# Numerical Modelling of Cylindrical Monopole Plasma Antenna Excited By Surface Wave

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**Abstract:** In the recent research on cylindrical monopole plasma antenna excited by surface wave, many meaningful results about plasmas antenna, e.g. the analysis of physical characteristics, numerical calculation methods, software simulation, diagnosis of parameters have been obtained. But the precise model of monopole plasma antenna according to its three-dimensional structure has not been proposed yet. There are few papers concerning this topic in previous academic journals. This paper analyzes three-dimensional distributions of electric and magnetic fields around monopole plasma antenna. The related formulas and equations of the model are derived by applying molecular dynamic theories and Maxwell-Boltzmann equations. In addition, numerical simulation methods are employed to verify the validity of proposed model.

Keywords: Cylindrical monopole plasma antenna, calculation, numerical modeling, surface wave.

## **1. INTRODUCTION**

In the present study of cylindrical monopole plasma antenna excited by surface wave, its precise calculation model has not been established yet. Experiments performed before have verified that monopole plasma antenna has many properties similar to metallic antenna. However, there are obvious differences between plasma antenna and the metallic one, with the state of plasma being unstable. The density presents linear variation or nonlinear distributions and the parameters of plasma antenna are also variable. So, the modeling methods on the research of monopole plasma antenna are very necessary on its further application research.

Some basic experiments on plasma antenna have already been proposed to aid in understanding of plasma antenna. The researchers of the plasma antenna laboratory in Australian National University have achieved some physical research and characteristics analysis of plasma antenna [1-3].

A.D. Cheetham studied the parameters of plasma antenna and the control system of plasma antenna excited by RF spiral wave energy. G.G. Borg researched the characteristics of monopole plasma antenna, such as the radiation efficiency, signal amplitude and noise amplitude. G.G. Borg and J.H. Harris' research work indicated that it has a wide varying range for the electrical parameters of plasma antenna. So, monopole plasma antenna was a good candidate for RF communication application [1]. They have verified that when the excited power is variable, current distribution of plasma antenna will change according to the excited power. J.P. Rayner and M. Hargreave presented the experimental results of plasma density distribution of plasma antenna, and radiation pattern in their review articles [4-7]. The experimental investigations show that the reduced of radiation efficiency is due to the lower conductivity of plasma column. The loss can be easily made up through boosting the propagation wave power. The cone shaping distribution of density and conductivity of plasma antenna have little impact on its radiation capability.

The preliminary and basic model research results have been given out in the academic journals [8-10]. However, the modeling method is too simple in articles mentioned above, the further model research is essential and urgent for the plasma antenna.

The study of the regularity and abnormal characteristics of plasma antenna are carried out by using modeling method in this paper. Only when the relationships between particle motion and properties of plasma antenna are obtained, can we get a thorough understanding of the monopole plasma antenna. The modeling research of the plasma antenna is to form a method, which can predicate the state of plasma antenna. The relationships between the electron motion and plasma parameters also can be obtained through proposed model. The analysis of three-dimensional model based on the molecular dynamics theory and Maxwell-Boltzmann equations is also presented in our paper. The average statistical results of the particle motion can be obtained using statistical physics method and mathematical tools.

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Fig. (1). The cylindrical monopole plasma antenna excited by surface wave.

### 2. ANALYSIS OF ELECTRICMAGNEIC FIELD

In order to obtain the precise analysis of plasma distribution, the cylindrical coordinate should be employed according to the characteristics of excitation power. The macroscopic collision and microscopic collision are taken into consideration in proposed model method, respectively. The calculations of density distribution of plasma, dielectric coefficient and electric conductivity of the plasma antenna can be achieved.

Fig. (1) illustrates the cylindrical RF coupling structure. In order to simplify the discussion process, the assumptions are made as follows. r represents the radius of antenna; l represents the length of plasma antenna; n is the number of the RF coil; i denotes the RF current and  $\omega$  denotes frequency of excited power.

The cylindrical monopole plasma antenna excited by surface wave is illustrated in Fig. (1). In three-dimensional coordinate system, the electric field and the inductive magnetic field are cylindrical symmetrical distribution.

