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RESEARCH ARTICLE

Optimal Allocation and Sizing of Capacitors for Distribution Systems Reinforcement Based on Minimum Life Cycle Cost and Considering Uncertainties

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Abstract:

Background:

The power and voltage losses have been greatly concerned in the distribution system planning and operation because low normal voltage, high impedance and load density of the system lead to increases in losses and decreases in bus voltages.

Method:

Therefore, an optimization model for the distribution system reinforcement improving efficiency of distribution systems by integrating capacitors is proposed. The optimal allocation and sizing of capacitors are determined concurrently with optimal upgrading sizing and timeframe of feeders and substations. The objective function of model is to minimize life cycle cost over reinforcement scheme including investment and operation costs of equipment in distribution systems (including feeders, substations and capacitors) and costs for purchasing energy from the electric market.

Conclusion:

The typical load curves for each day and season with uncertainties modeled by a Probability Distribution Function (PDF) and Time of Use (TOU) price applied to enhance accuracy of calculation results and suitability for real operation conditions. General Algebraic Modeling System (GAMS) is applied to undertake calculations in a test system.

Keyword: Distribution System, Optimization, Capacitor, Life cycle cost, GAMS, Uncertainties.

1. INTRODUCTION

Distribution systems serve as connections between distribution substations and retail customers with low normal voltage, high feeder impedance and load density [1, 2]. Hence, there will be a rise in power and voltage losses that are particularly considered in planning, designing and operating problems of distribution systems.

Capacitors are able to reduce power losses and delay upgrading of feeders and source substations because of reduction in transmission power on devices. Many new technologies in capacitor designs have been introduced with less leakage powers and the price recently. Additionally, the investment in construction and operation is low-cost and installation places are also unlimited [3, 4]. Hence, capacitors can be easily integrated in distribution systems and improve the economic and technical criteria. Many researches have been conducted to choose locations and capacity of capacitors to reduce power and voltage losses as well as operation cost in the distribution systems [5, 6]. The objective function to minimize power losses and voltage limits on load buses are introduced in [7, 8]. Furthermore, costs of devices are also considered in the objective function including total cost of energy losses and capacitor investment in

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order to enhance accuracy of calculation results [9 - 11]. Constraints on capacity of equipment (such as feeders and transformers) and bus voltages are utilized to ensure the requirements of operations.

In above researches, capacitors are often considered in problems of seeking optimal setting and sizing in distribution systems. It is assumed that the structures and parameters of the systems remain unchanged. However, the distribution systems are always reformed and upgraded by increasing the section of feeders and the capacity of transformer substations to meet developments of loads. In other words, the peak load is utilized to calculate the operation of distribution systems that demand strongly changes by time in day and season per year. Therefore, the error of calculation can be significantly big, which causes the over compensation in time of low demand and the increase in power loss at this time.

Moreover, the load of the distribution systems depends on demands of customers and weather that have arbitrary characteristics. Hence, the normal probability density function (PDF) is chosen for modeling the uncertainties of the load at each bus [13, 14]. In researches [15 - 18], the system's loads are assumed to follow the hourly load shape divided into different levels using a clustering technique with various probabilities. Therefore, the probability forecasting on load distribution becomes a significant issue that notably affect the operation and reinforcement of the distribution systems. However, the optimal compensation calculation of capacitor at the same time with the upgrading of feeders and transformer substations has not been mentioned in above researches. Uncertainties of loads are not validated because the demand in each hour is always set to be unchanged.

In recent years, the electricity price usually determined by Time-of-Use (TOU) is the simplest dynamic price. The objective of TOU price is to encourage the reduction of energy consumption during peak hours and the electricity price varies each day. TOU price is currently based on three periods of energy use as shown in [19 - 22] as follows: i) Peak - the highest demand; ii) Intermediate - the moderate demand; iii) Base - the lowest demand.

The mathematical optimization is widely applied in planning and reinforcement of the distribution systems with a variety of new models and methods [23]. Georgilall *etal.* and Tung *etal.* [24, 25] present models for the planning of the distribution systems with concurrent participation of distributed generators or capacitors. An optimal model in [26] is implemented with the objective function minimizing cost for investment feeder, transformer substation, distributed generator, capacitor and energy consumption from the electrical market in planning period. Similarly [27], introduces two stage planning models aim at decreasing the quantity and time of calculation. The objective function that seeks minimum value of investment and operation cost is used in associated with constraints in two stage in order to ensure the operation of system.

