New Evidence on the Excess Smoothness of Consumption

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Abstract: The purpose of this paper is to revisit the evidence on the excess smoothness of consumption within the permanent income model, by using recently available monthly data. Two formulations of the univariate process of personal disposable income are adopted: in the levels and in the log-levels. More than one sample is studied. Three different impacts are defined and measured. In theory, the three of them should be equal. The conclusion that is strongly supported is that these three impacts are significantly different from each other, implying that excess smoothness is still a feature of the data. However a weak version of the permanent income hypothesis is endorsed which is that consumption changes by the annuity value of revised expectations of future income. In other terms, permanent income innovations have a significant, although relatively small, effect on consumption.

Keywords: Permanent income hypothesis, excess smoothness of consumption, ARIMA models, revision of expectations, income innovation, consumption on non-durables, US evidence.

INTRODUCTION

Many macroeconomic theories postulate that consumption is smoother than income. Over seventy years ago, Keynes [1] introduced his famous marginal propensity to consume which is meant to be less than one, implying that the volatility of consumption is lower than the volatility of income. Over half a century ago Milton Friedman [2] published his seminal book on the theory of the consumption function, in which his major contribution was that consumption depends on permanent income. In turn, permanent income depends on life-time resources of financial and human capital. Since permanent income is by definition smoother than actual income, then consumption ought to be smoother than actual income. After Keynes and Friedman other economists have studied the permanent income hypothesis and added to our knowledge of the behavior of consumption. Hall [3, 4] is another pioneer. His research led to the theoretical formulation of the random walk behavior of consumption. This formulation relies on the maximization of discounted future utility of consumption with a wealth constraint. Assuming a quadratic preference function, Hall derives the first-order Euler Eq. that implies that consumption should follow a random walk: the change in consumption is orthogonal to any information known in the current period. However, as Flavin [5] demonstrates, the change in consumption depends on the innovation in income, which is produced by a revision in expectations of the annuity value of future income, and which is nothing else but the revision in expectations of future permanent income: this is rightly called the Random Walk Permanent Income Hypothesis (RWPIH).

If income follows an ARIMA(1,1,0) data generation process, with a partial autoregressive coefficient \( \lambda \), then consumption \( (C) \) will react to the income innovation \( \varepsilon_{t+1} \) in the following manner [6–11]:

\[
\Delta(C_{t+1}) = \frac{(1+r)}{(1+r-\lambda)} \varepsilon_{t+1}
\]

(1)

where \( r \) is the (constant) interest rate, \( t \) is the time period, and \( \Delta \) is the first-difference operator. If \( \lambda \) is positive, as it is usually the case with quarterly and annual data, then the multiplier of the income innovation \( \varepsilon_{t+1} \) is greater than one. The empirical results in Campbell and Deaton [7] show that, with quarterly data, \( \lambda \) is equal to 0.442, making the multiplier to the left of the income innovation \( \varepsilon_{t+1} \) in Eq. (1) around 1.79 with a zero interest rate, and 1.75 with a 12% interest rate. Malley and Molana [8, Table 2, 1031] find \( \lambda \) to be 0.315 with annual data, implying a multiplier of \( \varepsilon_{t+1} \) equal to 1.46, with a zero interest rate, and 1.39, with a 12% interest rate. From this it is obvious that, contrary to theoretical expectations, the inferred change in consumption is a multiple of the size of the income innovation. This means that the RWPIH implies theoretically that the standard deviation of the change in consumption is much higher than the standard error of the univariate income process: both Keynes and Friedman are hence contradicted. Actual quarterly figures place the ratio of the volatility of the change in consumption to the standard error of the income process in a range centered at 0.64. This is the paradox of the excess smoothness of consumption, or sometimes called the Deaton paradox, whereby consumption is too smooth compared to what can be inferred from the RWPIH. Such excess smoothness rehabilitates Keynes and Friedman.

There are essentially three methods to test for the RWPIH, as will be explained more formally in the next sec-
tion. The first is theoretical and calculates the magnitude of the theoretical impact of an income innovation on the change in consumption, in the same spirit as that in Eq. (1). The second and actual method is to calculate the actual ratio of the standard deviation of the change in consumption to the standard error of the income innovation. The third, which is the empirical approach, is by estimating a regression of the change in consumption on the income innovation. These three methods should yield the same size of the impact, and all must be equal if the RWPIH is true. Often the income process and the empirical regression of the income innovation on consumption are estimated separately, although some authors have estimated the model jointly, making it possible to test for the equality of the theoretical and empirical impacts, like [9] for example. Both procedures are followed in this paper.

