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The Open Electrical & Electronic Engineering Journal, 2015, *9*, 107-116

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Differentiation Coherence Algorithm for Steady Power Flow Control

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Abstract: This paper researched steady power flow control with variable inequality constraints. Since the inverse function of power flow equation is hard to obtain, differentiation coherence algorithm was proposed for variable inequality which is tightly constrained. By this method, tightly constrained variable inequality for variables adjustment relationships was analyzed. The variable constrained sensitivity which reflects variable coherence was obtained to archive accurate extreme equation for function optimization. The hybrid power flow mode of node power with branch power was structured. It also structured the minimum variable model correction equation with convergence and robot being same as conventional power flow. In fundamental analysis, the effect of extreme point was verified by small deviation from constrained extreme equation, and the constrained sensitivity was made for active and reactive power. It pointed out possible deviation by using simplified non-constrained sensitivity to deal with the optimization problem of active and reactive power. The control solutions for power flow for optimal control have been discussed as well. The examples of power flow control and voltage management have shown that the algorithm is simple and concentrated and shows the effect of differential coherence method for extreme point analysis.

Keywords: Inequality constraint, constrained sensitivity, power flow control, optimal power flow (OPF).

1. INTRODUCTION

The steady-state power flow control is the analysis about the power flow control by use of adjustable capacity of AC power. The voltage quality and voltage security are a concentrated expression of the inequality condition of the node voltage, where the economical operation is presented as the operating optimization objective. The steady-state power flow control is a power flow problem with operating conditions, also considered as the basis of analysis for discrete or discontinuous variables, which can be described [1-2]. Optimal Power Flow belongs to the steady-state power flow control problem, and the basic form is:

$$\min \quad z=f(x,y) \tag{1-1}$$

$$s.t \quad g(x,y)=0 \tag{1-2}$$

$$x_{\min} \le x \le x_{\max} \tag{1-3}$$

$$y_{\min} \le y \le y_{\max} \tag{1-4}$$

$$y \in R^m, g \in R^n, x \in R^n.$$

In 1960s, Carpentier proposed the description of OPF with mathematical equations. He then summarized all the engineering problems into a form of theoretical research and promoted various researches concerning the solution methods for extreme point equations [3-6]. In 1968, simplified gradient method which was proposed by Dommel and Tinney for solving OPF cleared the direction of the continuous variable optimization solution, and established the basic form of the equation with extreme conditions in (1-1), the simultaneous equation forms of equality constraint in (1-2), and the approach of the control variable inequality for the domain in (1-4) [3, 6]. In 1948, Sun, Tinney *et al.* [7] established that the large-scale OPF problems have practical value, through the application of Newton method in quadratic convergence and sparse technology to solve the equation of extreme points [5, 8].

Soon afterwards, research and trial of multi-aspects including the penalty function towards inequality constraints have been made.

With the development of optimal theory, in 1986, Karmarkar proposed the interior point method. The method is suitable for solving large-scale, multi-variable constraint problems. In 1991, Clements [9] and Quitana [10] proposed the theory of using interior point method to deal with inequality constraints in the power system, maintaining the variables in feasible region and making variable corrections with rules, with the convergence being far superior to the simple form of algorithm [9, 11].

Interior point method can effectively handle the calculation problems of tight constraints like (1-3), involving other problems in (1), and promote the development and application of the OPF problem [12, 13].

With the development of OPF, some challenging problems [14] which were proposed by Momoh in 1997 were still worth considering, e.g.: How to improve the transparency of the optimization, so that we can find the direction and measures of power system optimization.

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The problems of (1) involve complex nonlinear systems analysis of inequality constraints containing both feasibility and optimization [14]. Compared with the conditional extreme problem of equality constraints, the focus problems caused by inequality and tight constraints of the variable x in (1) are: How to determine the value of y, when $x_i = x_{max}$, or $x_i = x_{min}$, or x_i are constant? How to seek the solution for the mixed variables. Meanwhile the value of x and y are both required to be found out? What kind of constraints sensitivity can be measured and used in the extreme condition equation when x is under the condition of tight constraints.