Besides, the objective function based on life cycle cost is also widely utilized in the problems of planning and operation of the distribution systems. The model with the objective function minimizing the life cycle cost in entire distribution system planning period is introduced in [28 - 29]. These models aim at finding the best distribution system planning scheme to maximize overall benefits and costs in the life cycle of the system. Constraints on maximum allowable capacity of equipment (feeder and transformer) and balanced power flow of system are considered to guarantee the economic and technical indicators. Similarly, the life cycle cost is persistently used in objective functions in [30 - 31] to determine the optimal transformer capacity. The model integrates the dynamic change of money with time, environmental cost, preventive test cost, sensitivity coefficient and the parameters of transformers.

In this research, an optimal model combining the uncertainties of demand described by a Probability Distribution Function (PDF) will be proposed. The optimal displacement, sizing and installation period of capacitors are simultaneously determined with optimal sizing and timeframe of feeders and transformer substations. The objective function is minimum life cycle cost of the planning scheme as well as the technical constraints are utilized to guarantee operation of the distribution systems and capacitor. The electricity price in the system and the electrical market is TOU price.

Next sections of the paper are organized as follows: an uncertainty modeling of demand is introduced in Section 2. A mathematical model with objective function and constraints are presented in Section 3. Section 4 shows calculated results from the IEEE 9-bus test distribution system. Finally, the conclusion is demonstrated in Section 5.

2. MODEL OF LOAD UNCERTAINTIES

The uncertainties of load at each bus are stochastic parameters often modeled by the normal PDF as shown in equation (1) with mean of the distribution μ, standard deviation σ and variance σ^2 [13 - 16].

$$P(X = x \mid \mu, \sigma^2) = f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad (1)$$

In other words, there are a number of states of load in each time segment. The selection of the number of states plays an important role because a small number of states may affect badly accuracy of calculations while a large number increases the complexity of the problem and calculation time. Hence, in this research, the requirement of the distribution systems is hypothesized to follow the hourly load characteristics of the IEEE-RTS using a clustering technique [17 - 18]. In each time segment, the load is divided into ten levels (states) with different probabilities to guarantee a reasonable balance between accuracy and speed as shown in Table **1**.

Table 1. Load data in states.

State	% Peak load	Probability	State	% Peak load	Probability
1	100	0.01	6	58.5	0.163
2	85.3	0.056	7	51	0.163
3	77.4	0.1057	8	45.1	0.0912
4	71.3	0.1654	9	40.6	0.0473
5	65	0.1654	10	35.1	0.033

3. THE MATHEMATICAL MODEL

The uncertainty of the electrical demand considered in this research causes an increase in the complication of the problem. The mathematical model must take an uncertain load including many states into account with the different values of load and probabilities in each state. The proposed model simultaneously examines cost for upgrading of feeders, transformer substations and investment capacitors. Furthermore, the active and reactive power flow constraints in each state are proposed to calculate power flow and losses of system in all operation states with high accuracy. The model applies binary variables to represent the cost characteristic as well as investment and upgrading decision of equipment. Details of the model are presented as follows:

3.1. Objective Function

In the study, an objective function minimizing the life cycle cost of the investment project during calculation period is proposed as written in equation (2).

$$J = Min\sum_{t=1}^{T} \frac{1}{(1+r)^t}.\left(CF_t + CT_t + CC_t + CL_t + CE_t + RN\right) \qquad \forall t \in T \quad (2)$$

where: the life cycle cost is calculated at the base year by discount rate r; the upgrading cost of feeders CF_t; the upgrading cost of transformer substations CT_t; the investment cost of capacitors CC_t; the loss cost in the capacitor itself CL_t and the energy purchased cost CE_t and residual value of equipment at the end of the reinforcement period.

The upgrading cost of feeders including fixed capital cost C_{F0} and the variable capital cost C_F is presented in equation (3) with the binary variable $\alpha_{ij,t}$ to represent the cost characteristic and decision variable for feeder upgrading $F_{ij,t}$.

$$CF_t = \sum_{i=1}^{N}\sum_{j=i}^{N} L_{ij}(C_{F0}.\alpha_{ij,t} + C_F.F_{ij,t}) \qquad \forall ij \in N, i \neq j, t \in T \quad (3)$$

where N is total number of system buses; T is total number of planning year; i and j are indices for all buses in the system; t is index for examination year.