If the strong form of the RWPIH does not hold and the three approaches do not yield the same magnitude of the impact, then a weaker hypothesis can be tested which is whether consumption responds positively and statistically significantly to an income innovation, i.e. whether the empirical impact is positive and statistically significant.

As a quick prelude to the results of this paper, and despite the fact that \( \lambda \) in Eq. (1) is observed to be negative with monthly data, the theoretical impact, while less than one, is consistently higher than the actual impact, which, in turn, is consistently higher than the empirical impact. The evidence of excess smoothness is still noticeable in the data, although consumption is found to be smoother than income by all the three above-mentioned impacts.

The paper is organized as follows. The second section presents the theoretical foundations. The third section is the empirical part. In it new time series models for income, using monthly data, are estimated, and the effects of the income innovation on consumption are measured and compared to those in the literature. The final section concludes.

**THE THEORY**

Mathematically, and assuming certainty as Hall [3] did, the maximization is as follows:

\[
\text{Maximize } U = E_t \left( \sum_{j=0}^{\infty} \frac{1}{1+\omega} \right)^j U(C_{t+j})
\]

(2)

where \( U(\cdot) \) is a time-separable and time-additive utility function, \( C \) is consumption, \( \omega \) is the discount rate of the utility, \( E_t \) is the expectation operator conditional on information at time \( t \), and the subscript denotes the period. Imposing the following budget constraint [5]:

\[
A_{t+j} = (1+r)(A_{t+j} + Y_{t+j} - C_{t+j})
\]

(3)

where \( A \) is the stock of financial wealth, \( Y \) is labor income, and \( r \) is the (constant) interest rate, then, by solving Eq. (3) forward and assuming that the present value of wealth approaches zero as \( t \) tends to infinity, one gets the following Eq.:

\[
\frac{1}{1+r} \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j C_{t+j} = \frac{1}{1+r} \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j Y_{t+j} + A_t
\]

(4)

Eq. (4) holds unconditionally and conditionally relative to current expectations. Taking expectations of Eq. (4) one gets:

\[
\frac{1}{1+r} \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j E_t C_{t+j} = \frac{1}{1+r} \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j E_t Y_{t+j} + A_t
\]

(5)

If consumption follows a random walk, which occurs if the utility function is quadratic, or if marginal utility is linear, and if, in addition, the interest rate \( r \) is equal to the discount rate of the utility \( \omega \) then Eq. (5) collapses to:

\[
C_t = \frac{r}{1+r} \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j E_t Y_{t+j} + rA_t
\]

(6)

Taking expectations at time \( t+1 \) of Eq. (6) and subtracting the result from Eq. (6), the following holds [5] [6] [7] and [10]:

\[
\Delta(C_{t+j}) = \frac{r}{1+r} \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j (E_{t+j} - E_t) Y_{t+j}
\]

(7)

Eq. (7) states that consumption changes only due to the discounted annuity of the revision of expectations of future income. Campbell and Deaton [7] solve Eq. (7) by including logs, and obtain the following:

\[
\frac{\Delta(C_{t+j})}{Y_t} = \sum_{j=0}^{\infty} \rho^j (E_{t+j} - E_t) \Delta \log(Y_{t+j})
\]

(8)

where \( \rho \) is equal to \((1+\mu)/(1+r)\), and \( \mu \) is the average growth rate of labor income.

Assuming that labor income \( Y_t \) follows an ARIMA process [5]:

\[
\Phi(L) Y_t = \alpha + \varepsilon_t \text{ with } \Phi(L) = 1 - a_1 L - a_2 L^2 - \ldots - a_p L^p
\]

(9)

\( L \) being the lag operator, and \( \alpha \) being a constant, then Eq. (7) becomes:

\[
\frac{\Delta(C_{t+j})}{Y_t} = \frac{1}{1 - \sum_{j=0}^{\infty} \frac{a_j}{(1+r)^j}} e_{t+j} = \theta_0 e_{t+j}
\]

(10)

If, instead, \( \Delta \log(Y_t) \) follows an AR process \( \Psi(L) \) of order \( q \) in \( b_j \), with \( \Psi(L) = 1 - b_1 L - b_2 L^2 - \ldots - b_q L^q \), then Eq. (8) becomes [7]:

\[
\frac{\Delta(C_{t+j})}{Y_t} = \frac{1}{1 - \sum_{j=0}^{\infty} \frac{b_j}{(1+r)^j}} v_{t+j} = \theta_2 v_{t+j}
\]

