Tax Shocks, Sunspots and Tax Evasion[§]

Francesco Busato, Bruno Chiarini* and Enrico Marchetti

Department of Economics, University of Naples "Parthenope", Naples, Italy; Via Medina 40, 80133 Naples, Italy

Abstract: This paper shows that an increase in corporate/labor/income tax rates may push an economy with tax evasion into an expansionary pattern, under increasing returns to scale. These effects would be reversed when the steady state is saddle-path stable. This model does not undertake a full identification. The interesting feature of our results is that fiscal policy in an economy with a significant underground sector may provide inadvisable outcomes. Thus, tax policies can generate counterproductive results in an economy characterized by existence of aggregate increasing returns to scale and underground activities.

Keywords: Dynamic general equilibrium models, fiscal policy, tax evasion and underground activities, indeterminacy and sunspots.

1. INTRODUCTION

It is a well know fact that underground activities characterize many economies, and there are significant indications that this phenomenon is large and increasing.[1] The estimated average size of the underground sector (as a percentage of total GDP) over 1996-97 in developing countries is 39 percent, in transition countries 23 percent, and in OECD countries about 17 percent, as [1] suggests. For the United States, the average size of underground activities ranges between 5 percent of GNP (in the Seventies, as from [2] and 9 percent of the GDP in the Eighties and early Nineties, as from [3], or, more recently, from [1]). Other studies such as [1, 4] and OECD (2000), indicate that industrialized economies allocate a great deal of resources (labor and capital) in non-reported activities. More recent studies confirm the important role of the underground economy at a macroeconomic level (i.e. [5-7]).

The equilibrium effects and the role of fiscal policy in dynamic general equilibrium models have been the object of thorough investigations in the last decade. Significant work has been done in fiscal policy analysis within neoclassical growth models.[2] Fiscal policy implications have been investigated also in the context of dynamic equilibrium models when fluctuations are induced by sunspot shocks under increasing returns to scale at the aggregate level. In this case, particular attention has been devoted to the impact of changes in the steady state levels of tax rates on the topological properties of the model's attractor [8, 9].[3]

Therefore, it is important to jointly study the fiscal policy reactions of regular and underground sectors because households and firms, typically, simultaneously choose the resource allocation across them. This paper would, hopefully, enhance our understanding of economic behavior and the feasibility of the policy target in dynamic models with underground activities and aggregate increasing returns to scale. These features are of relevant interest also to policymakers, since the phenomenon presents an important challenge to the theory and practice of fiscal policy that builds on the idea of market activities.

This paper analyzes a one-sector dynamic general equilibrium model in which there are three agents: firms, households and a government. Assume, then, that there exist an homogeneous consumption good. The government levies proportional taxes on corporate revenues, labor and capital income flows, payroll taxes on labor services and balances its budget (in expected terms) for each period. Firms and households, being subject to distortionary taxation, use the underground labor market to evade taxes. Government faces tax evasion originating from the underground sector, and coordinates strategies to address abusive trust schemes. The model display endogenous fluctuation due to aggregate increasing returns to scale.

*Address correspondence to this author at the Department of Economics, University of Naples "Parthenope", Naples, Italy; Via Medina 40, 80133 Naples, Italy; Email: bruno.chiarini@uniparthenope.it

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[1]There is no universal agreement on what defines the underground economy, and obviously, the difficulty in defining the sector extends to the estimation of its size. There exist several synonyms for describing underground activities: underground activities, shadow or hidden economy. We are concerned with the size of the underground economy as encompassing activities which are otherwise legal but go unreported or unrecorded.

[2][16]] and [18] are seminal contributions sharing an emphasis on the supply-side response of labor and capital to shifts in government demand and tax rates. Recent related contributions are [19] and [20].

[3][22] study the dynamic response to changes in government spending under increasing returns to scale, when the attractor is still a saddle point.

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The main finding of this paper is that contractionary fiscal policies (i.e. an increase in corporate, income or labor tax rates) may induce expansionary effects for a class of dynamic general equilibrium models augmented with increasing returns to scale in production and underground activities; these effects would be reversed, *ceteris paribus*, when the steady state is saddle path stable. The paper's general idea is to show that in this context, characterized by underground activity and indeterminacy, "counterintuitive" responses of the main endogenous variables to fiscal shocks can arise.

It is known that the system's dynamic reactions to shock in the fundamentals are more complex to investigate when indeterminacy prevails; thus our aim is to offer a discussion of a possible outcome - based on the specific phenomenon of evasion/underground activities - in which (positive) shocks in the exogenous tax rates induce an initial (expansionary) effect, which is of opposite sign to that of the saddle-stable case.

In particular, under indeterminacy the discussion/ identification of fiscal policy implications are subject to an additional difficulty, due to the fact the equilibrium path may be indeterminate. Technically, the point here is that we cannot interpret our impulse response functions in a proper manner whenever the stochastic tax structure is perturbed: Our impulse response functions are just one of the possible dynamic outcomes that can emerge from this type of model. In this model we do not undertake a full identification in the spirit of [10], while examining just a non-identified model formulation. The further step - i.e. to implement the [10] procedure - is left to future research.

In this context we discuss selected empirical implications of tax policy for two types of models with an underground sector: a fairly standard neoclassical one and a model with increasing returns to scale.

The paper is organized as follows. Section 2 details the theoretical model, while Section 3 presents the topological properties of stationary state and discusses conditions for indeterminacy. Section 4 presents and discusses the model's response to fiscal policy shocks. Finally Section 5 concludes.

2. THE MODEL

2.1 Firms' Sector

Production technology for the homogenous good $y_{j,t}$ uses three inputs: physical capital, regular labor services, and underground labor services. The production function of firm reads:

$$y_{j,t} = A_t k_{j,t}^{\alpha} (n_{j,t}^{M})^{1-\alpha-\rho} (n_{j,t}^{U})^{\rho}, \quad 0 < \alpha + \rho < 1, \tag{1}$$

where $k_{j,t}$ denotes capital stock, $n_{j,t}^{M}$ is regular labor, $n_{j,t}^{U}$ represents irregular labor, and the quantity

$$A_t = \underbrace{\left\{ K_t^{\alpha} (N_t^{M})^{1-\alpha-\rho} \right\}^{\eta}}_{\text{``Regular? Ext.}} \ \underbrace{\left\{ (N_t^{U})^{\rho} \right\}^{\zeta}}_{\text{``Underground? Ext.}} , \tag{2}$$

represents an aggregate production externality: it passes through aggregate-average level of output (K,N^M and N^U are the economy-wide levels of the three inputs) and has two different sources.

The quantity $\left\{ K_t^{\alpha} (N_t^{M})^{1-\alpha-\rho} \right\}^{\eta}$ (the regular externality) is related to an external effect to that of standard one-sector models (e.g. [11]). The quantity $\left\{ (N_t^{U})^{\rho} \right\}^{\zeta}$ (the underground externality) is specifically related to underground activities.[4] Externality parameter for regular labor η can be different from that of the underground one (ζ). This formulation adds generality to the analysis: when $\eta = \zeta$ and there are neither tax evasion nor distortionary taxation, the model reduces to Farmer and Guo's one.