For the above questions, the differential analysis method has been used in this paper. Under the demand to determine y from x, which is required by tight constraints, and with the differential relationship of dx and dy, dy can be determined by dx. With the method that the definite value of y_1 meets the inequality constraints demand of x, the constraint sensitivity reflects the differential co-ordination that can be acquired. Then the problem for power flow caused by tight constraints on the variable x are analyzed and solved. Only the differential analysis can be left incomplete for the correctness and reasonableness of the analysis. An example for illustration and verification has been provided.

2. DIFFERENTIATE COHERENCE METHOD

The Differential Correlation Method used to solve the tight constraint problems for x is:

1) According to $g(x, y) = 0$, incidence matrix D for dx and dy shall be

$$dg = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy = 0 \tag{2}$$

Setting $dy_i = 1$, $dy_j = 0$ (i=1, 2, … , m,), solving successively:

$$\frac{\partial g}{\partial x}dx^i = -\frac{\partial g}{\partial y}\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \tag{3}$$

The following can be obtained:

$$\frac{\partial x}{\partial y} = \begin{bmatrix} \frac{\partial x}{\partial y_1} & \frac{\partial x}{\partial y_2} & \cdots & \frac{\partial x}{\partial y_m} \end{bmatrix} = \begin{bmatrix} dx^1 & dx^2 & \cdots & dx^m \end{bmatrix} = D$$

D is the unconstrained sensitivity matrix and the rate of change for the system status x to the control variable y, and reflects the inherent characteristics of the system.

Thus, the relationship of dx and dy is

$$dx = Ddy \qquad x \in R^n, y \in R^m$$

2) Determining Δy_1 which is required to balance Δx_1,

Setting $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$,

Where x_1 is a tight constraint variable, and x_2 is a loose constraint variable. y_1 is a variable that needs to make corresponding control in order to make the definite value for x_1. y_1 is conditional control variable, and y_2 is optimal control variable which can be used to optimize the objective.

When the tight constraints of x_1 meet the optimization objective and y_1 makes an optimal effect, then y_2 is optimized better and vice versa.

With the same meaning as the KKT equations, the solution of (1) is to make constraint condition equations, optimal extreme condition equations and power flow equation, $g(x, y) = 0$, is simultaneous equation through which the variables x and y can be solved. The quantity of the simultaneous equations is the same as the quantity of variables x and y. The optimization for extreme conditions must be reduced when the conditions of the constraint conditions increase.

Since $dx_1 = 0$($x_{1.i} = x_{max.i}$ or $x_{1.i} = x_{min.i}$), then

$$\begin{bmatrix} dx_1 = 0 \\ dx_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}\begin{bmatrix} dy_1 \\ dy_2 \end{bmatrix} \tag{4}$$

$$D_{11}dy_1 + D_{12}dy_2 = 0 \tag{5}$$

Setting, $dx_1' = D_{12}dy_2$,

$$D_{11}dy_1 = -dx_1' \tag{6}$$

1) There is no solution for dy_1 when the dimension of x_1 is more than the dimension of y_1, and there are infinitely many solutions in redundancy situation that the dimension of x_1 is less than the dimension of y_1.

Taking the N-dimensional of dy_1 corresponding to one - dimensional of dx_1, setting the components of dy_1 as the typical linear form of weighted average, then dy_1 can be

$$dy_1 = \begin{bmatrix} dy_{1.1} \\ \mathrm{M} \\ dy_{1.N} \end{bmatrix} = -\begin{bmatrix} d_1 \\ \mathrm{M} \\ d_N \end{bmatrix} dx_1'$$

d_i is the weight value. e.g., setting weighted average as the form

$$d_k = \frac{D_{11.k}}{\sum_{j \in y_1} D_{11.j}} \text{ or } d_k = \frac{y_{\max.k}}{\sum_{j \in y_1} y_{\max.j}} \text{ or } d_k = \frac{1}{N}$$

From the definite weight value, and determining one correction, other corrections can be calculated, which are expressed as:

$$dy_{1.k} = -d_k dx_1' \tag{7-1}$$

Equation (5) in the redundancy situation is

$$\begin{bmatrix} D_{11.1} & \cdots & D_{11.N} \end{bmatrix} \begin{bmatrix} dy_{1.1} \\ \vdots \\ dy_{1.N} \end{bmatrix} = -D_{12} \begin{bmatrix} dy_{2.1} \\ \vdots \\ dy_{2.M} \end{bmatrix}$$

So $D_{11}\dfrac{\partial y_1}{\partial y_2} = -D_{12}$, $D_{11.1}\dfrac{\partial y_{1.1}}{\partial y_{2.i}} + \cdots + D_{11.N}\dfrac{\partial y_{1.N}}{\partial y_{2.i}} = -D_{12.i}$.