(11)

This paper will estimate the theoretical values of \( \theta_1 \) and \( \theta_2 \) in Eqs. (10) and (11) for four different samples. These theoretical values will then be compared to (1) the actual ratio of the standard deviation of the left-hand side of Eqs.
The third subsection presents the univariate ARIMA models of income and the log of income (Tables 2, 3). Once the process for income is identified the income innovation can be retrieved. It is a feature of the data that with monthly values, income is negatively auto-correlated. This runs against the evidence with quarterly and annual data where income is positively auto-correlated [7, 8]. This has also implications on the effect of the income innovation on the change in consumption (i.e. Eq. (1)), where \( \lambda \) becomes negative, rendering the theoretical effect of the innovation of income less than one in value. The reason why income is found to be negatively correlated on a monthly basis is due maybe to the use of seasonally adjusted data.

The fourth subsection gives the evidence on the theoretical size, the actual size, and the empirical size for the effect of the income innovation, as discussed above. In the same subsection a joint estimation of the model is implemented.

The results support generally the hypothesis of excess smoothness. Therefore the initial finding of excess smoothness endures. Nevertheless the weaker version of the RWPIH that the income innovation includes new information in the determination of the behavior of consumption is not rejected. In other terms, \( \beta_0 , \beta_1 \), and \( \beta_2 \) are all positive and statistically significant.

**DEFINITION OF THE SAMPLES**

The whole sample is monthly and spans the period from 1959:1 to 2008:2. The data is taken from the web site of the Federal Reserve Bank of Saint Louis. The income series is for real personal disposable income per capita, and the consumption series is for real consumption expenditures on non-durables per capita. All series are adjusted seasonally at annual rates.

The whole sample is a natural candidate for estimation, but incorporates the undesirable assumption that behavior of market participants has remained stable for half a century. The sample from 1990:1 to 2008:12 is selected because it occurs after the publication of the original paper of Campbell and Deaton [7] on excess smoothness. It might be reasonable to think that this paper has, perhaps, prompted policy makers to take as granted Campbell and Deaton’s behavioral relation, and to adopt policies based on it. However the use of a behavioral relation by policy makers is likely to alter this behavior. This is the essence of the Lucas critique. Any way there is evidence that macro policy underwent indeed change in the early 1990s. The next sample is from 1959:1 to 1989:12 and it is the sample prior to the date of the above publication. Another sample is from 1959:1 to 1984:12, and corresponds approximately to the sample used in [7] and in [14], which is quarterly from 1953:1 to 1984:IV. The final sample is from 1985:1 to 2008:2 and corresponds to the sample posterior to the one studied by Campbell and Deaton [7] and Flavin [14]. *Ex post* this sample gives results very similar to those for the sample from 1990:1 to 2008:2. For this reason it was disregarded from the analysis.

A Chow test for parameter stability for a break at end-1984 in the whole sample rejected parameter stability of the income process of \( \Delta(Y_t) \) and \( \Delta(\text{Log}(Y_t)) \) at probability levels of 0.00017 and 0.00001 respectively. This denotes that there is a significant break at end-1984, which means that the sample that Campbell and Deaton [7] and Flavin [14] used is significantly different from the whole sample. The data generation process of income is essentially a random walk before end-1984, but is negatively auto-correlated after end-1984. The reason for such a break remains mysterious.

Another Chow test for parameter stability of the income data generation process was conducted for a break in end-
1989 in the sample between 1985:1 and 2008:2. The F-tests for \( \Delta(Y_t) \) and \( \Delta(\log(Y_t)) \) have lower-tailed probabilities of 0.3283 and 0.4414 respectively, failing to reject parameter stability. Therefore there is no break in end-1989. However since the empirical results are quasi the same for the sample from 1984:12 to 2008:2 and the sample from 1990:1 to 2008:2, then the latter sample is selected for further study. This is especially reasonable if the Lucas critique applies as theoretically it should.