Similarly, the upgrading cost of transformer substation CT_t including fixed capital cost C_{T0} and variable capital cost C_T is presented in equation (4) with a binary variable $\gamma_{i,t}$ to perform the cost characteristic, transformer upgrading decision variable $\Delta S^T_{i,t}$-N_T. N_T is total number of transformer substation buses.

$$CT_t = \sum_{i=1}^{N_T}(C_{S0}.\gamma_{i,t} + C_S.\Delta S^T_{i,t}) \qquad \forall i \in N_T, t \in T \quad (4)$$

The equation (5) represents a new investment cost of capacitor in year t including fixed capital cost C_{C0} and variable capital cost C_C with the binary variable $\beta_{i,t}$ to represent the cost characteristic and decision variable for capacitor investment $Q_{i,t}^{C}$ [12]. $Q_{i,t}^{C}$ is affected by voltage and determined as (10). N_C is total number of capacitor installation buses.

$$CC_t = \sum_{i=1}^{N_C} (C_{C0}.\beta_{i,t} + C_C.Q_{i,t}^C) \qquad \forall i \in N_C, t \in T \tag{5}$$

The loss cost in the capacitor itself integrated uncertainties in year t are shown in equation (6) including decision variable for investment capacitor $Q_{i,t}^{C}$.

$$CL_t = \sum_{i=1}^{N_C}\sum_{s=1}^{N_S}\sum_{h=1}^{N_H} d^s.\rho_h^P.k_C.Q_{i,t}^C \quad \forall i \in N_C, t \in T, s \in N_S, h \in N_H \tag{6}$$

where k_C is the active power loss factor of the capacitor itself; d^s is the total number of days in each season; N_H is total number of hours in each day; h is index of each hour and N_s is total seasons in a year.

The energy purchased cost CE_t considering uncertainties of loads is presented in equation (7) with active power $P_{i,t,s,h,g}$ and reactive power $Q_{i,t,s,h,g}$ purchased through transformer substation.

$$CE_t = \sum_{i=1}^{N_T}\sum_{s=1}^{N_S}\sum_{h=1}^{N_H}\sum_{g=1}^{N_G} d^s.k_\rho.\lambda_g(\rho_h^P.P_{i,t,s,h,g} + \rho_h^Q.Q_{i,t,s,h,g}) \tag{7}$$
$$\forall i \in N_T, t \in T, s \in N_S, h \in N_H, g \in N_G$$

where P_h^{P}, P_h^{Q} are active power purchased price at hour h; K_p is increase coefficient of electrical price in year t; The load probability in state g; λ_g; N_G is total number of states; g is index of each state.

The life cycle of each equipment is different from each other and the equipment is not concurrently invested so at the end of the calculation period the residual value of them is presented in equation (8). Hence, the residual value is the current value calculated at the base year in objective function.

$$RN = \frac{(t_{inv}^F - T_L^F)}{T_L^F}.CF_t + \frac{(t_{inv}^T - T_L^T)}{T_L^T}.CS_t + \frac{(t_{inv}^C - T_L^C)}{T_L^C}CC_t \qquad \forall\, t \in T \tag{8}$$

where t_{inv}^{F}, T_L^{F} are installation time and life cycle of feeder; t_{inv}^{T}, T_L^{T} are installation time and life cycle of transformer substation; t_{inv}^{C}, T_L^{C} are installation time and life cycle of capacitor.

3.2. Constraints

The technical constraints are utilized to guarantee the operability of the distribution systems and capacitor including alternating current (AC) power flow, feeder upgrading section, transformer substation upgrading capacity, limitation of bus voltage and capacitor size.

3.2.1. Constraints for Power Flow

Considering uncertainties in this research will lead to calculating the power flow for every state of load. Therefore, an AC nonlinear power flow model supplemented a state index g is written in equation (9). The effects of reactive power on the calculation of power and voltage losses are interested in the model because the accuracy of calculation results is improved.