Variable constraint sensitivity exists, but infinitely has many solutions.

2) y_1 becomes minimum and y_2 becomes maximum when the dimension of *x1* is the same as the dimension of y_1, then

$$dx_1' = D_{12}dy_2,\ dy_1 = -D_{11}^{-1}dx_1' \tag{7-2}$$

In these conditions, control methods have the same dimensions out of which y_1 is the most sensitive whereas the diagonal elements of the D_{11} are the biggest under different choice of y_1.

Thus, adherence problem can be solved according to the demand of △x1 and the correction △y1 of y_1 on the basis of (7) can be determined.

3) Constraint sensitivity of the variable can be acquired in accordance with Matrix D. According to (7-2) and (4):

$$dy_1 = -D_{11}^{-1}D_{12}dy_2 \tag{8-1}$$

$$dx_2 = (D_{22} - D_{21}D_{11}^{-1}D_{12})dy_2 \tag{8-2}$$

Setting $dy_{2i} = 1$, $dy_{2j} = 0$ ($j \neq i$) , then

$$\frac{\partial y_1}{\partial y_2} = -D_{11}^{-1}D_{12} \tag{9-1}$$

$$\frac{\partial x_2}{\partial y_2} = D_{22} - D_{21}D_{11}^{-1}D_{12} \tag{9-2}$$

Thus, the constraint sensitivity of variable concerning the tight constraints has been formed. Constraint sensitivity is the rate of change of x to y_2 under the coordinated conditions of y_1, and it reflects the special characteristics of the system.

4) The constraint sensitivity of objective function z to variable y_2 makes a differential, then:

$$dz = \left(\frac{\partial f}{\partial x_2}\right)^T dx_2 + \left(\frac{\partial f}{\partial y_2}\right)^T dy_2 + \left(\frac{\partial f}{\partial y_1}\right)^T dy_1 \tag{10}$$

Extreme condition equation of optimization function is

$$\frac{\partial z}{\partial y_{2i}} = \frac{\partial f}{\partial y_{2i}} + \left(\frac{\partial f}{\partial x_2}\right)^T \frac{\partial x_2}{\partial y_{2i}} + \left(\frac{\partial f}{\partial y_1}\right)^T \frac{\partial y_1}{\partial y_{2i}} = 0 \quad i \in y_2 \tag{11}$$

Connecting to the variable constraint sensitivity in (9), equation (11) is extreme condition equation concerning inequality tight constraints. Extreme equations reflects the connection of variables y_1 to y_2, x_2.

5) The constraint sensitivity of the objective function to the control mode, according to the differential coefficient of optimization function is:

$$dz = \begin{pmatrix} \left(\dfrac{\partial f}{\partial x_2}\right)^T (D_{22} - D_{21}D_{11}^{-1}D_{12}) - \left(\dfrac{\partial f}{\partial y_1}\right)^T \\ D_{11}^{-1}D_{12} + \left(\dfrac{\partial f}{\partial y_2}\right)^T \end{pmatrix} dy_2 \tag{12}$$

$$\frac{\partial z}{\partial y_{2i}} = \left(\left(\frac{\partial f}{\partial x_2}\right)^T (D_{22} - D_{21}D_{11}^{-1}D_{12}) - \left(\frac{\partial f}{\partial y_1}\right)^T D_{11}^{-1}D_{12} + \left(\frac{\partial f}{\partial y_2}\right)^T\right) \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

Then the constraint sensitivity of the objective function to the control mode should be:

$$\Delta z = \sum_{i \in y_2} \frac{\partial z}{\partial y_{2i}} \tag{13}$$

Current extreme point is acquired in accordance with the extreme conditions of (11), and must be tested whether it is the global optimum. For D11, which is in other control methods or y_1 (i.e. D22, D12, D21) which intends to be selected respectively and be substituted in equation (13), the results are as follows: If other control methods are taken for optimization, the deviation sensitivity of the corresponding extreme value to the current extreme value is the constraint sensitivity of the objective function to the control mode.