**UNIT ROOT TESTS**

Four unit root tests are undertaken (Table 1). These are the Augmented Dickey-Fuller [15], the Phillips and Perron [16], the Ng and Perron MPT [17], and the KPSS [18] tests. They show that the first-differences of \( Y_t \), \( C_t \), \( \log(Y_t) \), and \( \log(C_t) \) are all stationary for all samples at probability levels less than 0.001. The findings for the Ng and Perron MPT test [17] in Table 1 also show that the levels of \( Y_t \), \( C_t \), \( \log(Y_t) \), and \( \log(C_t) \) all follow an integrated process of order 1. The other tests show that in some cases the hypothesis of non-stationarity is rejected for these level variables. This is true for the ADF [15] and Phillips and Perron [16] tests of \( Y_t \), \( C_t \), \( \log(Y_t) \), and \( \log(C_t) \) for the sample between 1990:1 and 2008:2. Therefore the evidence in Table 1 that \( Y_t \), \( C_t \), \( \log(Y_t) \), and \( \log(C_t) \) are not integrated of order 1 for the aforementioned sample is rather weak, and may be due to chance.

**ARIMA MODELS FOR \( Y_t \) AND \( \log(Y_t) \)**

Table 2 reproduces the estimated ARIMA processes for \( Y_t \) for the four sample periods, while Table 3 reproduces the same estimations for \( \log(Y_t) \). The ARIMA processes for the two series are quite comparable. The ARIMA processes for the first sample period (from 1959:1 to 1984:12) are random walks for both series. The ARIMA processes for the second sample period (from 1959:1 to 1989:12) have both two significant auto-regressive lags. The ARIMA processes for the last two samples have all three significant auto-regressive lags. Moreover, and for both series, all the coefficients on the auto-regressive lags are negative. The prob-

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<td>( \log(C) )</td>
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<td>(2) 0.748</td>
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<td>( \log(Y) )</td>
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Notes: \( C \) stands for real per capita consumption expenditure on non-durables, and \( Y \) stands for real per capita personal disposable income. \( \log \) is the natural logarithm. The symbol \( \Delta \) stands for the first-difference operator. All series are monthly. The null hypothesis for tests (1), (2) and (3) is non-stationarity, while the null hypothesis for test (4) is stationarity.
abilities of the Ljung-Box Q-statistics reported in Tables 2 and 3 for 12 and 24 lags all denote absence of further serial correlation in the ARIMA residuals except for minor random cases, like the Q(24) for \( Y_t \) for the sample between 1959:1 and 1989:12 and the Q(12) for \( \log(Y_t) \) for the sample between 1990:1 and 2008:2. Q-statistics for the other lag of these same sample periods are not lower than 0.05. Such discrepancies arise naturally in empirical work ([19]: 126).

The fact that the ARIMA processes do not have MA components is not a surprise, as this is the usual case in the literature. What is a surprise are the negative coefficients on the auto-regressive lags which may be due to the use of sea-

Table 2. Box-Jenkins ARIMA Models for \( Y_t \), Real Personal Disposable Income Per Capita

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \phi(L)(1-L)Y_t = \alpha + \epsilon_t )</th>
<th>Q(12)</th>
<th>Q(24)</th>
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<tr>
<td>1959:1</td>
<td>( \alpha = 0.036525 (4.538) )</td>
<td>0.2932</td>
<td>0.1969</td>
</tr>
<tr>
<td>1984:12</td>
<td>( \phi(L)=1 ) standard error = 0.00014195</td>
<td></td>
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<tr>
<td>1959:1</td>
<td>( \alpha = 0.04758 (5.451) )</td>
<td>0.4985</td>
<td>0.0363</td>
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<td>1989:12</td>
<td>( \phi(L)=1 + 0.1592 L + 0.1483 L^2 ) ( (3.087) (2.875) ) standard error = 0.00015887</td>
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</tr>
<tr>
<td>1990:1</td>
<td>( \alpha = 0.077595 (4.522) )</td>
<td>0.0426</td>
<td>0.3629</td>
</tr>
<tr>
<td>2008:2</td>
<td>( \phi(L)=1 + 0.3818 L + 0.2128 L^2 + 0.2109 L^3 ) ( (5.713) (3.028) ) standard error = 0.00023403</td>
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<tr>
<td>1959:1</td>
<td>( \alpha = 0.061808 (7.209) )</td>
<td>0.2345</td>
<td>0.3199</td>
</tr>
<tr>
<td>2008:2</td>
<td>( \phi(L)=1 + 0.3931 L + 0.2147 L^2 + 0.2101 L^3 ) ( (5.881) (3.044) ) standard error = 0.00019161</td>
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Notes: \( L \) is the lag operator, i.e. \( LY_t = Y_{t-1} \), \( L^2 Y_t = Y_{t-2} \) etc. T-statistics are in parenthesis. Q(k) is the probability of the Ljung-Box Q-statistic for a total number of lags k.