The electric field can be expressed as equation (1).

$$E_{rf} = [E_r(a,z)e_r + E_z(a,z)e_z]e^{-i\omega r}$$
(1)

The inductive magnetic field is expressed as equation (2).

$$B_{rf}(a,t) = \left[B_r(a,z)e_r + B_z(a,z)e_z\right]e^{-j\omega t}$$
<sup>(2)</sup>

The boundary condition of the equations (1) and (2) are shown as equations (3) and (4).

$$\begin{cases} E_r(a,0) = E_a(z) \\ E_r(\infty,0) = 0 \\ E_z(a,l) = E_a(z) \cdot \alpha \\ E_z(a,\infty) = 0 \end{cases}$$
(3)

$$\begin{cases} B_z(a,0) = B_a(z) \\ B_z(\infty,z) = 0 \\ B_r(a,l) = B_a(z) \cdot \beta \\ B(a,\infty) = 0 \end{cases}$$
(4)

 $E_a(z)$  is magnetic field and  $B_a(z)$  is electric field respectively in the boundary of the plasma antenna,  $E_a(z)$  and  $B_a(z)$  are determined by the RF electric field. Here, *a* represents the variable distance,  $\alpha$  and  $\beta$  are the attenuation factors of the magnetic and electric fields around the plasma antenna,  $\alpha$  and  $\beta$  are determined by the excitation power, the coils can be regarded as a series of current sources, as for electric field,  $E_r(a,z)$  presents the distribution of Bessel functions.

When the plasma frequency and collision frequency are different, the surface current distribution will be different, which will result in different distribution of the electric field. The surface current distribution presents attenuation distribution with low plasma frequency and high particle collision frequency. With increase of the plasma frequency and reduction of the plasma collision, the axial current presents cyclical alternation, and then axial electrical distribution of the plasma antenna can be obtained.

The physical parameters such as conductivity and the permittivity are determined by the movement of electron and ions in the plasma. The numerical values of physical parameters, such as the conductivity and the dielectric factor, can be calculated by using the Boltzmann-Maxwell equations.

### **3. FORMULATION AND ANALYSIS**

When the plasma is in low temperature state, the velocity distribution function of the electron satisfies the Boltzmann Equation. We assume that electrons in the plasma antenna satisfy the Boltzmann equation as below.

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{F}{m} \frac{\partial f}{\partial v} = S(f,q)$$
(5)

f = f(x,v,t) represents the electron velocity function, x = (x,y,z) represents the electron position vector;  $v = (v_x, v_y, v_z)$  represents the electron velocity vector; *t* represents time parameters;  $\vec{F}$  represents sum of the force; *m* represents the quality of the electron; S(\*) represents the distribution function caused by collisions; *q* represents velocity distribution function of electron in the equilibrium state; *f* represents the velocity distribution function of electron in the non-equilibrium state.

The forms of the collision integral are very complex, and the numerical calculation is very difficult to perform. If the distribution functions are expanded into power progression by using variation principle, collision integral can be calculated under certain approximate extent. If the former two terms of the progression are adopted in the calculation process, the complex calculation can be avoided; the linearization equation can be expressed as (6).

$$f = f_0 + f_1 \tag{6}$$

The Boltzmann equations can be written as (7).

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} - \frac{eE_f}{m_e} \cdot \frac{\partial f}{\partial v} = S(f)$$
<sup>(7)</sup>

$$f(r,v,t) = f_0(\varepsilon) + f_1(r,v)e^{-i\omega t}$$
(8)

 $f_0(\varepsilon)$  is electron energy distribution function (EEDF);  $f_1(r,v)$  represents the disturber distribution caused by the outer energy field,  $|f_1| << |f_0|$ . By substituting equation (6) to equation (5), the interference distribution can be obtained.  $f_1(r,v)$  satisfies equation (9).

$$(v - i\omega)f_1^{\pm} \pm v_r \frac{\partial f_1^{\pm}}{\partial x} = ev_{\varphi}E_{\varphi}\frac{df_0}{d\varepsilon}$$
(9)

Where the sign  $\pm$  denotes  $v_r > 0$ ,  $v_r < 0$ , the *r* and  $\varphi$  respectively represent the direction of *r* and  $\varphi$  in the cylindrical coordinate system;  $v_r$  and  $v_{\varphi}$  represent the velocity in the direction *r* and  $\varphi$ ;  $\varepsilon = mv^2/2$  represents the kinetic energy of the electron. In order to simplify the calculation, the new variable function (10) is used here.