$$\begin{aligned}
P_{i,s,t,h,g} - PD_{i,s,t,h,g} + k_C.Q_{i,s,t,h,g}^C &= \sum_{j=1}^{N}\left|Y_{ij,t}\right|.\left|U_{i,s,t,h,g}\right|.\left|U_{j,s,t,h,g}\right|.\cos(\theta_{ij,t} - \delta_{j,s,t,h,g} - \delta_{i,s,t,h,g}) \\
Q_{i,s,t,h,g} - QD_{i,s,t,h,g} + Q_{i,s,t,h,g}^C &= -\sum_{j=1}^{N}\left|Y_{ij,t}\right|.\left|U_{i,s,t,h,g}\right|.\left|U_{j,s,t,h,g}\right|.\sin(\theta_{ij,t} - \delta_{j,s,t,h,g} - \delta_{i,s,t,h,g})
\end{aligned} \tag{9}$$
$$\forall i, j \in N, s \in N_S, h \in N_H, t \in T, g \in N_G$$

where $PD_{i,t,s,h,g}$, $QD_{i,s,t,h,g}$ are active and reactive power of load i; $U_{i,\ s,\ t,h,g}$, $\delta_{i,s,t,h,g}$ are module and angle of bus voltage; $Y_{ij,\ t}$, $\theta_{ij,\ t}$ are magnitude and angle of admittance matrix element formulated by the feeder impedances in year t; $Q^{C}_{\ \ i,s,t,h,g}$ is reactive power of capacitor at each state as equation (10).

$$\begin{aligned} &Q^C_{i,t} = \max_{s,h,g}\{Q^C_{i,s,t,h,g}\} \\ &Q^C_{i,s,t,h,g} = 2\Pi f.C.U^2_{i,s,t,h,g} \\ &\forall t \ge 1, i \in N_C, s \in N_S, t \in T, h \in N_H, g \in N_G \end{aligned} \quad \textbf{(10)}$$

3.2.2. Limits on Dimension of Capacitors

The participation of capacitor changes the power flow and affects the technical and economic indicators of distribution systems. To ensure annual installation of capacity corresponding to the equipment parameters, the size of capacitor invested at each bus in each year needs to follow constraints in (11). Additionally, the planning period is short-term; therefore, at each load bus, only a capacitor system is chosen in overall the plan horizon to reduce the installation cost as shown in equation (12).

$$Q^C_{i,t} - Q_{\min}.\beta_{i,t} \ge 0; \qquad Q^C_{i,t} - Q_{\max}.\beta_{i,t} \le 0 \qquad \forall t \ge 1, i \in N_L, t \in T \quad \textbf{(11)}$$

$$\sum_{t=1}^{T} \beta_{i,t} \le 1 \qquad \forall i \in N_L, t \in T \quad \textbf{(12)}$$

where Q_{max}, Q_{min} are maximum and minimum capacity limit of capacitor; $\beta_{i,t}$ is binary variable.

3.2.3. Upgrading Section Constraints of Feeder

The power carrying capability of feeders is imposed by thermal limits taken into the consideration of new feeder investments. The constraints for feeder upgrading are represented in equation (13) with capacity supplemented at each year, $\Delta S_{ij,t}^{\ F}$. $\Delta S_{ij,t}^{\ F}$ is set when the capacity value is greater than the capacity limit used at year *t-1*. Thus, the feeder capacity needs to be upgraded at year t to meet in order to supply power of the load as equation (14).

$$\begin{aligned} &S^{\max}_{ij,t} \le (S^{*F}_{ij,t-1} + \Delta S^F_{ij,t}); \Delta S^F_{ij,t} \ge \Delta S^F_{\min}.\alpha_{ij,t}; \qquad \Delta S^F_{ij,t} \le \Delta S^F_{\max}.\alpha_{ij,t} \\ &\forall t \ge 1, ij \in N, t \in T \end{aligned} \quad \textbf{(13)}$$

$$S^{*F}_{ij,t} = S^{*F}_{ij,t-1} + \Delta S^F_{ij,t} \qquad \forall t \ge 1, ij \in N, t \in T \quad \textbf{(14)}$$

where $\Delta S_{max}^{\ F}$, $\Delta S_{min}^{\ F}$ is maximum power running in feeder at year t; $\Delta S_{max}^{\ F}$ are maximum and minimum capacity limit added of feeders; $S_{ij,\ t}^{\ *F}$ is standard size of feeder and $\alpha_{ij,t}$ is binary variable.

