orbit models of object motion in Newton mechanics as shown in Fig. (**5**).

In the model, the cognitive orbits represent the development of students' mental calculation ability, including the acceleration, *etc*. Any change in students' acceleration, such as turning from the positive direction to the negative, can be observed in the model as well.

Fig. (**6**) displays an example of the mental calculation acceleration. The trends of students' acceleration can be observed in the figure, like observing object motion in physics.

Analysis of Cognitive Topological Structure

Cognitive topological structures are the spaces of cognitive dynamics or cognitive orbit observation. The cognitive topological structures are also the foundation of the cognitive dynamics analysis. In our research approaches, the cognitive topological spaces are described by tree-based networks. The key and challenging part of the research is how to automatically obtain the topological structures in tree-based networks from the BSM platform. By investigating the man-machine interaction in students' testing as a case study, the automatic methods are focused that can build the topological structures in tree-based networks. Based on the tree-based network model of cognitive structures, it is assumed that every question in a test is an instance (**Cobj**) of a concrete cognitive point. These instances (**Cobj**) are continuously abstracted to be high-order cognitive objects in order to create a multi-layered cognitive structure. With the help of vectorization technologies, such as SVG (Scalable Vector Graphics), a visualization model is built for cognitive tree-based networks.

Based on the BSM platform and the process of testing, the cognitive object collection, structure extraction and analysis are completed in the following steps:

1) Test paper vectorization-markers are the instances of cognitive objects: the test papers are scanned and answers are collected at the places reserved for the answers. Taking the question 3+9=? as an example, students are required to input their answers at the place where the symbol "?" is in a pre-defined manner. The answers can be easily perceived
by a computer, no matter the answer is 12 or 21. All answers are considered as the testing data for the question (Fig. **7**).

2) Association of testing points and high-order structures-build abstract high-order cognitive objects and their structures: according to the objectives of education, testing points in a testing paper are abstracted as high-order knowledge structures or cognitive structures. The high-order cognitive structures can be identically considered as structures at the second layer over the test paper. Apparently, alternative structures would be given by professors, indicating their different understandings of the test (Fig. **8**).

3) Analysis of testing points and cognitive structures in disciplines: in (1) the test vectorization and (2) the determination of the testing content and high-order cognitive structures, (3) the testing points are associated with the cognitive structures. Undoubtedly, alternative testing points would be associated with different cognitive points, and one testing point would be associated with different cognitive points. (Fig. **9**).

4) Automatic collection for the statuses of cognitive structures: after associating the knowledge structures with testing points, automatic analysis can be applied to the properties and relations of the multi-layered cognitive structures. For example, the testing results are obtained for each testing point, (Fig. **10**), and then the rate of correct/wrong is determined, and then these are marked in red, yellow and green to indicate the status of students' understanding.

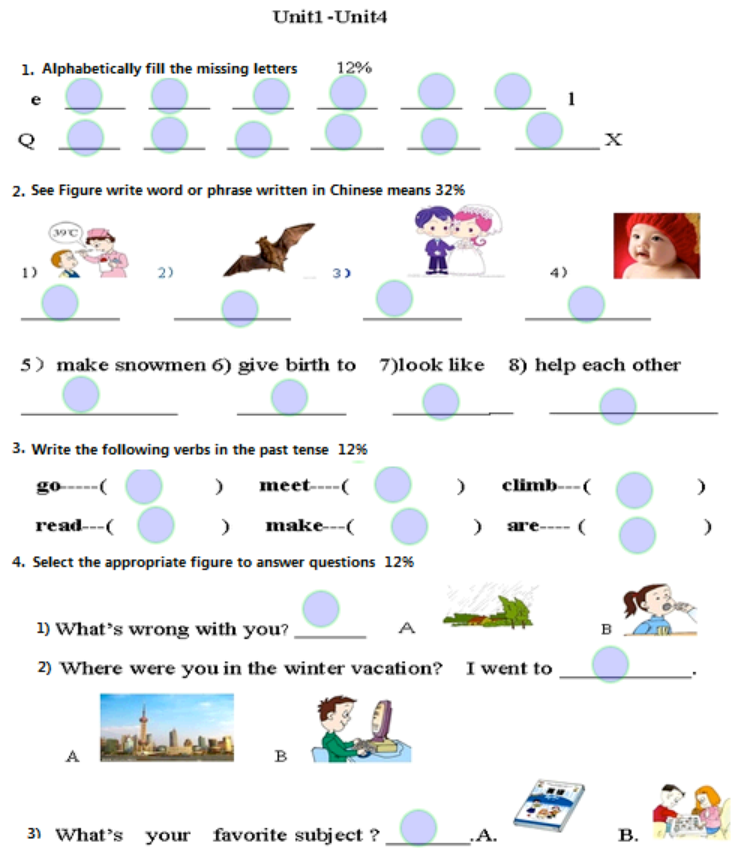

Fig. (7). An example of test paper vectorization and answer collection.