As firms are homogeneous, overall level of output for a given (and equal for all firms) level of inputs utilization is given by:

$$\begin{aligned} Y_t &= A_t \int_j \left\{ k_{j,t}^{\alpha} (n_{j,t}^{M})^{1-\alpha-\rho} (n_{j,t}^{U})^{\rho} \right\} dj \\ &= K_t^{\alpha(1+\eta)} (N_t^{M})^{(1-\alpha-\rho)(1+\eta)} (N_t^{U})^{\rho(1+\zeta)} \end{aligned} \tag{3}$$

Increasing returns to scale are a pure aggregate phenomenon (as the former equation suggests), and returns to scale are constant at firm level, as each firm takes K, N^M and N^U and as given.

Firms evade taxes on total revenues and on labor services, by allocating labor demand to underground labor market. Firms, however, may be detected evading, with probability p (within the 0,1 interval), and forced to pay the statutory tax rates on revenues and the payroll tax rate on labor (τ_t^{Π} and τ_t^{N} respectively), increased by a surcharge factor, $s>1$, applied to the standard tax rate.[5]

When a firm is not detected evading (with probability *1-p*), its profit are denoted with $\pi_{j,t}^{ND}$. If detected evading (with probability p), we denote firm's profits as $\pi_{j,t}^{D}$; both are defined below:

$$\begin{array}{lll} \pi_{j,t} \rightarrow \underset{\sim p}{\textbf{Detected}} & & \pi_{j,t}^{D} = (1 - s\tau_t^{\Pi}) y_{t,j} - (1+\tau_t^{N}) w_t^{M} n_{j,t}^{M} - (1 + s\tau_t^{N}) w_t^{U} n_{j,t}^{U} - r_t k_{j,t} \\ \searrow & & \\ & \underset{\sim(1-p)}{\textbf{Not Detected}} & \pi_{j,t}^{ND} = y_{t,j} - (1+\tau_t^{N}) w_t^{m} n_{j,t}^{M} - w_t^{U} n_{j,t}^{U} - r_t k_{j,t}, \end{array} \tag{4}$$

[4]Underground labor services use the same capital stock that is used by regular labor. We could imagine that the same firm produces in the regular economy in the day, and in the underground economy by night, by using the same employees. This production scheme is denoted as *moonlighting* production. [17] loosely defines the underground economy to be all economic activity that contributes to value added and goes unreported by a society's measurement technique. This author outlines several different activities that would lead to a distorted measure of national measurements: moonlighting is defined as failure to report income from a second job; or profit-businesses that are paid in cash and do not report this additional income i.e. hair dressers may report fewer clients than they really service; the failure to report interest earnings and barter.

[5]The hypothesis that firms always evade is related to the use of underground labor. It must be noticed that such hypothesis is encapsulated in the definition of the production function: in order to have nonzero production, $n_{j,t}^{U}$ must be positive in equilibrium.

where w_t^M is the regular sector wage, w_t^U is the underground sector wage and r_t is capital remuneration rate. Tax rates can be hit by random shocks. Expected profits are computed by taking linear projection, i.e. $E\pi_{j,t} = (1-p)\pi_{j,t}^{ND} + p\pi_{j,t}^{D}$:

$$E\pi_{j,t} = (1 - sp\tau_t^{\Pi})y_{j,t} - \left(1+\tau_t^N\right)w_t^M n_{j,t}^M + \\ -\left(1+sp\tau_t^N\right)w_t^U n_{j,t}^U - r_t k_{j,t}. \tag{5}$$

The following condition, then, ensures a non-zero production:

Condition 1 $\left(1 - sp\tau_t^{\Pi}\right) \geq 0$

For the rest of the analysis we will assume that Condition 1 holds (in particular, for our parameterizations the quantity $\left(1 - sp\tau_t^{\Pi}\right)$ is positive). As markets are competitive, firm's behavior is described by the first order conditions for the (expected) profit maximization, with respect to $k_{j,t}$, $n_{j,t}^M$ and $n_{j,t}^U$:

$$\alpha(1 - sp\tau_t^{\Pi})\frac{y_{j,t}}{k_{j,t}} = r_t$$
$$(1-\alpha-\rho)(1 - sp\tau_t^{\Pi})\frac{y_{j,t}}{n_{j,t}^M} = (1+\tau_t^N)w_t^M \tag{6}$$
$$\rho(1 - sp\tau_t^{\Pi})\frac{y_{j,t}}{n_{j,t}^U} = (1+sp\tau_t^N)w_t^U.$$

Concavity of the production function (recall that firms take A_t as a constant) ensures the existence of a unique solution.

2.2. Households

Suppose that there exist a continuum of households, uniformly distributed over the unit interval. The $j-th$ household's preference are represented by the following momentary utility function:

$$u(c_{j,t}, n_{j,t}, n_{j,t}^U) = \ln c_{j,t} - B_0(n_{j,t}) - B_1(n_{j,t}^U), \tag{7}$$

where $c_{j,t}$ denotes household's consumption flow, $n_{j,i}$ and $n_{j,t}^U$ denote aggregate and underground labor supplies; the term $B_0(n_{j,t})$ with $B_0 \geq 0$, represents the overall disutility of working, while the last term, $B_1(n_{j,t}^U)$ with $B_1 \geq 0$, reflects the idiosyncratic cost of working in the underground sector[6].

Aggregate labor supply equals sum of regular and underground labor flows:

$$n_{j,t} = n_{j,t}^M + n_{j,t}^U. \tag{8}$$

The household evades income taxes by reallocating labor services from regular to underground sector. Underground-produced income flows $w_t^U n_{j,t}^U$ are, therefore, not subject to distortionary income tax rate τ_t^Y, as the feasibility constraint below suggests:

$$c_{j,t} + i_{j,t} = (1-\tau_t^Y)\left(w_t^M n_{j,t}^M + r_t k_{j,t}\right) + w_t^U n_{j,t}^U. \tag{9}$$

Capital stock is accumulated according to a customary state equation, i.e.

$$k_{j,t+1} = (1-\delta)k_{j,t} + i_{j,t}, \tag{10}$$

where $i_{j,t}$ represents net investments, and δ denotes a quarterly capital stock depreciation rate.

Imposing a constant subjective discount rate $0 < \beta < 1$, and defining $\mu_{j,t}$ as the costate variable, the Lagrangian of the household control problem reads:

$$\mathbf{L}_{0,j} = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{j,t}, n_{j,t}, n_{j,t}^U) + \\ +E_0 \sum_{t=0}^{\infty} \mu_{j,t}[(1-\tau_t^Y)\left(w_t^M n_{j,t}^M + r_t k_{j,t}\right) + w_t^U n_{j,t}^U - c_{j,t} + \\ -k_{j,t+1} + (1-\delta)k_{j,t}], \tag{11}$$

and the first order conditions obtain:

$$\beta^t c_{j,t}^{-1} = \mu_{j,t}$$
$$\beta^t B_0 = \mu_{j,t}(1-\tau_t^Y)w_t^M$$
$$\beta^t B_0 + \beta^t B_1 = \mu_{j,t} w_t^U \tag{12}$$
$$E_t\left\{\mu_{j,t+1}\left[(1-\delta) + (1-\tau_{t+1}^Y)r_{t+1}\right]\right\} = \mu_{j,t}$$
$$\lim_{T\to\infty} E_T \mu_{j,T} k_{j,T} = 0.$$

2.3. Government

The government budget, balanced in each period, is given by:

$$\tau_t^Y(w_t^M N_t^M + r_t K_t) + sp\tau_t^{\Pi} Y_t + sp\tau_t^N w_t^U N_t^U + \\ +\tau_t^N w_t^M N_t^M = E_t REV_t = G_t, \tag{13}$$

where $E_t REV$ denote expected government revenues, which are allocated to government expenditure G_t. Notice that government balances its budget in expected terms since tax revenues collected from the underground sector depend on the probability of being detected p. Government expenditure is assumed to be wasteful, as the fiscal authority collects taxes in corporate sector after that production takes place.