$$\begin{cases} F^{+} = \frac{1}{2}(f_{1}^{+} + f_{1}^{-}) \\ F^{-} = \frac{1}{2}(f_{1}^{+} - f_{1}^{-}) \end{cases}$$
(10)

Then the equations (11) and (12) are obtained.

$$\begin{cases} (v - iw)F^{*} + (v_{x}\frac{\partial}{\partial x} + \omega_{c}\frac{\partial}{\partial \phi})F^{-} = \frac{eE_{y}}{m_{c}}\frac{\partial f_{0}}{\partial v_{y}} \\ (v - i\omega)F^{-} + (v_{x}\frac{\partial}{\partial x} + \omega_{c}\frac{\partial}{\partial \phi})F^{*} = 0 \end{cases}$$
(11)

$$f_1(r, z.v, t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left[ \frac{ev_{\varphi} E_{nk} \sin(k_n r) \sin(k_k z)}{v - i\omega + ik_n v_r} \right] \frac{df_0}{d\varepsilon} e^{-i\omega t}$$
(12)

Where

$$k_n = n\pi/2r$$
,  $E_{nk} = \int_{-r}^{r} \int_{0}^{h} [E_{\varphi}(r,z)\sin(k_nz)]drdz$ 

If the disturb function is obtained, the density current can be worked out.

$$j_{\varphi}(r,z,t) = -\frac{1}{2\pi} \left(\frac{m_e}{2}\right)^{\frac{3}{2}} en_0 \int dv f_1(r,z,v,t)$$
  
$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left[\sigma_{nk} E_{nk} \sin(k_n r) \sin(k_k z)\right] e^{-i\omega t}$$
(13)

The surface current and electric field distribution can be worked out by means of the proposed model. The conductivity in the certain part of plasma antenna in Fourier space can be obtained, then, dielectric constant and magnetic conductivity of whole plasma antenna can be worked out through integral calculation. The EEDF can also be obtained using the proposed equations above. By using the proposed model, the surface current distribution of the plasma antenna can also be worked out. Thereby, the parameters such as the radiation pattern and gain can be obtained as well [11].

$$v_c = 1.52 \times 10^7 P_{\rm v} T_e$$
 (14)

The relationship between the collision frequency and the gas pressure is written as (14), where  $v_c$  represents the collision frequency, *P* represents the inner pressure of the plasma antenna and the  $T_c$  represents the electron temperature of the plasma antenna. With increase of the pressure, the collision frequency of plasma will be increased.

$$f_p = \frac{\omega_p}{2\pi} \approx 8.98 \sqrt{N[m^{-3}]}$$
(15)

The relationship between the plasma frequency and the density of the plasma is shown in the equation (15), where  $f_p$  represents the frequency of the plasma;  $\omega_p$  represents the angle frequency of the plasma; *N* represents the density of the plasma which is governed by the pressure and excitation power [12, 13].

Under the nonmagnetic condition, the relationship between the excitation frequency and the conductivity of plasma antenna is shown in equation (16),  $\sigma$  represents the conductivity of the plasma [14].

$$\sigma = \frac{\varepsilon_0 \omega_p^2}{(j\omega + v_c)} \tag{16}$$

It is known that the conductivity of plasma changes according to the frequency of excitation field, the permittivity will also change with the variation conductivity shown as the (17), where  $\sigma_{ij}$  and  $\mathcal{E}_{ij}$  represents conductivity and the permittivity of the plasma antenna in certain state, separately.

$$\left\|\boldsymbol{\sigma}_{ij}\right\| = j\omega\boldsymbol{\varepsilon}_{0}\left(\left\|\boldsymbol{\varepsilon}_{ij}\right\| - \left\|\boldsymbol{I}\right\|\right) \tag{17}$$

The radiation pattern of monopole plasma antenna is determined by its surface current distribution, the physical parameters, such as the conductivity, dielectric constant, surface current of the plasma antenna are all dominated by the EEDF of the plasma antenna [14, 15].

#### 4. NUMERICAL SIMULATION AND DISCUSSION

The EEDF reflects the state of plasma and governs the parameters of plasma antenna. In this section, the relationship between the EEDF and the excited parameters of the plasma antenna are discussed. The numerical simulation method of this model is adopted to predicate and analyze the state of plasma antenna.