3.2.4. Addition Capacity Constraints for Substation

In order to use the maximum of existing substations capacity and satisfy upgrading power corresponding to the equipment parameters. The constraint (15) is utilized to set substation sizes added in year t, $\Delta S_{i,t}^{\ S}$, with binary variable $\gamma_{i,t}$.

$$\begin{aligned} &S^{\max}_{i,t} \le (S^{*S}_{i,t-1} + \Delta S^S_{i,t});\ \Delta S^S_{i,t} \ge \Delta S^S_{\min}.\gamma_{i,t}; \qquad \Delta S^S_{i,t} \le S^S_{\max}.\gamma_{i,t} \\ &\forall t \ge 1, i \in N_T, t \in T \end{aligned} \quad \textbf{(15)}$$

where $S_{max}^{\ S}$, $\Delta S_{min}^{\ S}$ are capacity limit added of transformer substation; $S_{i,t}^{\ *S}$ is standard size and $S_{i,t}^{\ max}$ is maximum capacity running in the transformer at year t.

3.2.5. Constraints on Bus Voltages

The distribution systems are connected to the transmission systems by transformer substations so the voltages at substation bus are usually stabilized and assumed to be constant. At load buses, the voltages are usually changed according to parameters of the equipment and the value of the loads. Thus, the limits on load bus voltage are shown in equation (16).

$$U_{\min} \leq \left|U_{i,s,t,h,g}\right| \leq U_{\max} \qquad \forall i \in N_L, s \in N_S, t \in T, h \in N_H, g \in N_G \tag{16}$$

where N_L is total number of load buses; U_{min}, U_{max} are maximum and minimum limit of bus voltage.

The proposed model is a mixed integer nonlinear programming (MINLP) calculated in GAMS environment with BONMIN solver to find out an optimal solution [36].

4. CALCULATIONS AND DISCUSSIONS

4.1. Parameters of Distribution Systems and Assumptions in Analysis

The IEEE 9-bus radial structure with 22kV connected to grid by transformer substation is utilized to investigate the proposed model in Fig. (**1**) [32, 33]. Parameters changed to match the problem are presented in appendix. The total active power and reactive power at the base year are 15,024.2kW and 10,641.8kVAr, respectively. The hourly load demand of four seasons is computed by fact of peak demand as shown in Fig. (**2**). The uncertainties of load at each bus are stochastic parameters as introduced in section 2.

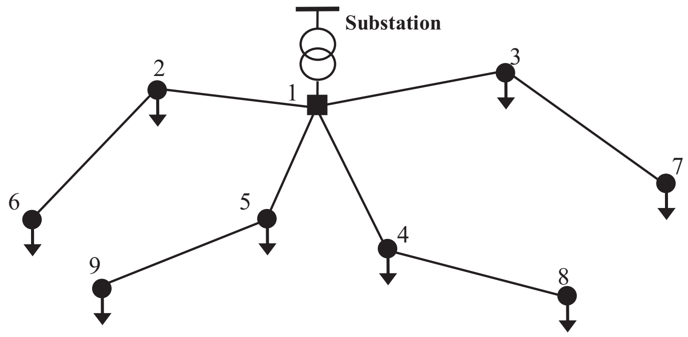

Fig. (1). Diagram of the test distribution system.

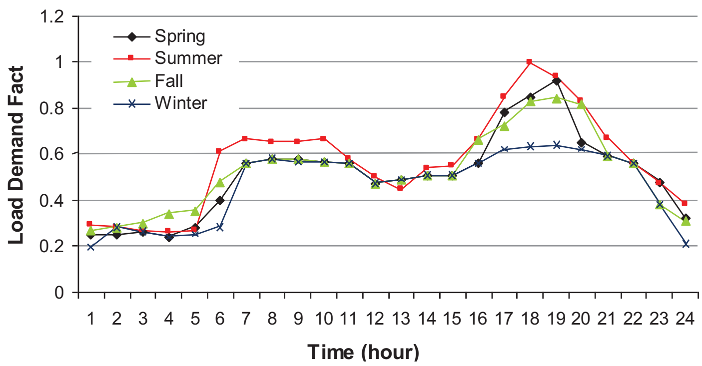

Fig. (2). The hourly load demand for four seasons.

The following assumptions are utilized in this research including:

- The calculation period is selected to be 5 years with the annual developing rate of demand 10% per year. The typical characteristics of demand at the entire load are set to be the same as in Fig. (**2**). The electrical energy price is TOU price as shown in Fig. (**3**) [20, 21, 34].
- The fixed and variable upgrading costs of transformer substation are 0.2M$ and 0.05M$/MVA, respectively. Similarly, the upgrading costs of 22kV feeders consist of 0.15M$/km and 0.001M$/MVA.km. The assumption lifetime of transformer substation and feeder is 20 years. The areas for upgrading of substation transformers and feeders are assumed to be unlimited.
- The fixed and variable investment cost of capacitors assumed in this research are 1000$ for each installed system and 20$/kVAr as shown in [7, 11, 12]. The active power loss factor of the capacitor itself k_c is

determined 0.02W/kVAr and the life of capacitor are 20 years [35]. The capacitors can be selected to install at all of load locations because of quick installation and small spaces of occupation.