Tax shocks - which are a source of intrinsic uncertainty - are defined by the following set of stochastic difference equations, where hat variables denote percentage deviations from the stationary state:

$$\widehat{\tau}_{t+1} = Q\widehat{\tau}_t + e_{t+1}, \tag{14}$$

where $\widehat{\tau}_{t+1} = \left[\widehat{\tau}_t^Y, \widehat{\tau}_t^N, \widehat{\tau}_t^{\Pi}\right]'$. Q is a matrix with elements $\left[\phi_{ii}^V\right]$, $i = 1,2,3$; $V = \{Y, N, \Pi\}$ on the principal diagonal

[6]Specifically, this cost may be associated with the lack of any social and health insurance in the underground sector.

and zeroes elsewhere; e is a 3×1 vector of i.i.d. random shocks and the covariance matrix Σ is a diagonal matrix $\left[\sigma(e)_{ii}^{V}\right]$, $i = 1,2,3;$ $V = \{Y, N, \Pi\}$ where hat variables denote percentage deviation from stationary state.

3. MODEL'S TOPOLOGICAL PROPERTIES

3.1. Stationary State

Proposition 1 shows that the model has a unique stationary state for capital stock, consumption, regular and underground labor services.

Proposition 1 *There exists a unique stationary equilibrium for regular labor supply* N_M^* *, for underground labor supply* N_U^**, and for capital stock and consumption (*K^**,* C^* *>0) such that:*

$$N_M^* = \frac{(1-\bar{\tau}^Y)(1-\alpha-\rho)\left(\frac{r^*}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}}}{(1+\bar{\tau}^N)B_0\Gamma}$$

$$N_U^* = \frac{\rho\left(\frac{r^*}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}}}{(1+sp\bar{\tau}^N)\left(B_0+B_1\right)\Gamma}$$

$$K^* = \left(\frac{r^*}{\alpha}\right)^{\frac{1}{\phi_1-1}}\left[\left(N_M^*\right)^{\phi_2}\left(N_U^*\right)^{\phi_3}\right]^{\frac{1}{1-\phi_1}}$$

$$C^* = \Gamma\left[\left(N_M^*\right)^{\phi_2}\left(N_U^*\right)^{\phi_3}\right]^{\frac{1}{1-\phi_1}},$$

where $\Xi = \left(\left(1-\bar{\tau}^Y\right)\left(\frac{(1-\alpha-\rho)}{(1+\bar{\tau}^N)}+\alpha\right)+\frac{\rho}{1+s\rho\bar{\tau}^N}\right)$

and

$\Gamma = \Xi\left(\frac{r^?}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}} - \delta\left(\frac{r^?}{\alpha}\right)^{\frac{1}{\phi_1-1}}$ *are two positive constants and* $r^* = \frac{\beta^{-1}-1+\delta}{1-\bar{\tau}_Y}$ *is the stationary value for the rate of return on capital.*

Proof See Appendix.

3.2. Conditions for Indeterminacy

For our parameterization (see Section II-3 below), the model's attractor is a sink: the eigenvalues characterizing the stability properties of the unique stationary state are equal to 0.8211±0.2123i. The topological properties of the model's attractor depend upon some crucial parameters; we restrict our attention to those characterizing the labor inputs' heterogeneity: particularly on η and ζ and the tax rates. It can be shown that the model can display indeterminacy only if the following conditions are satisfied (see [14]):

Condition **NC** (Necessary):

$$\beta < (1+\eta)\left\{\begin{array}{l}\left(1-\left(1-\alpha-\rho\right)\frac{\bar{\tau}^N}{1+\bar{\tau}^N}\right)+ \\ \rho\left(\frac{1}{\left(1-\bar{\tau}^Y\right)\left(1+sp\bar{\tau}^N\right)}-1\right)\end{array}\right\}$$

Condition **SC** (Sufficient):

$$\max\left\{\frac{1}{\beta(1-\delta)}, \frac{\underline{\mathbf{R}}}{\underline{\mathbf{R}}-1}\right\} < \varepsilon_{\widehat{N}^U}^{L_M^D} + \varepsilon_{\widehat{N}^M}^{L_U^D} < \frac{\bar{\mathbf{R}}}{\bar{\mathbf{R}}-1}.$$

Condition **SC** is expressed in terms of the cross-elasticities of the (inverse) inputs demand functions; from firms first order conditions we have:

$$\begin{aligned}\varepsilon_{\widehat{N}^U}^{L_M^D} &= \frac{\partial\widehat{w}_t^M}{\partial\widehat{N}_t^U} = \frac{\partial\widehat{r}_t}{\partial\widehat{N}_t^U} = \left(1+\zeta\right)\rho, \\ \varepsilon_{\widehat{N}^M}^{L_U^D} &= \frac{\partial\widehat{w}_t^U}{\partial\widehat{N}_t^M} = \frac{\partial\widehat{r}_t}{\partial\widehat{N}_t^M} = \left(1+\eta\right)(1-\alpha-\rho);\end{aligned} \tag{15}$$

The quantities $\bar{\mathbf{R}}$ and $\underline{\mathbf{R}}$ read:

$$\bar{\mathbf{R}} = \frac{\delta(M-s_I)\left[1-\beta(1-\delta)\right]\left(1-\alpha(1+\eta)\right)+2\left[\delta\alpha(1+\eta)M+s_I(2-\delta)\right]}{\delta(M-s_I)\left[1-\beta(1-\delta)\right]\left(1-\alpha(1+\eta)\right)+2\left[\delta\alpha(1+\eta)M+s_I(1-\beta(1-\delta))\right]} > 1,$$

$$\underline{\mathbf{R}} = \frac{\delta s_I - \delta\alpha(1+\eta)M}{s_I\left[1-\beta(1-\delta)\right]-\delta\alpha(1+\eta)M} > 1,$$

while $M = 1 - G^*/Y^* = s_I + s_C$; $s_I = I^*/Y^*$; $s_C = C^*/Y^*$.