3. STEADY - STATE POWER FLOW CONTROL PROBLEM

The steady-state power flow control problem means that formula (14-1) had solution with the conditions of (14-6), simultaneously meeting the operating conditions equations (14-3) (14-4), (14-5) and the desired optimization objective function (14-2)

$$g(x,y) = y - h(x) = 0 \quad (14\text{-}1)$$

$$\min \quad f(x,y) \quad (14\text{-}2)$$

$$S(x) \geq 0 \quad (14\text{-}3)$$

$$x_j = x_j^s \quad (14\text{-}4)$$

$$x_{\min} \leq x \leq x_{\max} \quad (14\text{-}5)$$

$$y_{\min} \leq y \leq y_{\max} \quad (14\text{-}6)$$

A. The mathematical model of Power Flow Control

The transmission power of the tie line power or power flow control flow control meets $S(x) \geq 0$, and the active constraint is $P_{ij} + jQ_{ij} = P_{ij}^s + jQ_{ij}^s$. The load node is when the central point voltage of the voltage management $V_i = V_i^s$ becomes the PQV node while P_{Di}, Q_{Di}, V_i are definite values.

Load point voltage can be higher than the supply voltage in the power grid with k*>1 transformer (In the k*>1 model, the k*>1 means the secondary p.u. voltage is higher than the primary p.u. voltage). When the voltage sector V_i of the load node *i* extends to the tight constraints of the boundary value, the load node *i* turns to be PQV node. When the voltage sector V_i is out of the boundary of inequality, V_i is the loose bound variable, and the load node becomes the PQ node.

The generation is represented as PV node which has active power P_{Gi} and generator terminal voltage V_{Gi}, both being adjustable, and a slack bus with adjustable voltage set .

n is setting the grid node amount is, V_n is the slack bus voltage , PV node amount is m, PQ node amount is (n-m-1), making PQ node voltages to be V_D and θ_D, PV node and slack bus voltages being V_G and θ_G, and active power of PV node being P_G.

Voltage vector x and generation vector y' are respectively: $x = \begin{bmatrix} \theta_D^T & \theta_G^T & V_D^T \end{bmatrix}^T, y' = \begin{bmatrix} P_G^T & V_G^T & V_n \end{bmatrix}^T$.

Where $y = y_1 \cup y_2$, y_1 is the generation variable with adjustable capacity. y_2 is the un-adjustable constant generation variable.

Nodal power equations of PQ node and PV node are respectively:

$$g_{PDi} = P_{Gi}^S - P_{Di} - V_i \sum_{j=1}^{n} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 \quad (15\text{-}1)$$

$$g_{QDi} = Q_{Gi}^S - Q_{Di} - V_i \sum_{j=1}^{n} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 \quad (15\text{-}2)$$

$$g_{PGi} = P_{Gi} - P_{Di} - V_i \sum_{j=1}^{n} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 \quad (15\text{-}3)$$

The inequality constraint equation works while branch power and nodal voltage are the definite values, i.e.

$$P_{ij} = -V_i^2 G_{ij} + V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) - P_{ij}^s = 0 \quad (16\text{-}1)$$

$$Q_{ij} = V_i^2 B_{ij} + V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) - Q_{ij}^S = 0 \quad (16\text{-}2)$$

$$g_{Vk} = V_k^S - V_k = 0 \quad (16\text{-}3)$$

Equations (15) and (16) are the mixed power flow model, which is formed by the node power equations and the contributing inequality tight constraint equations are also the basic form of power flow control.