Table 3. Box-Jenkins ARIMA Models for \( \log(Y_t) \), the Natural Logarithm of Real Personal Disposable Income Per Capita

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \phi(L)(1-L)\log(Y_t) = \alpha + \epsilon_t )</th>
<th>Q(12)</th>
<th>Q(24)</th>
</tr>
</thead>
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<tr>
<td>1959:1</td>
<td>( \alpha = 0.0018462 (4.836) )</td>
<td>0.2846</td>
<td>0.2397</td>
</tr>
<tr>
<td>1984:12</td>
<td>( \phi(L)=1 ) standard error = 0.00067316</td>
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<tr>
<td>1959:1</td>
<td>( \alpha = 0.002205 (5.619) )</td>
<td>0.5312</td>
<td>0.1564</td>
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<tr>
<td>1989:12</td>
<td>( \phi(L)=1 + 0.1200 L + 0.1332 L^2 ) ( (2.322) (2.577) ) standard error = 0.00070898</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990:1</td>
<td>( \alpha = 0.002398 (4.569) )</td>
<td>0.0400</td>
<td>0.2811</td>
</tr>
<tr>
<td>2008:2</td>
<td>( \phi(L)=1 + 0.3931 L + 0.2147 L^2 + 0.2101 L^3 ) ( (5.881) (3.044) ) standard error = 0.00071749</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1959:1</td>
<td>( \alpha = 0.0023276 (7.161) )</td>
<td>0.6479</td>
<td>0.4088</td>
</tr>
<tr>
<td>2008:2</td>
<td>( \phi(L)=1 + 0.2263 L + 0.1414 L^2 + 0.0939 L^3 ) ( (5.498) (3.383) ) standard error = 0.00071914</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: \( L \) is the lag operator, i.e. \( LY_t = Y_{t-1} \), \( L^2 Y_t = Y_{t-2} \) etc. T-statistics are in parenthesis. Q(k) is the probability of the Ljung-Box Q-statistic for a total number of lags k.
sonally adjusted data. As already mentioned negative coefficients imply that $\lambda$ in Eq. (1) is negative, driving the whole Eq. to a value less than 1, contradicting the evidence that the theoretical impact is much larger than 1 [7]. However as will be seen below, and despite the fact that the theoretical impact is less than one, the actual and empirical impacts are even far below 1, preserving intact the existence of the excess smoothness paradox. Although consumption is smooth it is still excessively so.

**IMPACTS RELATED TO $\Delta(C_t)$**

Table 4 presents the evidence on the effect of the income innovation on $\Delta(C_t)$. Columns 2, 3, and 4 report the theoretical impact (i.e. $\theta_t$) with the interest rate $r$ being equal to 0%, 4%, and 6% per annum respectively. This impact is largest for the sample period between 1959:1 and 1984:12, where it is unity, and smallest for the sample period between 1990:1 and 2008:2, where it is between 0.554 and 0.556. The other impacts are in between these ranges. There is little change within a given sample, but more change between sample periods. Therefore the choice of the interest rate affects little the results. For the case where $r = 0\%$, all the theoretical impacts $\theta_t$, except for the random walk sample, are significantly lower than 1, with the lowest t-statistic being 5.165, and the highest being 12.450.

The actual ratio of the standard deviation of $\Delta(C_t)$ over the standard error of the income innovation (column 5 in Table 4) is between 0.2386 and 0.3776, again larger for the sample between 1959:1 and 1984:12 (0.3776) and smaller for the sample between 1990:1 and 2008:2 (0.2386). Therefore the actual ratio is still way below the theoretical impact ($\theta_t$), averaging approximately its half. The t-statistics for the difference between the two impacts range between -7.038 and -10.256, rejecting the null hypothesis of no-difference.

Authors like Diebold and Rudebusch [20] and Gali [21] [22], who considered that a theoretical impact ( $\theta_t$) lower than 1 is evidence of absence of excess smoothness, would all conclude erroneously that excess smoothness is absent. In fact, as Patterson [23] and Patterson and Sowell [24] note, excess smoothness is present if the ratio of the standard deviation of $\Delta(C_t)$ ($\sigma_{AC}$) on the theoretical impact ($\theta_t$) and on the standard error of the income innovation ($\sigma_r$) is less than one. They argue correctly that $\theta_t$ is a measure of income persistence and not of excess smoothness.