The **Condition NC** suggests that in order to have indeterminacy it is necessary that the term $\frac{s_I+s_C}{1-\bar{\tau}^Y} = \left(1-\left(1-\alpha-\rho\right)\frac{\bar{\tau}^N}{1+\bar{\tau}^N}\right)+\rho\left(\frac{1}{\left(1-\bar{\tau}^Y\right)\left(1+sp\bar{\tau}^N\right)}-1\right)$, augmented by the regular externality parameter η, must be *large enough*; in other words, distortionary taxation should not drain too much resources away from the private sector, in order to allow the latter to form self-fulfilling beliefs.[7]

Taking $\bar{\tau}_Y$ as given, the higher the probability of being detected evading (and/or the penalty surcharge factor), the more difficult it becomes to allocate resources to the underground sector, and the smaller the quantity $\left(\frac{1}{\left(1-\bar{\tau}^Y\right)\left(1+sp\bar{\tau}^N\right)}-1\right)$ gets. Consider the extreme case where tax evaders are punished with an infinitely large penalty (that is $s \to \infty$). The condition NC reads: $\beta < (1+\eta)\left\{\left(1-\left(1-\alpha-\rho\right)\frac{\bar{\tau}_N}{1+\bar{\tau}_N}\right)-\rho\right\}$, suggesting that the parameter region for indeterminacy shrinks when tax evasion becomes extremely costly, and it fails if labor taxes are too high ($\bar{\tau}_N > \frac{\beta-(1+\eta)(1-\rho)}{\alpha(1+\eta)-\beta}$, for example).[8] Indeed, when tax evasion is extremely costly/risky, and when taxes are higher than a certain threshold, they *tax away* the externality, in the spirit of [8].

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[7]Recall that government expenditure is wasteful in the model. It is difficult to figure out, off hand, whether our results still hold if government were rebating tax revenues either as consumption of private goods, or as investment to augment private inputs' productivity. This analysis is left for future investigation.

[8]The NC condition fails when $\beta > (1+\eta)\left\{\left(1-\left(1-\alpha-\rho\right)\frac{\bar{\tau}^N}{1+\bar{\tau}^N}\right)-\rho\right\}$. This inequality can be recast in terms of $\bar{\tau}^N$ obtaining $\bar{\tau}^N > \frac{\beta-\left(1-\rho\right)\left(1+\eta\right)}{\alpha\left(1+\eta\right)-\beta}$. Notice, moreover, that the quantity $\frac{\beta-\left(1-\rho\right)\left(1+\eta\right)}{\alpha\left(1+\eta\right)-\beta}$ is quite small for reasonable parameters' value.

The picture is different if we take $\overline{\tau}_N$ as given. An increase in income tax rate $\overline{\tau}_Y$ monotonically increases the quantity $\left(\frac{1}{(1-\overline{\tau}^Y)(1+sp\overline{\tau}^N)}-1\right)$, easing, by this hand, the necessary condition NC. That happens because there is no probability of being detected evading income taxation (on the households' side). Therefore, the higher the income tax rate, the higher would be the underground labor supply; in this sense, resources would be reallocated toward an input that ensures a tax-free externality. In this case we cannot claim that higher income tax rates tax away the externality.

The **Condition SC** is more enlightening about the nature of the economic process at the basis of indeterminacy in our model. It suggests that the labor demand schedules should have a sufficiently large response to changes in equilibrium employment (i.e. the "lower inequality" in the condition SC), but, at the same time, that this response should not be too large (that is the "upper inequality" in the condition SC). Rewriting the upper inequality in Condition SC in terms of elasticity of labor demand schedules to changes in capital stock ($\varepsilon^{L_M^D}_{\widehat{K}}$ and $\varepsilon^{L_U^D}_{\widehat{K}}$), yields:

$$\varepsilon^{L_U^D}_{\widehat{K}},\varepsilon^{L_M^D}_{\widehat{K}} > \frac{s_I}{s_I+s_C}\left\{1+\frac{(1-\delta)(1-\beta)}{\delta}\left(\varepsilon^{L_M^D}_{\widehat{N}^U}+\varepsilon^{L_U^D}_{\widehat{N}^M}\right)\right\}. \qquad (16)$$

This suggests that regular and underground labor demands should react relatively more to expected changes in capital stocks rather than changes in labor services. In other words, the shifts of labor demands driven by changes in capital stock should be larger than those driven by changes in labor services. Both labor demand schedules are well behaved (in the sense that they slope down), compared to standard one-sector economy models where labor demand is upward sloping. Just observe that:

$\frac{\partial \widehat{w}_t^M}{\partial \widehat{N}_t^M}=(1+\eta)(1-\alpha-\rho)-1$ and

$\frac{\partial \widehat{w}_t^U}{\partial \widehat{N}_t^U}=(1+\zeta)\rho-1$

are both negative, for our parameterization.[9]

3.3. Parameterization

The model is parameterized for the U.S. economy; parameterization is based on seasonally adjusted series from 1970 to 2001, expressed in constant 1995 prices. Actual data for the United States economy are drawn from [12]. The system of equations we use to compute the dynamic equilibria of the model depends on a set of 15 parameters; a starred parameter denotes the precise calibrated value.

The technology parameters α and δ are set to standard figures in the literature; precisely α*=0.23 and δ*=0.088.

The choice of parameters that are more closely related to tax evasion and the underground economy deserve more attention.[10] For the *probability of being detected p*, we rely on [13], which estimate that the probability of auditing in the US ranges between 4.6% and 5.7%. We choose the higher value, $p^*=0.057$, but results do not significantly change if we consider the lower value 4.7%.

As for η and ζ, they are calibrated by using the regular labor demand schedule and the aggregate production function. More precisely, rewrite these two equations in terms of the empirically-known macroeconomic ratios (the capital/regular labor ratio and the share of underground labor out of regular labor); we, then, take a logarithmic transformation and solve for η and ζ. We obtain $\eta^*=0.44$ and ζ*=0.28.

The disutility parameters B_0 and B_1 are set to calibrate the steady state values of the equilibrium regular labor services to 32 percent of labor force and the steady state value of the ratio N^*_U/N^*_M to 0.10. This is equivalent to imposing that the underground labor services equal the 8.8 percent of aggregate labor services, as the data suggest.

To stop abusive trust promoters, the Internal Revenue Service undertakes a national coordinated strategy to address abusive trust schemes.[11] Violations of the Internal Revenue Code may result in civil penalties, which includes a fraud penalty up to 75% of the underpayment of tax attributable to the fraud in addition to the taxes owed. Therefore we set the *surcharge factor* $s^*=1.75$.[12]

As for the average-long run levels of taxation, the effective income tax rate $\overline{\tau}^Y$ and corporate tax rate $\overline{\tau}^\Pi$ are computed from the Effective Tax Rates, 1979-1997, Table H-1a, prepared by the Congressional Budget Office; social security tax rate is taken from www.socialsecurity.com; we choose the values applying for the 1990s and later, which equal to $\overline{\tau}^Y=0.1186$, $\overline{\tau}^\Pi=0.355$ and, $\overline{\tau}^N=0.153$.

The externality parameters are set following [15]; aggregate level of returns to scale equals 1.42, which is lower than the original of [11] calibration (1.61).

Shocks to tax rates are assumed to be permanent, that is the autocorrelation coefficients in the shocks' processes equal unity: $\varphi_{11}^Y=\varphi_{22}^N=\varphi_{33}^\Pi=1$. It is therefore assumed that unexpected increases in tax rates is maintained in all subsequent periods, i.e. fiscal policy is based on a commitment to a non-transitory tax increase. Such hypothesis is not particularly restrictive, as the impulse response would maintain analogous qualitative features also

9This result is robust to a large set of parameters, as long as the regular externality is sufficiently small. The quantity $(1-\alpha-\rho)$ is in fact positive and small. If η gets too big, the slope $\frac{\partial \widehat{w}_t^M}{\partial \widehat{N}_t^M}=(1+\eta)(1-\alpha-\rho)-1$ becomes positive. The same can happen to $\frac{\partial \widehat{w}_t^M}{\partial \widehat{N}_t^M}=(1+\zeta)\rho-1$ when ζ is high enough.