In the experiments, the discharge gas is composed of hydrogen, plasma density is about  $1.5 \times 10^{20}$  / cm<sup>3</sup>, *R* is radius of monopole plasma antenna, *L* represents antenna length and the internal pressure of plasma antenna is different, *n* is the number of RF coils and external excitation RF power is controllable.

Firstly, the analysis of the effect of excitation power on the EEDF of the plasma is made, Fig. (2) illustrates the influence of gas pressure on the EEDF, the frequency of excited power is 13.56 MHz and the gas pressure is variable.



Fig. (2). The influence of gas pressure on EEDF, other parameters: R = 1 cm, L = 10 cm, I = 10 A, n = 1, F = 13.56 MHz



Fig. (3). The influence of driving frequency on EEDF, other parameters: R = 1 cm, L = 10 cm, I = 10 A, n = 1, P = 15 pa



Fig. (4). The influence of height of the plasma antenna on the EEDF, other parameters:  $\omega = 2\pi \times 13.56 MHz$ , p = 15Pa, I = 10A, n = 1 R = 2cm



Fig. (5). The influence of radius of the plasma antenna on the EEDF, other parameters:  $\omega = 2\pi \times 13.56$  MHz, p = 15Pa, I = 10A, n = 1, L = 10cm.



Fig. (6). The influence of RF current amplitude of the plasma antenna on the EEDF, other parameters:  $\omega = 2\pi \times 13.56 MH_z$ , p = 15Pa, R = 1cm, L = 10cm.



Fig. (7). The influence of turns number of the plasma antenna on the EEDF, other parameters:  $\omega = 2\pi \times 13.56 MH_z$ , p = 15Pa, R = 1cm, L = 10cm.



Fig. (8). The experiment results of surface current of the plasma antenna.



Fig. (9). The model calculation results of surface current of the plasma antenna.



Fig. (10). The experiment results of electric field around the plasma antenna.



Fig. (11). The calculation results of electric field around the plasma antenna.

Fig. (3) shows the effects on different frequencies, it can be seen that with the decrease of gas pressure and increase of discharge frequency, the number of high-energy electrons increases gradually. The distribution function of electronic energy presents Maxwell distribution and these results are consistent with the related plasma research conclusions.

The influence of length and radius of plasma antenna on the electronic energy distribution function are as shown in Fig. (4) and Fig. (5), it can be found that with the length or the radius of plasma antenna increased and other parameters are fixed, the electron energy will be reduced simultaneously and the effects of radius are more apparent than the effects of plasma length. So the radius of plasma antenna has a greater influence on the plasma state, and it has been verified through related experiments of the plasma antenna.

The strength of current in the RF coil reflects the external excitation power directly, and it determines the power coupled into plasma antenna as shown in Fig. (6) and Fig. (7), the EEDF changed with the strength of current in the coil. When the current of the coil and the number of coil increased, the electron energy will increase accordingly, because the increased excitation power will leads to the increase of absorption power in plasma. Therefore, the state of plasma can be controlled by changing the number of coils and RF currents in the coupling coils.

The measured results and calculation results of the plasma antenna are shown in Fig. (8), Fig. (9), Fig. (10) and Fig. (11), the measurement results of the plasma antenna

experiment are shown in Fig. (8) and Fig. (10). From these figures, we can see that the measurement results have the same tendency as the numerical simulation results of the three-dimensional model. If we compare the numerical simulation results with the measured results in the Fig. (8), Fig. (9), Fig. (10) and Fig. (11), we can conclude that the proposed three-dimensional model can be used in the control and predication of the monopole plasma antenna.

#### **5. CONCLUSIONS**

In this paper, EEDF of cylindrical monopole plasma antenna excited by surface wave has been analyzed. The relationships between EEDF and gas pressure, antenna size, coupling method and external excited energy are discussed by using numerical simulation method. The numerical results and experimental results show that the proposed model method can predict and analyze the characteristics of monopole plasma antenna. The calculation results of the model can reflect the electromagnetic field distribution around the antenna and the distribution characteristics of power density and surface current.

The numerical model proposed in the paper can present a qualitative guidance for the plasma antenna research, but the electrostatic field distribution in the plasma and the effect of sheath layer around inside the antenna are not taken into consideration. So if the researchers want to make more precise analysis and prediction of the plasma antenna, a more reasonable and mixed model should be put forward. None declared.

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