Patterson and Sowell [24] estimate $\theta_t$ to be either 0.7100 or 0.7722 (their Table 3, p. 1251) for their Z income series which is real personal disposable income per capita. These estimates are close to the estimates in this paper. For example, depending on the sample chosen and for the case where $r = 0\%$, these estimates are 1, 0.7649, 0.5539, and 0.6265 (see column 2 of Table 4).

Moreover, although Patterson and Sowell [24] estimate ARFIMA models for the income series, i.e. ARIMA models with fractional integration, their estimates of the fractional integration parameter for their Z series is either close to 1 (their Table 1, p. 1250) or insignificantly different from 1 (their Table 2, p. 1250). This gives support to the assumption in this paper that the income series is integrated of order 1, and needs first-differencing.

The empirical effect $\beta_0$ obtained by regressing $\Delta(C_t)$ on the income innovation, with an AR(1) process for the residuals, is less than the theoretical impact $\theta_t$, and also less than the actual impact (see column 6, Table 4). The actual impacts are statistically much higher than the empirical impacts, the t-statistics for the difference between the two ranging between 14.640 and 22.836.

**Table 4. Theoretical Impacts (Columns 2 to 4), Actual Impacts (Column 5), Empirical Impacts (Column 6) and a Hypothesis Test of $\theta_t = \beta_0$ (Column 7)**

<table>
<thead>
<tr>
<th>Sample</th>
<th>$r = 0%$ $\theta_t$ in the Text (t-Statistics)</th>
<th>$r = 4%$ $\theta_t$ in the Text</th>
<th>$r = 6%$ $\theta_t$ in the Text</th>
<th>Ratio of the Standard Deviation of $\Delta(C_t)$ on the Standard Error</th>
<th>Effect of the Income Innovation on $\Delta(C_t)$ with an AR(1) Model $\beta_0$ (t-Statistics)</th>
<th>t-Statistic for the Hypothesis $\theta_t = \beta_0$ ($r = 6%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959:1 1984:12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.3776</td>
<td>0.1129 (6.398)</td>
<td>6.852</td>
</tr>
<tr>
<td>1959:1 1989:12</td>
<td>0.7649 (16,804)</td>
<td>0.7658</td>
<td>0.7662</td>
<td>0.3303</td>
<td>0.0880 (6.296)</td>
<td>7.423</td>
</tr>
<tr>
<td>1990:1 2008:2</td>
<td>0.5539 (12,336)</td>
<td>0.5553</td>
<td>0.5561</td>
<td>0.2386</td>
<td>0.0322 (2.284)</td>
<td>3.437</td>
</tr>
<tr>
<td>1959:1 2008:2</td>
<td>0.6265 (18,554)</td>
<td>0.6279</td>
<td>0.6286</td>
<td>0.2809</td>
<td>0.0596 (6.150)</td>
<td>7.938</td>
</tr>
</tbody>
</table>

Notes: the standard error is that of the income innovation estimated in Table 2. The last column is a result of a joint estimation which assumes an ARIMA(1,1,0) process for $Y_t$ and includes the first lag of the dependent variable, i.e. the first lag of $\Delta(C_t)$. Higher lags for the income process were found to be statistically insignificant.
IMPEARCE RELAYE TO $\Delta(\log(C_i))$ \(\Delta(C_i) / Y_{t-1}$

Table 5 presents the results of estimating the effect of an income innovation, from all the ARMA models of $\Delta(\log(Y_t))$, on the ratio $\Delta(C_i) / Y_{t-1}$. The estimates of the theoretical impact (i.e. $\theta_2$) are between 0.550 and 1. The sample period from 1959:1 to 1984:12 has the highest impact while the sample period between 1990:1 and 2008:2 has the lowest. This is in conformity to Table 4. Three estimates of $\theta_2$ are reported: when the interest rate $r$ is equal to $\mu$, the average growth rate of the income process, and when the interest rates are respectively 4%, and 6% (see columns 2 to 4 of Table 5). The estimates for a given sample are all close to each other. The choice of the interest rate has little effect on the results. The estimates between samples differ. The highest is 1 for the random walk sample, and the lowest is 0.550 for the sample from 1990:1 to 2008:2. For the case where $\mu = r$, all these estimates, except for the random walk sample, are significantly less than 1, the t-statistics ranging between 4.128 and 13.277. However these estimated theoretical values are all higher than the actual ratios of the standard deviation of $\Delta(C_i) / Y_{t-1}$ over the standard error of the income innovation, which range between 0.239 and 0.407 (see column 5 of Table 5). Hypothesis testing shows that the actual impacts are significantly lower than the theoretical impacts $\theta_2$, the t-statistics for the difference ranging between -6.977 and -9.262.