10[14]] calibrate a model for the Italian Economy, and [21] calibrate a model for the Spanish economy

11For more details about the Internal Revenue Service policy regarding abusive trusts, refer to Internal Revenue Service Public Announcement Notice 97-24, which warns taxpayers to avoid abusive trust schemes that advertise bogus tax benefits.

12Violations may also result in criminal prosecution; in this case there are penalties up to five years in prison for each offense.

in the case of purely temporary shocks ($\varphi_{11}^{Y} = \varphi_{22}^{N} = \varphi_{33}^{\Pi} = 0$); the qualitative nature of the economy's response only depends upon the topological properties of the attractor. In all exercises the size of each shock equals one unit standard deviation.

4. RESULTS: SELECTED FISCAL POLICY IMPLICATIONS

4.1. Impulse Response Functions

Fig. (**1**, Panels **1-4**) includes model's response to a permanent increase in the three tax rates (specifically, after a positive one unit standard deviation innovation in $\hat{\tau}^{\Pi}$, $\hat{\tau}^{Y}$ and $\hat{\tau}^{N}$). In order to highlight the role of indeterminacy on the model's response to fiscal policy shocks, we confront impulse responses to increases in tax rates when the equilibrium is a sink (for $\eta = 0.45,\ \zeta = 0.2$) and when there is saddle path stability ($\eta = \zeta = 0,$ but results holds also for nonzero values of and guaranteeing saddle stability).

Under indeterminacy an increase in tax rates may push the economy into an expansion, while the economy plummets into recession when equilibrium is saddle-path stable. Fig. (**1**) shows indeed that GDP, consumption, capital stock and aggregate equilibrium labor services may increase; the smaller impact of the tax rate on profits is related to the fact that in the model, the expected reduction in profits due to an increase in statutory corporate tax must be reduced by a factor *sp*. Under indeterminacy the Sufficient Condition SC ensures that an increase in input demand functions is capable to offset the recessionary impulse that would be generated by the increase in tax rates. On the contrary, when there are no aggregate externalities, this condition is not satisfied anymore, and a tax shock has just a recessionary effect.

An increase in one of the tax rates induces agents to allocate resources to the underground sector; agents in turn could forecast that such a reallocation may cause an economic expansion. But when there is just one transitional path to the stationary equilibrium, the reallocation to the underground sector is not capable to expand the economy and the expansionary prophecy would not be fulfilled. When

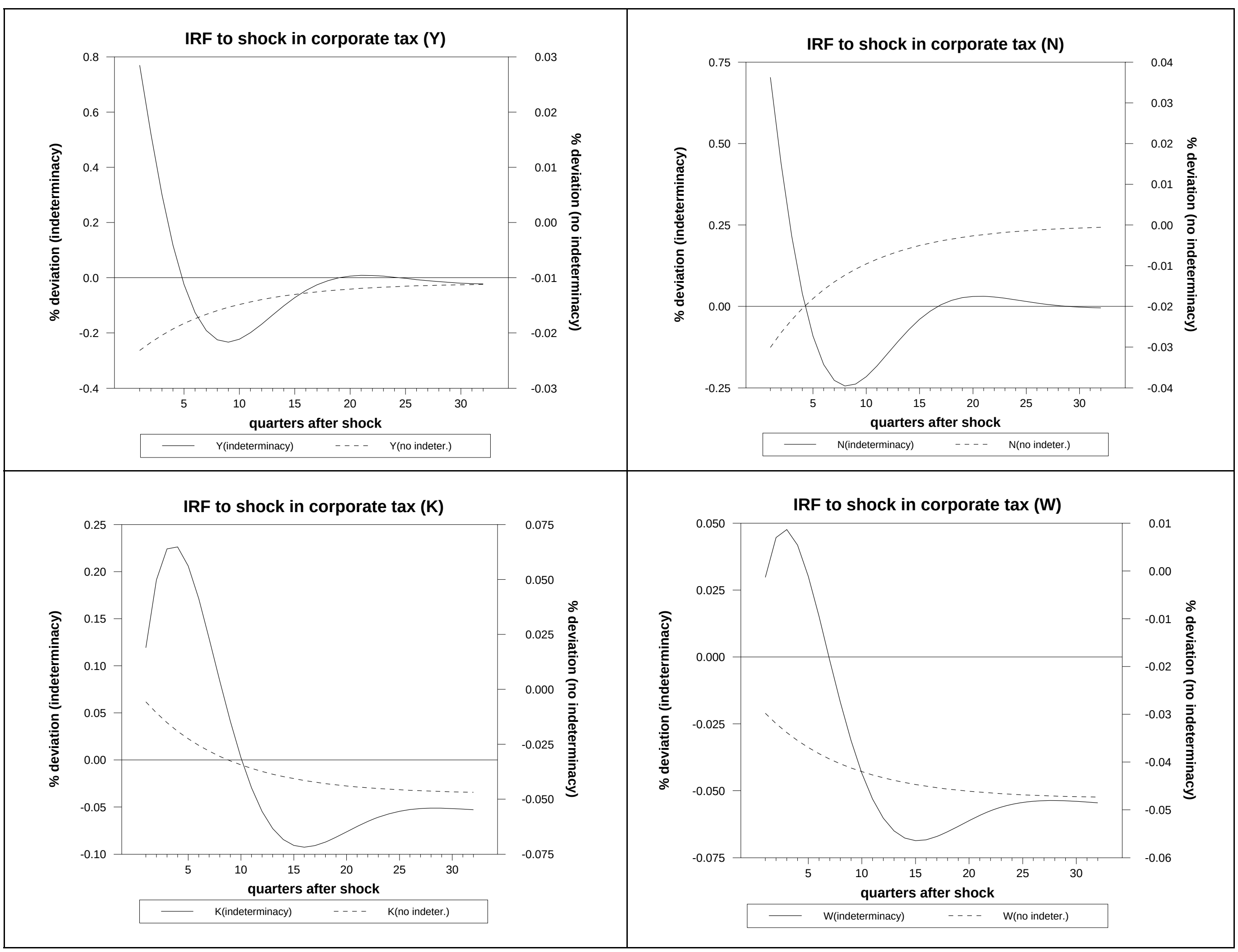

Fig. (1). Predicted impulse response of output, capital stock, labor, and wage rate to a permanent increase in corporate tax rate, with (η= 0.45; ζ= 0.2) and without indeterminacy (η= 0.00; ζ= 0.00). Solid Lines are responses under indeterminacy; dashed lines are responses when steady state is saddle-path stable.