B. Unconstrained sensitivity for grid power flow model

The specific forms of (15), and (2) turn out to be:

$$\begin{bmatrix} \dfrac{\partial g_{PD}}{\partial \theta} & \dfrac{\partial g_{PD}}{\partial V_D} \\ \dfrac{\partial g_{PG}}{\partial \theta} & \dfrac{\partial g_{PG}}{\partial V_D} \\ \dfrac{\partial g_{QD}}{\partial \theta} & \dfrac{\partial g_{QD}}{\partial V_D} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ \vdots \\ d\theta_{n-1} \\ dV_1 \\ \vdots \\ dV_{n-m-1} \end{bmatrix} = - \begin{bmatrix} \sum\limits_{i=n-m+1}^{n} \dfrac{\partial g_{PD1}}{\partial V_i} dV_i \\ \vdots \\ \sum\limits_{i=n-m}^{n} \dfrac{\partial g_{PDn-m-1}}{\partial V_i} dV_i \\ dP_{Gn-m} + \dfrac{\partial g_{PGn-m}}{\partial V_{n-m}} dV_{n-m} \\ \vdots \\ dP_{Gn-1} + \dfrac{\partial g_{PGn-1}}{\partial V_{n-1}} dV_{n-1} \\ \sum\limits_{i=n-m}^{n} \dfrac{\partial g_{DD1}}{\partial V_i} dV_i \\ \vdots \\ \sum\limits_{i=n-m}^{n} \dfrac{\partial g_{DDn-m-1}}{\partial V_i} dV_i \end{bmatrix} \quad (17)$$

Connecting (17) according to the method of (3), we obtain:

$$\frac{\partial x(x,y)}{\partial y} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \quad (18)$$

C. The constraints sensitivity of the variable

The constraints sensitivity of the variable is the form of (9).

D. Constrained extremum equation of the active power in the slack bus

The node power equation of the grid active power loss is:

$$f = P_L = \sum_{i=1}^{n} (P_{Gi} - P_{Di}) = \sum_{i=1}^{n-m-1} P_{Gi}^s + \sum_{i=n-m}^{n-1} P_{Gi} + P_{Gn} - \sum_{i=1}^{n} P_{Di} \quad (19)$$

Where the slack bus power P_{Gn} is :

$$P_{Gn} = -V_n \sum_{j=1}^{n} V_j (G_{nj} \cos\theta_j - B_{nj} \sin\theta_j) \quad (20)$$

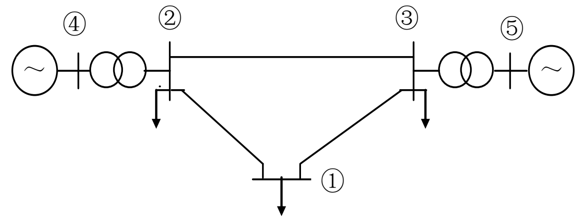

Fig. (1). IEEE 5 buses system.

According to (11), the extreme equation for the active power optimization is

$$l_{PGi}=1+\left(\frac{\partial P_{Gn}}{\partial V_D}\right)^T\frac{\partial V_D}{\partial P_{Gi}}+\left(\frac{\partial P_{Gn}}{\partial \theta}\right)^T\frac{\partial \theta}{\partial P_{Gi}}+\left(\frac{\partial P_{Gn}}{\partial V_G}\right)^T\frac{\partial V_{G.y1}}{\partial P_{Gi}}=0 \tag{21}$$

The extremal equation for the reactive power optimization is:

$$l_{VGi}=\left(\frac{\partial P_{Gn}}{\partial V_D}\right)^T\frac{\partial V_D}{\partial V_{Gi}}+\left(\frac{\partial P_{Gn}}{\partial \theta}\right)^T\frac{\partial \theta}{\partial V_{Gi}}+\left(\frac{\partial P_{Gn}}{\partial V_G}\right)^T\frac{\partial V_{G.y1}}{\partial V_{Gi}}=0 \tag{22}$$

In (21) and (22), the partial derivatives to P_{Gn}, $\partial P_{Gn}/\partial V_D$, $\partial P_{Gn}/\partial\theta$, $\partial P_{Gn}/\partial V_G$ can be obtained by (20), and the constraint partial derivative of y_1, x_2 to y_2 can be achieved by (9).

4. THE SOLUTION TO THE POWER FLOW CONTROL EQUATION

Due to the large number of voltage inequality equations and uncertainty of out-of-limit, according to (7), voltage inequality constraints serve as a check and deal with more limit. Only the simultaneous solution of flow equations and the branch power condition equations with mixed variables and PQV node are solved. The steps of Newton method for solving are as follows:

Step1: Request the correction amount Δx.