Patterson and Sowell [24] estimate $\theta_2$ to be between 0.4342 and 0.4454 for the log of their $Z$ variable, estimates that are lower than those in this paper, but not that much lower. Moreover, although Patterson and Sowell [24] estimate ARFIMA models for the log of the income series their estimates of the fractional integration parameter for the log of their $Z$ series is either close to 1 or marginally different from 1. The assumption in this paper that the log of the income series is integrated of order 1, and needs first-differencing, is supported.

The empirical effect ($\beta_1$) is even smaller than the actual one, ranging between 0.0348 and 0.1168, with all impacts statistically significant (see column 6, of Table 5). The hypothesis tests for the difference between the actual impacts and the empirical impacts all reject the null hypotheses of no-difference, the t-statistics ranging between 14.298 and 22.215. For the case where $\mu = r$, the joint estimates of $\theta_2$ and $\beta_1$ are all statistically different (see column 7 of Table 5), the lowest t-statistic being 3.591.

Again three conclusions emerge. These are the same conclusions reached in the previous section. One, excess smoothness is still a feature of the data. Two, the consumption-derivation regressions suffer from serial correlation. Three, the weak form of the RWPH stands: the income innovations explain statistically significantly the change in consumption. In other terms, the empirical impact ($\beta_1$) is positive and statistically significant.

IMPACTS RELATED TO $\Delta(\log(C_i))$}

Table 6 documents the results when the change in the log of consumption is taken as the dependent variable. It is to be noted that theoretically this is wrong because the model in logs developed by Campbell and Deaton [7] has $\Delta(C_i) / Y_{t-1}$ as the dependent variable and not $\Delta(C_i) / C_i$ or $\Delta(\log(C_i))$. However Dejuan [11], Dejuan et al. [12] utilize the changes in the log of consumption. For comparability purposes the same is done here.

Column 2 of Table 6 reproduces column 2 of Table 5, which is $\theta_2$, for the case where $\mu = r$. The 3rd column of Table 6 gives the actual ratio of the standard deviation of $\Delta(\log(C_i))$ over the standard error of the innovation in $\Delta(\log(Y_i))$. All these actual figures are close to one, the range being between 0.873 and 1.228. These actual figures are all close to the estimates of $\theta_2$, in Dejuan [11], and Dejuan et al. [12]. In fact, in their joint estimation, Dejuan [11], and Dejuan et al. [12] fail to reject the equality between $\theta_2$ and $\beta_1$ (Eq. 14). When actual impacts are compared to theoretical impacts there is false evidence of excess volatility of consumption.

For the case where $r = 6\%$, the joint estimation of the income process with the process for $\Delta(C_i)$ results in rejection of the hypothesis of equality of the theoretical ($\theta_1$) and empirical ($\beta_0$) impacts for all sample periods: the lowest t-statistic being 3.437 (see column 7, Table 4). Bagliano and Bertola [6, 17] report an actual ratio of 0.64 for quarterly data, much higher than the one estimated with the data in this paper. Malley and Molana [8: Table 3, p. 1032, and Table 5, p.1036] estimate a similar model for $\Delta(C_P)$, with empirical impacts ranging between 0.315 and 0.369 for annual data. Dejuan et al. [9] find statistically different theoretical ($\theta_1$) and empirical ($\beta_0$) impacts for Germany, with a range for $\theta_1$ between 0.907 and 1.284, and a range for $\beta_0$ between 0.093 and 0.351. These estimates for $\beta_0$ are mostly higher than the empirical estimates of this paper which range between 0.0322 and 0.1129 (see column 6, Table 4). The estimates for $\theta_1$ in the literature are themselves mostly higher than the ones in this paper, which range between 0.554 and 0.766, ignoring the peculiar random walk sample.