- The voltage at load buses is allowed to change from 0.9pu to 1.1pu in order to guarantee the operation of equipment. At the substation bus, the voltage usually is stabilized and assumed to be 1.05pu.

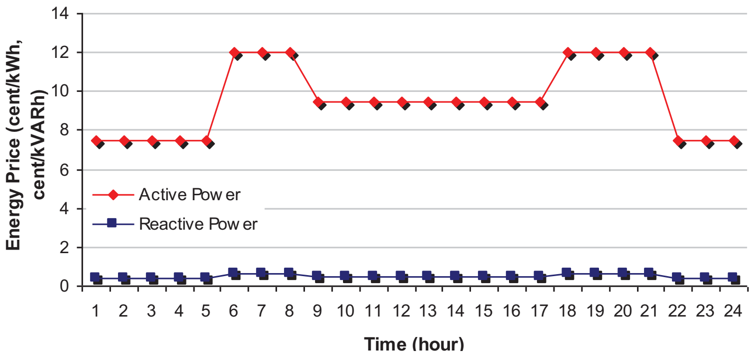

Fig. (3). Energy prices.

Table 2. Calculated Cases.

Case	Certainty	Uncertainty	Capacitor
1	x		
2		x	
3	x		x
4		x	x

4.2. Results

The efficiency of proposed model is investigated by four cases as listed in Table **2**. The plan of distribution systems is undertaken to determine the upgrading capacity and time of feeders, substations with fixed and stochastic load in each hour in case 1 and case 2, respectively. The influence of capacitor to distribution system planning is calculated in case 3 that system parameters are similar case 1. In case 4, the stochastic demand is simultaneously considered with participation of capacitor in planning problem.

The calculation results with the above assumption parameters determined upgrading time and capacity of feeders and substations as well as investment time and capacity of capacitor as shown in Table **3**. All of four cases, the substation capacity is selected 10MVA but the upgrading time is different. The transformer substation is upgraded at 4th year in case 1 and 2 while case 3 and 4 is deferred to 5th year because of the capacitor reducing power flow in this case. Similarly, in cases 1 and 2, feeders 1-2 and 1-3 need to be upgraded 5th year with capacity 8MVA while remaining feeders are not upgraded during computation period. Because of the support of capacitor in case 3 and case 4, the feeder 1-3 is deferred and is not upgraded in both cases while the only feeder 1-2 is upgraded at case 4.

Table 3. The investment and upgrading decided of equipment.

No	Equipment	Bus	Base parameter	Upgraded or Invested Time (year)														
				1	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*
				Case 1, 2					Case 3					Case 4				
1	Capacity upgraded of feeder i-j (MVA)	1-2	6.67					8										8
		1-3	6.67					8										
2	Capacity upgraded of transformer substation i (MVA)	1	10				10						10					10

(Table 3) contd.....

No	Equipment	Bus	Base parameter	Upgraded or Invested Time (year)														
				1	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*
				Case 1, 2					Case 3					Case 4				
3	Capacity invested of Capacitor i (MVAr)	2	0										0.5					
		3											0.5				0.5	
		6	0										0.5					
		7	0						0.5									0.5

Table 4. The investment and upgrading decided of equipment.

No	Cost	Case 1	Case 2	Case 3	Case 4
1	Total life cycle cost (M$)	32.46	20.61	32.22	20.03
2	Feeder and substation upgrading cost (M$)	1.97	1.97	0.53	0.98
3	O&M and electrical energy cost (M$)	32.36	20.03	32.16	19.98
4	Investment capacitor cost (M$)	0	0	0.09	0.04

In this research, the effect of investment project is evaluated by life cycle cost during examination period. The calculation results in Table **4** show that the efficiency of distribution systems is always enhanced with participation of capacitor. The life cycle cost of project in case 3 in comparison with case 1, the fixed load parameters, reduces 0.24M$, the corresponding 0.74%. Similarly, the life cycle cost of project for investment the capacitor in case 4 compared with case 2, the uncertainties load parameters, decreases 0.58M$ in proportion to 1.8%. The cost for investment and upgrading equipment also reduces from 1.97M$ of case 2 to 1.02M$ of case 4. Moreover, stochastic load with many states and different probabilities considered leads to reduce the electrical energy purchased from grid. The comparison between case 1 and case 2 shows that the energy cost reduces to 12.33M$, the corresponding 38.10%. Similarly, the energy cost also decreases to 12.18M$ in proportion to 37.87% when the case 4 is compared with case 3.