The correction equation of power flow model without regarding the voltage inequality conditions should be:

$$\begin{bmatrix}\Delta g_{PD}\\ \Delta g_{PG}\\ \Delta g_{QD}\\ \Delta P_{Line}\\ \Delta Q_{Line}\end{bmatrix}=\begin{bmatrix}H_{DD} & H_{DG} & N_{DD} & 0 & 0\\ H_{GD} & H_{GG} & N_{GD} & \frac{\partial g_{PG}}{\partial V_{Gy_1}} & \frac{\partial g_{PG}}{\partial P_{Gy_1}}\\ M_{DD} & M_{DG} & L_{DD} & \frac{\partial g_{QD}}{\partial V_{Gy_1}} & 0\\ \frac{\partial P_{Line}}{\partial \theta_{ij}} & 0 & \frac{\partial P_{Line}}{\partial V_{ij}} & 0 & 0\\ \frac{\partial Q_{Line}}{\partial \theta_{ij}} & 0 & \frac{\partial Q_{Line}}{\partial V_{ij}} & 0 & 0\end{bmatrix}\bullet\begin{bmatrix}\Delta\theta_D\\ \Delta\theta_G\\ \Delta V_D\\ \Delta V_{Gy_1}\\ \Delta P_{Gy_1}\end{bmatrix} \tag{23}$$

Solving Δx and line conditions control Δy_1 aiming at the unbalanced quantity, if ΔP_{line} and ΔQ_{line} are not out of limit, there is no need to make modification for $\Delta V_{Gy_1}, \Delta P_{Gy_1}$.

Step 2: Tight constraints determine the conditions correction to determine Δx_1 in tight constraints state:

$$\Delta x_{1i}=\begin{cases}0, & x_{i\min}<x_i^{(k+1)}<x_{i\max}\\ x_i^{(k)}+\Delta x_i-x_{\min i}, & x_i^{(k+1)}\le x_{i\min}\\ x_i^{(k)}+\Delta x_i-x_{\max i}, & x_i^{(k+1)}\ge x_{i\max}\end{cases}$$

Δx_1 compensated by Δy_1 gives the method of compensation:

A. Condition Control of N to N Linear Combination

According to (7-2), selecting appropriate combination of control Δy1, the correction of compensation Δx_1 required for conditions control can be obtained:

$$\Delta y_1=-D_{11}^{-1}\Delta x_1 \tag{24-1}$$

B. N to 1 Redundant Condition Control

According to (7-1), selecting appropriate control Δy_1, the condition control variables required for compensation Δx_1 can be acquired. E.g. the form of a weighted average is

$$\Delta y_{1.k}=-\frac{D_{kk}}{\sum_{j\in N}D_{jj}}\Delta x_i \tag{24-2}$$

C. One to One Condition Control

For the exceeding amount, selecting larger sensitivity D_{ij}, according to (7-2), Δy_1 becomes

$$\Delta y_{1.j}=-\Delta x_{1i}/D_{ij} \tag{24-3}$$

Step 3: Variable correction

The variable corrections are:

$$x^{(k+1)}=x^{(k)}+\Delta x^{(k)} \tag{25-1}$$

$$y^{(k+1)}=y^{(k)}+\Delta y_1^{(k)} \tag{25-2}$$

Comparing the above process with the conventional power flow calculations, the mixed power flow model replaces the grid power flow model in more than 2 steps of out-of-limit special process for x.

5. EXAMPLES AND ANALYSIS

Fig. (**1**) is the IEEE 5 buses system which is featured in both-side power supply, large impact of generation adjusting

Table 1. Solutions for extreme character check.

Scheme	Caption	The Setting For PQ Nodes And Slack Node
1	**V_1=0.93, optimize active and reactive power**	**P_{G4} =3.19680, V_{G4}=1.07323, V_{G5}=1.07828**
2	V_1=0.93, optimize reactive power, ΔPG4=-0.01	P_{G4} =3.18680, V_{G4} =1.07300, V_{G5}=1.07870
3	V_1=0.93, optimize reactive power , ΔPG4=＋0.01	P_{G4} =3.20680, V_{G4} =1.07346, V_{G5}=1.07786
4	V_1=0.93, optimize active power, ΔVG4=-0.01	P_{G4} =3.16732, V_{G4} =1.06323, V_{G5}=1.09307
5	V_1=0.93, optimize active power , ΔVG4=+0.01	P_{G4} =3.22654, V_{G4} =1.08323, V_{G5}=1.06352

to power flow change, and the step-up transformer making the load point voltage higher than the supply voltage. That is why the author chose it for the principle analysis in this paper.