Three conclusions emerge. One, excess smoothness is still a feature of the data, although the theoretical impacts $\theta_1$ are, in the majority of cases, less than 1. Two, the consumption-regressions suffer from serial correlation, which might be a consequence of omitted variables. Three, the weak form of the RWPH stands: the income innovations explain statistically significantly the change in consumption. In other terms, the empirical impact ($\beta_1$) is positive and statistically significant.
In Table 5 the empirical effect of an income innovation of $\Delta\{\log(Y_t)\}$ on the change in the log of consumption is higher than those in Tables 4 and 5 (see column 4 of Table 6). With the condition $\mu = r$, when the theoretical impact $\theta_2$ and the empirical impact $\beta_2$ are estimated jointly, they are significantly different from each other, the lowest t-
statistic being 3.021 (see column 5 in Table 6). While with the changes in the log of the consumption series the three impacts, theoretical, actual, and empirical are closer to each other, they are still different, and are contrary to the underlying theory as developed in Campbell and Deaton [7]. However, the income innovation has still a significant impact on the change in the log of consumption (i.e. \( \beta_2 \)) although the empirical Eq. suffers from serial correlation. As a general conclusion it is only the weak version of the RWPIH that receives support, by having \( \beta_2 \) positive and statistically significant. Otherwise, excess smoothness of consumption is still a salient feature of the data.

THE INTERNATIONAL EVIDENCE

It is worthwhile to carry out a comparison between countries on the validity of the permanent-income hypothesis. Tests of this hypothesis in the literature were conducted for the US [8,12,20,21], for the UK [23,24], for OECD countries [22], for UK regions [11], for US states [12], and for West German states [9]. There is no clear pattern in the findings. The evidence for the UK supports excess volatility more than excess smoothness [23,24]. The evidence for the US supports a high degree of uncertainty in the estimated coefficients [20], a highly volatile relation [25], and estimates of \( \beta_0 \) between 0.328 and 0.369 [8]. The evidence for six countries in the OECD, including the US, provides strong rejection of the hypothesis especially when consumption on durables is studied [22]. The evidence for UK regions [11] supports the empirical fact that \( \beta_2 \) is lower than \( \theta \). The evidence for US states [12] supports generally the permanent-income hypothesis. The evidence for West German states [9] supports the presence of excess smoothness. Overall, the results are disparate and no specific conclusion can be made.

CONCLUSION

The purpose of this paper is to update the evidence on the existence of excess smoothness in consumption within the permanent income model of consumption behavior based on the work of Friedman [2], Hall [3], Flavin [5], and Campbell and Deaton [7]. Two formulations of the model are tested, one in the levels of the income variable and the other one in the logs. More than one sample is selected. Three impacts are identified: the theoretical, the actual, and the empirical. The theoretical impact is the one implied by the univariate process of income and the Random Walk Permanent Income Hypothesis (RWPIH). The actual impact uses actual data on the variability of the consumption series, and other consumption-related variables, and on the standard errors of the income innovation. The empirical impact is determined by carrying out a regression of the change in consumption, and other consumption-related variables, on the income innovation. The three impacts are found to be statistically different, with the theoretical impact being the largest and the empirical impact being the smallest. Although the theoretical impact is, in the majority of the cases, less than one, it is argued that this is not evidence of the absence of excess smoothness. The conclusion is strong that excess smoothness is still a feature of the data, about 20 years after its discovery by Campbell and Deaton [7].

A disturbing finding which is also present with annual data (see [8]) is serial correlation of the residuals in the empirical regression, which may denote a problem of omitted variables, or other misspecification. Nonetheless a weak version of the RWPIH is supported, which is that the income innovation is a significant explanatory variable for the change in consumption and other consumption-related variables, like \( \Delta(C_t) / Y_{t-1} \) and \( \Delta(\log(C_t)) \). In other terms, the empirical impacts \( \beta_0 \), \( \beta_1 \) and \( \beta_2 \) are all positive and statistically significant.

It is guessed that the inherent problems of the RWPIH are its assumption of quadratic utility, or linear marginal utility, and probably its ruling out of uncertainty and precautionary saving. The latter depends on prudence, which in turn depends on the third derivative of the utility function, this being nil with quadratic utility. Future research might benefit from changing these assumptions, especially since other authors like Morley [25] have also found independently evidence of excess smoothness that supports, in Morley’s view, habit formation and precautionary saving. Precautionary saving, by including in the empirical regression the variance of the income innovation as an additional independent variable, may alleviate the misspecification of the model, in case the latter is caused by omitted relevant variables.

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New Evidence on the Excess Smoothness of Consumption


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