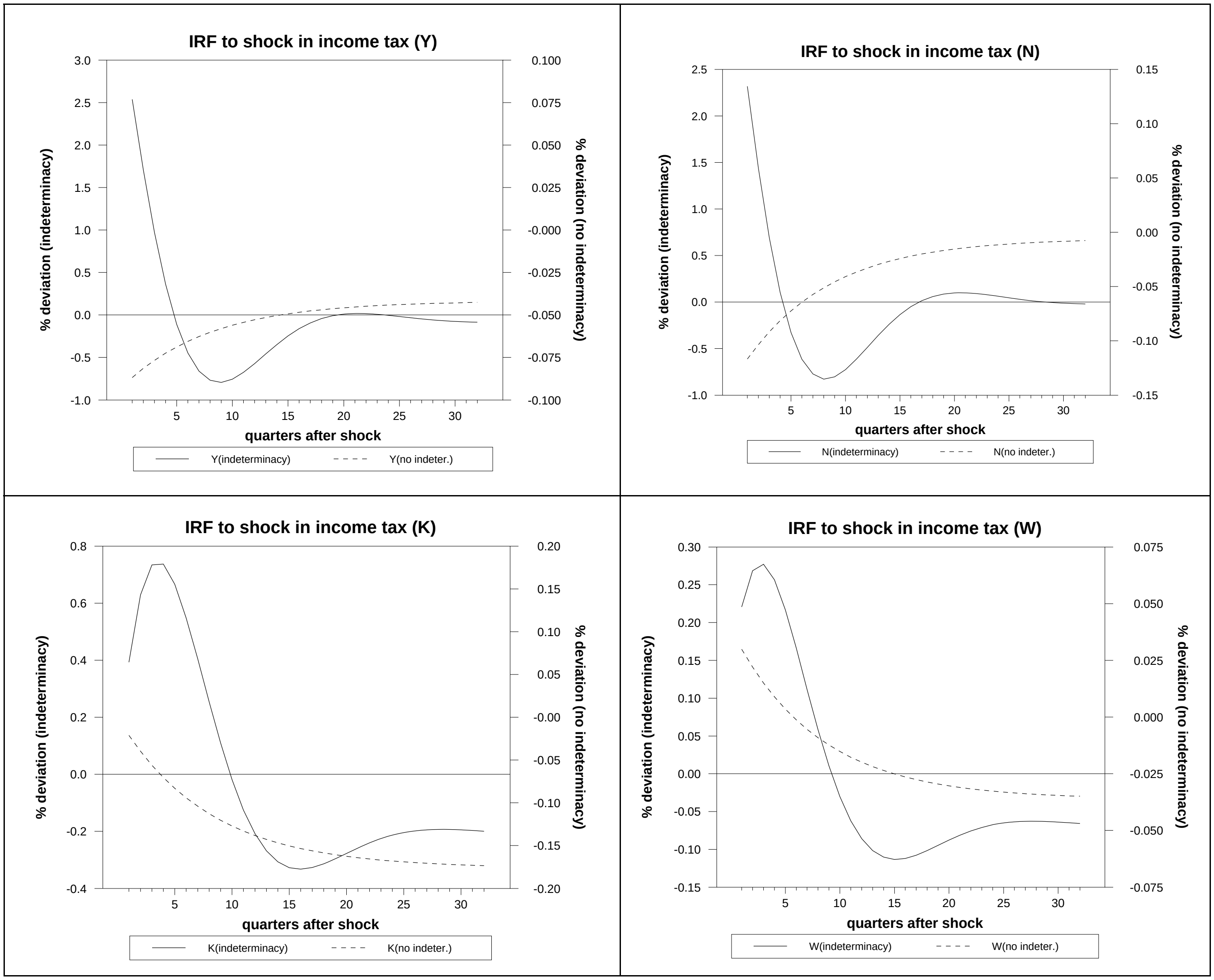

Fig. (2). Predicted impulse response of output, capital stock, labor, and wage rate to a permanent increase in income tax rate, with (η= 0.45; ζ= 0.2) and without indeterminacy (η= 0.00; ζ= 0.00). Solid Lines are responses under indeterminacy; dashed lines are responses when steady state is saddle-path stable.

instead there is indeterminacy, the expansionary prophecy can indeed be fulfilled; thus, the increase in tax rates can have an effect which is qualitatively similar to an expansionary sunspot shock.

The interesting feature of our results is that fiscal policy under underground economy may provide inadvisable outcomes. Thus, a tax policy designed to drive the economy on some dimensions (e.g. reduce tax evasion, boost the economy, etc.) can achieve counterproductive results. This happens because the underground activity may easily entail indeterminate equilibrium paths.

Now we show that when the underground economy is able to pick up certain features (discussed in the previous sections), the results can be rationalized by describing the theoretical mechanism operating in the model.

4.2. Theoretical Mechanism

The model response is driven by a distinctive mechanism that operates through the labor market, and differs from that operating in standard one-sector models. We analyze qualitatively the behavior of labor markets after an unexpected and permanent increase in one of the tax rates.

Consider an exogenous increase in corporate tax rate (figure below). The regular $\left(L_M^D\right)$ and the underground $\left(L_U^D\right)$ labor demand schedules shift left after an increase in corporate tax rate $\left(\uparrow \hat{\tau}^{\Pi}\right)$ (along the corresponding supplies), reducing both $\widehat{N}_t^M$ and $\widehat{N}_t^U$; the labor demand cross-elasticity induces a further inward shift up to $L^D(1)$.[13] The fall in $\widehat{N}_t^M$ and $\widehat{N}_t^U$, coupled with the increase $\widehat{\tau}^{\Pi}$,

[13]More formally, we have: $\downarrow \widehat{N}_t^M \Rightarrow \downarrow \widetilde{w}_t^U$ and $\downarrow \widehat{N}_t^U \Rightarrow \downarrow \widetilde{w}_t^M$, because $\frac{\partial \widetilde{w}_t^M}{\partial \widehat{N}_t^U} > 0$ and $\frac{\partial \widetilde{w}_t^U}{\partial \widehat{N}_t^M} > 0$.