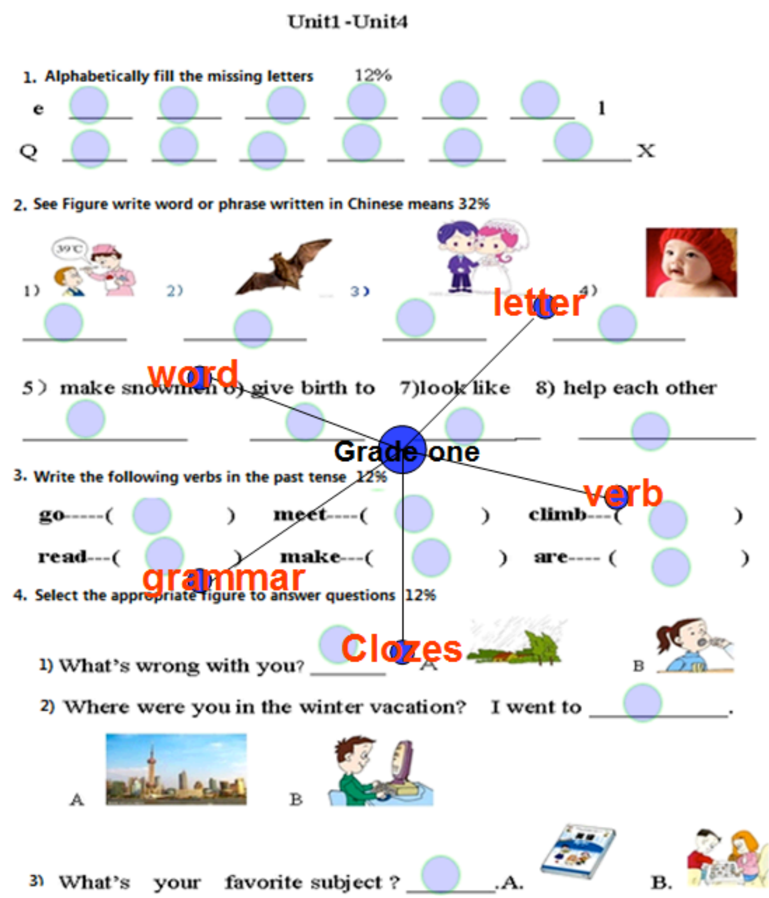

Fig. (8). High-order cognitive structures of testing points.

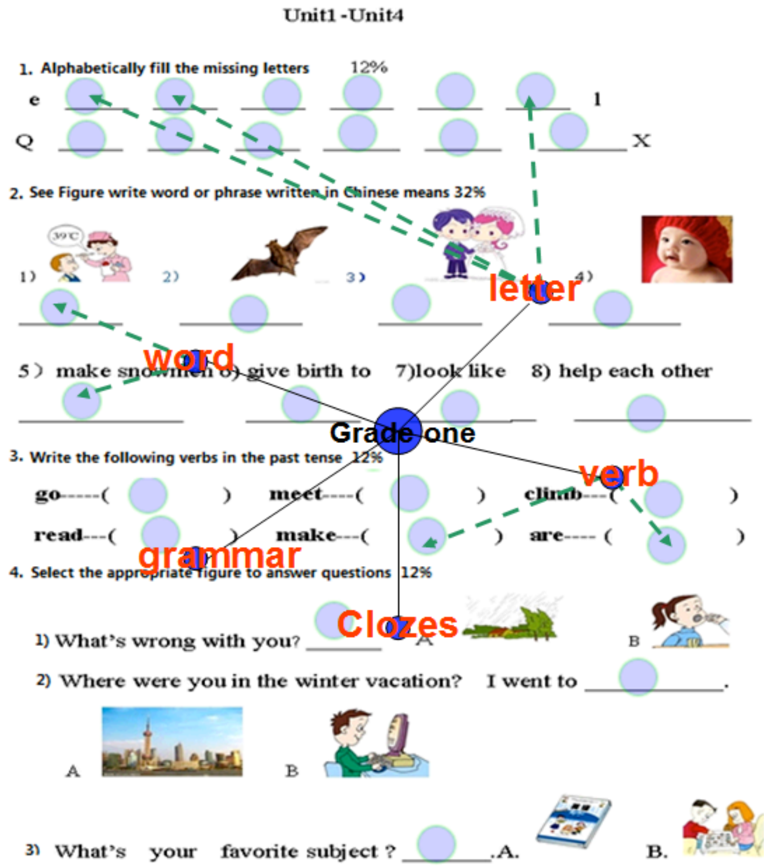

Fig. (9). Association of the knowledge structure and testing points.