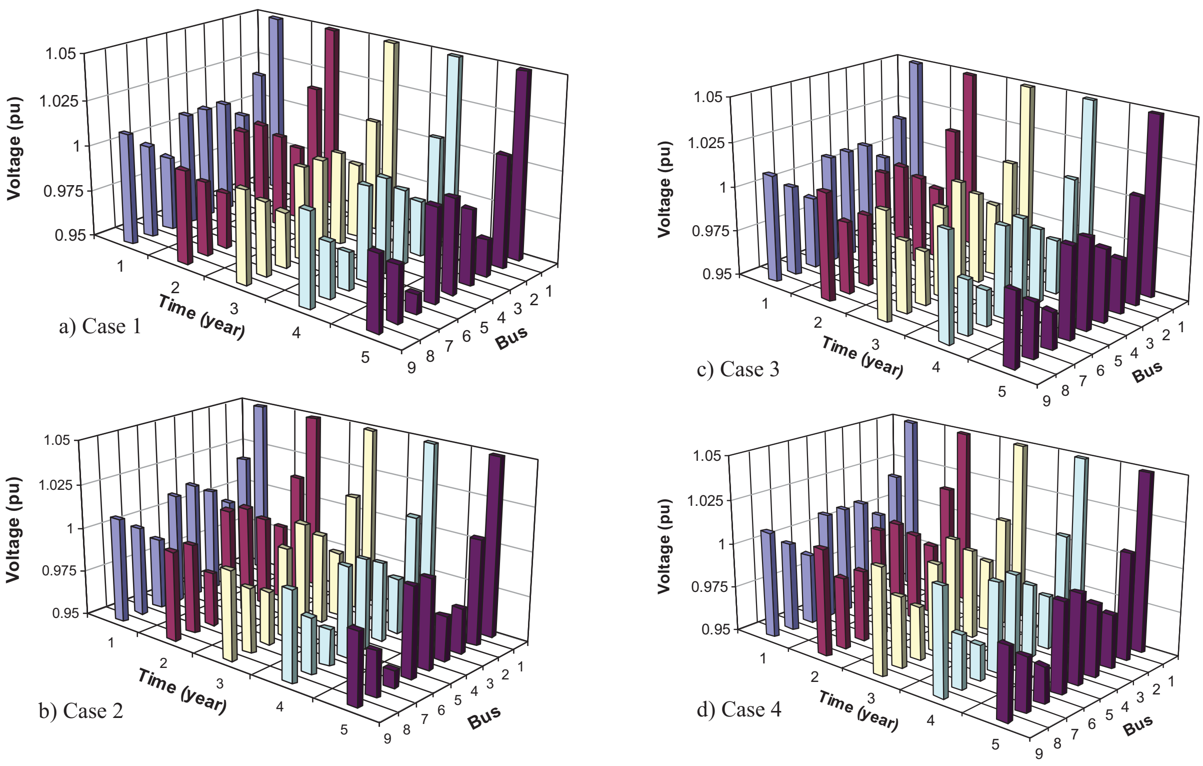

Fig. (4). Minimum voltage at buses.

The maximum power loss is determined at state that the demand is maximization present as shown in Fig. (**5**). In cases 1, 2 and 4, this value is 4.56% at the 1st planning year and increases 0.51% per year at next year depending on the rise of load demand. However, at the 5th year, the maximum power loss is 6.43% and only increases 0.33% compared with the 4th year in case 1 and case 2. This decrease is made due to feeders 1-2 and 2-3 upgraded decrease the resistors of the distribution system. In case 4, this loss is only 6.23% and increases 0.13% compared with the 4th year because the capacitor invested reduces power flow of feeders. Similarly, the capacitor installed in case 3 supported power loss at just 1st year with decrease 0.21% in comparison with case 1. The maximum difference of maximum power loss is 0.42% at the 5th year.