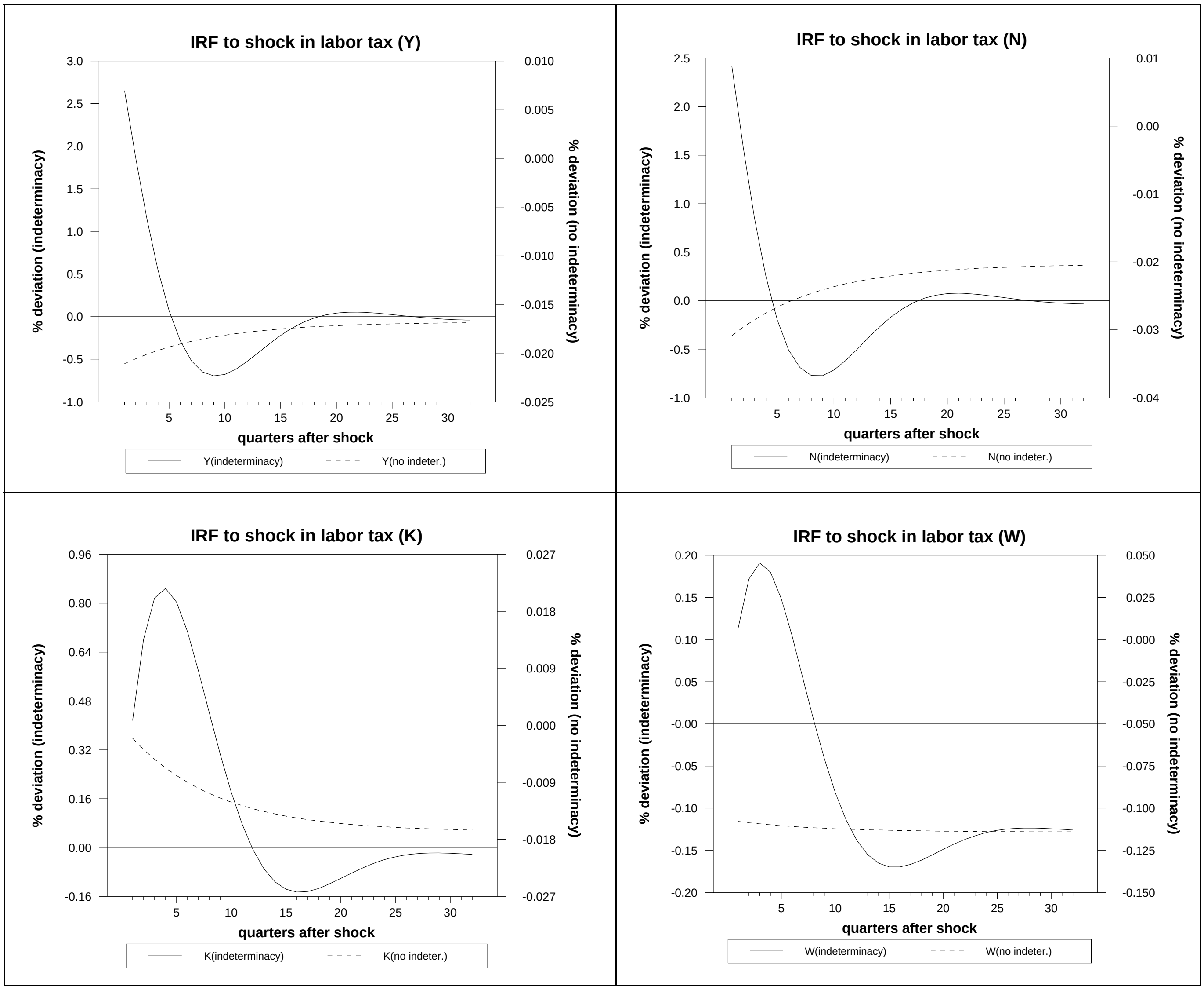

Fig. (3). Predicted impulse response of output, capital stock, labor, and wage rate to a permanent increase in labor tax rate, with (η= 0.45; ζ= 0.2) and without indeterminacy (η= 0.00; ζ= 0.00). Solid Lines are responses under indeterminacy; dashed lines are responses when steady state is saddle-path stable.

reduces the interest rate $\hat{r}_t$;[14] this produces two consequences. First, the relative prices of production inputs are now changed, as rental cost $\hat{r}_t$ of capital is lowered with respect to the wage rates $\hat{w}_t^M$ and $\hat{w}_t^U$; thus labor is relatively more costly than capital. Second, the downward pressure on $\hat{r}_t$ reduces the incentive to invest and makes consumption more attractive. The households could increase consumption, but they would need more resources to do so.

Here enters the crucial role played by tax evasion and the underground sector. In a perfect foresight equilibrium agents are aware that, allocating resources to the underground input, they can reduce their tax burden, increase their disposable income, and therefore afford a higher level of consumption. The resource reallocation toward the underground labor triggers an expansionary mechanism, because it increases returns to capital, regular labor services, and therefore equilibrium capital stock and equilibrium regular labor. The increase in the tax rate is an incentive to allocate more resources to the underground sector.

In addition, the sufficient condition for indeterminacy states that the elasticities of regular and underground labor demand schedule to capital stock should be sufficiently larger than a combination of the elasticities of both kinds of labor services.[15] In this context the capital stock's impact is

[14]This can be seen from the linearized demand for capital:

$$\hat{r}_t = \left[(1+\eta)\alpha - 1\right]\hat{K}_t + (1+\eta)(1-\alpha-\rho)\hat{N}_t^M + (1+\zeta)\rho\hat{N}_t^U - \left[\frac{sp\bar{\tau}^\Pi}{1-sp\bar{\tau}^\Pi}\right]\hat{\tau}_t^\Pi$$

[15]To see this more clearly, consider the inverse (linearized) demand for regular labor (from the firm's first order conditions):

$$\hat{w}_t^M = \left[(1+\eta)\alpha\right]\hat{K}_t + \left[(1+\eta)(1-\alpha-\rho)-1\right]\hat{N}_t^M + \left[(1+\zeta)\rho\right]\hat{N}_t^U + \hat{P}_M$$

i.e. as a function $\hat{w}_t^M = L_D^M\{\hat{N}_t^M;\hat{N}_t^U,\hat{K}_t,\hat{P}_M\}$, where $\hat{P}_M = \hat{P}_M(\hat{\tau}_t^\Pi,\hat{\tau}_t^N)$ represents a set of parameters and other exogenous variables large enough so that the labor demand schedules are shifted out up to $L^D(2)$; the consequent increase in income induces a raise in equilibrium consumption, leading to a higher value function for households.

An increase of income tax rate, as well as an increase in labor tax, still have an expansionary impact, as the operating mechanism is qualitatively similar to that previously presented. In summary, after a permanent increase in either corporate, income or labor tax rates the new equilibrium is characterized by a higher employment, higher wage rates, and higher interest rate on capital stock. This is a direct consequence of the self-fulfilling prophecies acting *via* capital accumulation and the cross elasticities of the three inputs demand schedules; a high-enough level of externality is needed for this kind of mechanism to be active.

CONCLUSIONS

This paper discusses selected fiscal policy experiments in a one-sector dynamic general equilibrium model augmented with tax evasion and underground activities. The model displays increasing returns to scale due to aggregate production externalities in regular and underground inputs, capable to induce sunspots and indeterminacy.

The paper analyzes the effect of exogenous shocks to taxation structure, while studying the dynamic response of the economy, and highlighting the differences with respect to the case of constant returns to scale. The interesting element for the discussion is that an increase in corporate, income or labor tax rates may have an expansionary effect under local indeterminacy of the equilibrium path (i.e. when aggregate production technology displays increasing returns to scale high enough for producing indeterminacy). Results are reversed under saddle-path stability. This difference is due to a specific economic mechanism, which is distinctive of the indeterminacy case and acts through the cross elasticities of the inputs' demand functions.

Before concluding, a caveat should be noted. The difficulty here is that the impulse response functions cannot be interpreted as in classical model with decreasing returns to scale. Our impulse response functions are just one of the possible dynamic outcomes that can emerge from this type of model - i.e. when a shock over an exogenous variable is simulated under indeterminacy. In other words, this paper does not undertake a full identification in the spirit of [10], while examining a non-identified model formulation. The further step - i.e. to imple-ment the procedure suggested in [10] - is left to future research.