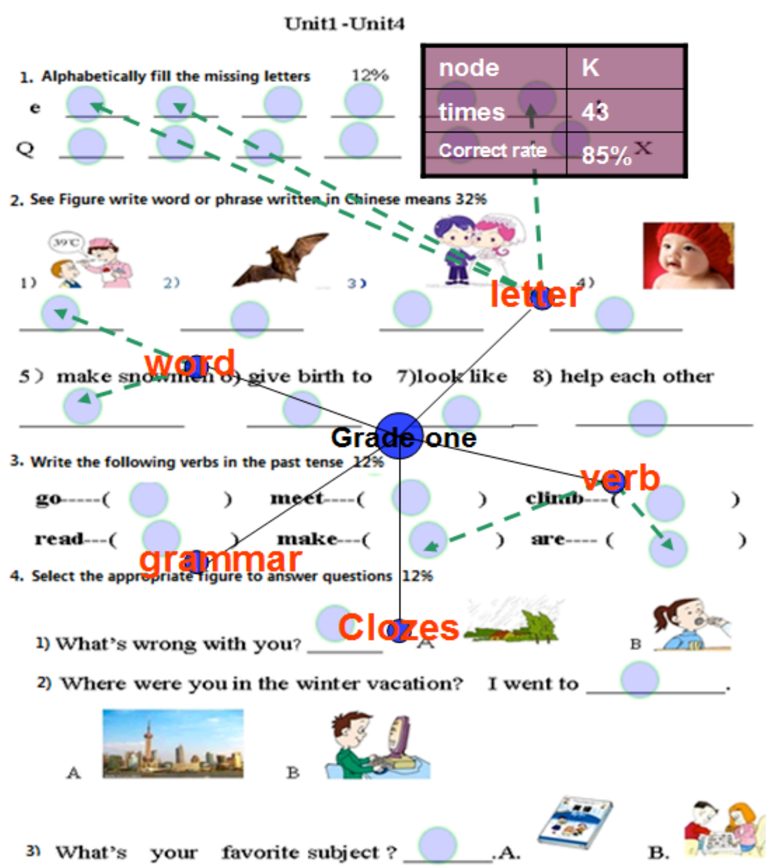

Fig. (10). Automatically collect testing results of the cognitive (knowledge) points.

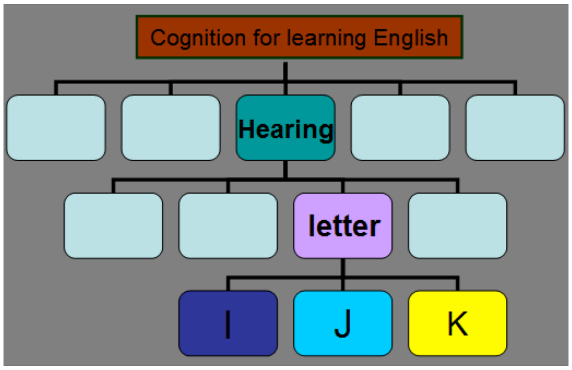

Fig. (11). Cognitive structure tree topological space.

5) Cognitive structure imaging for disciplines and orbit dynamics analysis: achieving cognitive topological structures in tree-based networks is a milestone, which indicates that a cognitive topological space is created, (Fig. **11**). In space, cognitive points and their relations can be built by cognition observation data. Moreover, cognitive orbits can be predicted and planned, and the process of cognitive dynamics can be analyzed in space. For example, the cognitive points were grouped in a cognitive structure by particular cognitive rules. Each ground represents a cognitive orbit.

In this section, a method is presented that builds a cognitive tree-based network based on BSM digital environments, and simple cognitive object generation and analysis methods. By utilizing the cognitive topological structure, the cognitive data are collected from the behaviors of learning and testing, estimating students' understanding statuses in the cognitive structures. Based on the analysis methods of cognitive topological tree-based structures, like X-ray imaging used by doctors in diagnosis, evidence of empirical research and automation for education and support for education decision and management are provided.

CONCLUSION

A novel man-machine-based cognitive coupling observation platform (BSM) is created, which allows users to observe the process of cognition and learning online, collecting a large amount of cognitive information and providing support to cognitive representation and empirical analysis of cognitive structures. Based on the BSM platform, high dimensional cognitive manifolds are mapped to low-dimensional tree-based manifolds for cognitive structure visualization.

The visualized cognitive structures not only support cognitive structure imaging and cognitive orbit research, but also provide cognition research "materialization and technicality". In cognitive topological structures, the cognitive orbits are designed and observed, and cognitive dynamics research is conducted.

The BSM-based cognitive structure observation not only helps in the in-brain cognitive structure imaging and empirical research of cognitive structures, but also provides a novel research scheme for cognitive observation, detection and prediction.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

The paper is supported by the following funds: the Education Ministry Fund in Fujian Province (JA12321, JA13318, JB05189), The Special Fund for Industry Development and Scientific Research in Nanping, Fujian (2011DJ04) and the Key Scientific Project Fund in Nanping, Fujian (N2012Z06-7), Wuyi University Research Fund for Young Teachers (xq201025). Wuyi University Research Fund (XQL05003).

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Received: June 10, 2015 Revised: July 29, 2015 Accepted: August 15, 2015