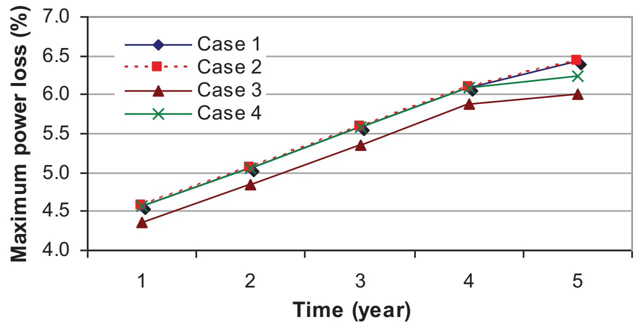

Fig. (5). Maximum power loss.

Moreover, the electrical energy loss always reduces during calculation period when considering to capacitor and uncertainties of loads as represented in Fig. (**6**). The electrical energy loss in case 3, the capacitor invested, is reduced 0.11% at just the 1st year in proportion to 7609.47kWh. This value increases 0.02% per year in 2nd, 3rd and 4th year. Nevertheless, it only is 0.13% in the 5th year because the feeders upgraded of case 1 decrease the resistors of the distribution system. Similarly, the capacitor installed at 4th year in case 4 decreases 0.05% of energy loss in proportion to 2150.05kWh compared with case 2. In 5th year, the capacity of capacitor is added 0.5MVAr so the electrical energy loss reduces 0.1%, the corresponding 4738.10kWh. Besides, the electrical energy loss in case 2 in comparison with case 1 always reduces from 1.02% to 1.78%. The maximum difference when the comparison between case 4 and case 3 is 1.66%. This result is made because of the important effect of the stochastic demand with less probabilities and load demand fact.

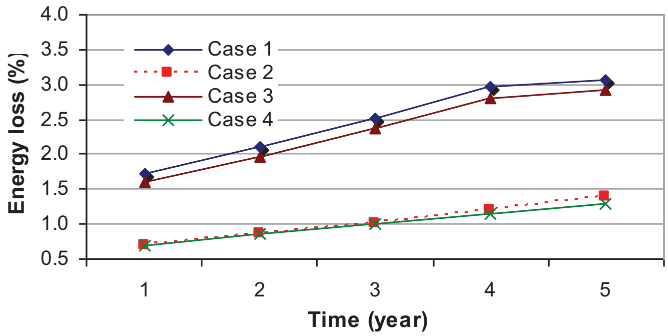

Fig. (6). Total energy loss of distribution system.

CONCLUSION

This paper presents a generalized formulation of distribution system reinforcement integrating the capacitor and uncertainties of loads. The problem is scheduling the upgrading equipment of distribution systems (feeders and transformer substation) together with installation capacitor to minimize life cycle cost of the investment project. The proposed model is based on mixed integer nonlinear programming (MINLP) with binary variables utilized to perform the nonlinear cost characteristics and installation size of the equipment. The proposed formulation is verified by the IEEE 9-bus radial structure with uncertainties of loads and TOU prices. The result of simulation shows that a significant saving of life cycle cost can be achieved in comparison with traditional schemes. Only upgrading of the feeders and transformer substation is demanded. The reduction of power loss and electrical energy loss as well as upgrading delay of equipment in during calculating period could be seen as an effect of capacitor and stochastic load to distribution system reinforcement. Moreover, the voltage at all buses is always ensured in technical requirements in all operation modes.

APPENDIX

See Table (**5** and **6**).

Table 5. Data of Loads.

No	Bus	PD0 (kW)	QD0 (kVAr)
1	1	-	-
2	2	2551.0	1857.7
3	3	2497.7	1935.1
4	4	2656.8	1616.6
5	5	2570.4	1542.5
6	6	1425.6	1110.2
7	7	1182.2	926.9
8	8	1041.8	798.2
9	9	1098.7	854.8
	Total	*15024.2*	*10641.8*

Table 6. Data of Feeder Parameters.

No	Bus i - Bus j	Smax.ij (MVA)	Lij (km)	Rfij (W)	Xfij (W)
1	1-2	6.67	3.3	2.55	1.42
2	1-3	6.67	6.2	4.79	2.66
3	1-4	6.67	5.3	4.10	2.27
4	1-5	6.67	4.5	3.48	1.93
5	2-6	6.67	2.7	2.09	1.16
6	3-7	6.67	3.2	2.47	1.37
7	4-8	6.67	4.2	3.25	1.80
8	5-9	6.67	2.2	1.70	0.94

CONSENT FOR PUBLICATION

Not applicable.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

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