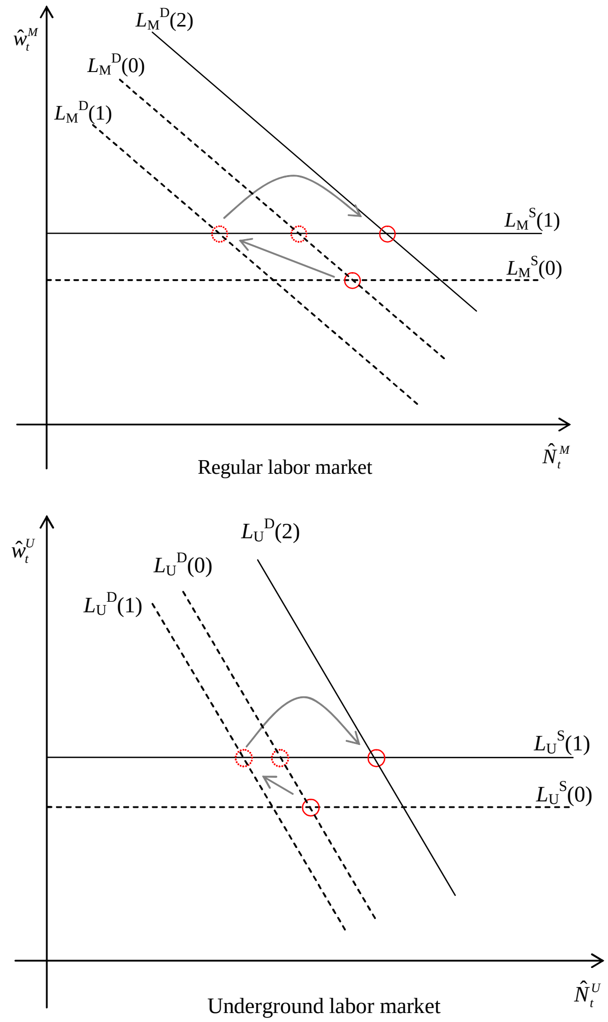

Fig. (4). Labor Markets' response to a permanent increase in corporate tax rate. The left panel presents the regular labor market equilibrium; right panel includes the underground labor market equilibrium. L_M^D, L_M^S denote, respectively, the regular labor demand and supply, while L_U^D, L_U^S denote underground labor demand and supply.

(mainly tax rates) and whose partial derivatives have the following signs: $\frac{\partial L_D^M}{\partial \widehat{N}_t^M} < 0, \frac{\partial L_D^M}{\partial \widehat{N}_t^U}, \frac{\partial L_D^M}{\partial \widehat{K}_t} > 0$. Symmetrically, the wage $\widehat{w}_t^U$ - the demand for underground labor - equals:

$$\widehat{w}_t^U = \left[(1+\eta)\alpha\right]\widehat{K}_t + \left[(1+\eta)(1-\alpha-\rho)\right]\widehat{N}_t^M + \left[(1+\zeta)\rho-1\right]\widehat{N}_t^U + \widehat{P}_U$$

and it is written as $\widehat{w}_t^U = L_D^U\{\widehat{N}_t^M; \widehat{N}_t^U, \widehat{K}_t, \widehat{P}_U\}$ where $\frac{\partial L_D^U}{\partial \widehat{N}_t^U} < 0, \frac{\partial L_D^U}{\partial \widehat{N}_t^M}, \frac{\partial L_D^U}{\partial \widehat{K}_t} > 0$. Now, the initial fall in each sector equilibrium labor services (that is a movement along each sector demand schedule) induces a further reduction in each sector employment through an inward shift of demand schedules (that is a schedule shift, induced by a change in the other-sector equilibrium employment).

APPENDIX

Evaluating the firms' and households' first order conditions at the stationary state yield:

$$\begin{aligned}
B_0 &= C^{-1}(1-\bar{\tau}_Y)w_M \\
C^{-1}w_U &= B_0 + B_1 \\
r^{?} &= \frac{\beta^{-1}-1+\delta}{1-\bar{\tau}_Y} \\
C &= \Xi K^{\phi_1}N_M^{\phi_2}N_U^{\phi_3} - \delta K \\
\tilde{w}_M &= (1+\bar{\tau}_N)w_M = (1-\alpha-\rho)K^{\phi_1}N_M^{\phi_2-1}N_U^{\phi_3} \\
\tilde{w}_U &= (1+sp\bar{\tau}_N)w_U = \rho K^{\phi_1}N_M^{\phi_2}N_U^{\phi_3-1} \\
r &= \alpha K^{\phi_1-1}N_M^{\phi_2}N_U^{\phi_3}
\end{aligned}$$

where $\Xi = \left(\left(1-\bar{\tau}_Y\right)\left(\frac{(1-\alpha-\rho)}{(1+\bar{\tau}_N)}+\alpha\right)+\frac{\rho}{1+sp\bar{\tau}_N}\right) > 0$ and $\phi_1 = \alpha(1+\eta)$, $\phi_2 = (1-\alpha-\rho)(1+\eta)$. The third equation shows the stationary value for the rate of return on capital $r^{?}$. By solving the last equation (demand for capital) w.r.t. K we obtain:

$$K = \left(\frac{r^{?}}{\alpha}\right)^{\frac{1}{\phi_1-1}}\left(N_M^{\phi_2}N_U^{\phi_3}\right)^{\frac{1}{1-\phi_1}}$$

Now, by plugging this latter value of K into the resource constraint $C = \Xi K^{\phi_1}N_M^{\phi_2}N_U^{\phi_3} - \delta K$ we obtain:

$$C = \Xi\left(\frac{r^{?}}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}}\left(N_M^{\phi_2}N_U^{\phi_3}\right)^{\frac{\phi_1}{1-\phi_1}}\left(N_M^{\phi_2}N_U^{\phi_3}\right) - \delta\left(\frac{r^{?}}{\alpha}\right)^{\frac{1}{\phi_1-1}}\left(N_M^{\phi_2}N_U^{\phi_3}\right)^{\frac{1}{1-\phi_1}}$$

which simplifies into:

$$C = \Gamma\left(N_M^{\phi_2}N_U^{\phi_3}\right)^{\frac{1}{1-\phi_1}}$$

where the coefficient $\Gamma = \Xi\left(\frac{r^*}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}} - \delta\left(\frac{r^*}{\alpha}\right)^{\frac{1}{\phi_1-1}}$ is positive.[16]

We can now substitute for regular labor,

$B_0 = (1-\tau_Y)w_M C^{-1}$, together with

$\tilde{w}_M = (1+\bar{\tau}_N)w_M = (1-\alpha-\rho)K^{\phi_1}N_M^{\phi_2-1}N_U^{\phi_3}$, so to obtain:

$$B_0 = \frac{(1-\bar{\tau}_Y)}{(1+\bar{\tau}_N)}\frac{(1-\alpha-\rho)\left(\frac{r^*}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}}\left(N_M^{\phi_2}N_U^{\phi_3}\right)^{\frac{\phi_1}{1-\phi_1}}\left(N_M^{\phi_2}N_U^{\phi_3}\right)}{\Gamma\left(N_M^{\phi_2}N_U^{\phi_3}\right)^{\frac{1}{1-\phi_1}}}N_M^{-1}$$

from which we can compute the stationary value for the regular labor input:

$$N_M^* = \frac{(1-\bar{\tau}_Y)(1-\alpha-\rho)\left(\frac{r^*}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}}}{(1+\tau_N)B_0\Gamma} > 0$$

We now repeat the procedure for the supply of underground labor $C^{-1}w_U = B_0 + B_1$, and obtain the stationary value:

$$N_U^* = \frac{\rho\left(\frac{r^*}{\alpha}\right)^{\frac{\phi_1}{\phi_1-1}}}{(1+sp\bar{\tau}_N)\left(B_0+B_1\right)\Gamma} > 0$$

Finally, by plugging $N_M^{?}$ and $N_U^{?}$ into the previous two equations the stationary values $K^{?}$ and $C^{?}$ are obtained.

[16] We do not prove the positivity of Γ in general, but it is straightforward to show that this parameter is indeed positive under our parameterization $\Gamma = 1{,}51$.

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