A. The inspection and application of the constraint sensitivity

For IEEE 5 buses system, the calculation cases for verifying the constraint sensitivity and its application are listed in Table **1** and are as follows:

According to power supply settings listed in Table **1**, making power flow calculation and constraint sensitivity calculation in (21) and (22), the result is listed in Table **2**.

The gird loss of constraint sensitivity ∂PL/∂PG, ∂PL/∂VG in Scheme (1) is close to zero, which shows that the state is: V1=0.93, the active power is optimized, and the reactive power is optimized for the operating point. Scheme (2) and Scheme (3) maintain V1 = 0.93 and the reactive power is optimized. Only the active power has a small deviation which shows that the after state has a slight deviation from Scheme (1). The value of ∂PL/∂PG increases, and the loss of the corresponding power is slightly larger than that of Scheme (1). Similarly, Scheme (4) and Scheme (5) are in reverse. The results of Table **2** show the points which satisfied (21), and the extreme points of (21) which can withstand the small deviation of calibration. Also, according to the state of the operating point, constrained extreme conditions can be the judgment and the basis for calculations.

According to (18), the calculations of the system's sensitivity matrix D for power flow state in Scheme (1) are listed in Table **3**. The array element of matrix D reflects the properties system without the constraints of the natural rate of change, among which, the sensitivity of the voltage state to the voltage source is much larger than the voltage state to the power source.

When x_1 equals to V_1 that is restricted and the corresponding variable y_1 equals to V_{G5}, divide matrix D into D_{11}, D_{12}, D_{21} and D_{22}; the variable constraint sensitivity calculated according to (9) is listed in Table **4**.

The constraint sensitivity in Table **4** highlights that state to constraint sensitivity of active power (P_G) is slightly different with the unconstrained sensitivity in Table **3**, but for constraint sensitivity of reactive power (V_G), it is quite different from the unconstrained sensitivity in Table **3**. This result shows that when addressing the voltage inequality constraints, adopting unconstrained sensitivity simply to make the active power characteristics analysis an approximation, and the deviation of reactive power characteristics is large.

B. Algorithms and calculation of the power flow control and voltage management

Table **5** sets an example that the transmission line power and load point voltage which are counted for the required value of the flow control, apply equation variable mode to solve and deal with the exceeding voltage in (24).

Table **5** gives example solutions of power control and voltage management,among which, multiple objective deterministic control of three different types of objectives including flow control, voltage management and reactive power optimization are proceeded in Scheme (8).

Table 2. Small derivation check of extreme point.

Scheme	4	2	1	3	5
	ΔV_{G4} -0.01	ΔP_{G4} -0.01	optimize active, reactive power	ΔP_{G4} +0.01	ΔV_{G4} +.001
θ_1/rad	-0.30349	-0.30783	-0.30666	-0.30550	-0.30996
θ_2/rad	-0.07599	-0.08110	-0.07924	-0.07738	-0.08261
θ_3/rad	-0.11138	-0.11362	-0.11344	-0.11326	-0.11559
θ_4/rad	-0.03330	-0.03888	-0.03691	-0.03493	-0.04062
V_1/p.u	0.93	0.93	0.93	0.93	0.93
V_2/p.u	1.09952	1.10829	1.10850	1.10872	1.11747
V_3/p.u	1.09783	1.08538	1.08497	1.08457	1.07213
V_4/ p.u	1.06323	1.07300	1.07323	1.07346	1.08323
V_5/ p.u	1.09307	1.07870	1.07828	1.07786	1.06352
PG4/p.u	3.16732	3.18680	3.19680	3.20680	3.22654
PG5/p.u	1.09307	4.21431	4.20431	4.19431	4.17513
Ploss/p.u	0.10169	0.10112	0.10111	0.10112	0.10168
$\partial P_L/\partial P_{G4}$	-8.14444×10^{-7}	-8.91308×10^{-4}	5.36430×10^{-8}	0.00089	-8.20054×10^{-7}
$\partial P_L/\partial V_{G4}$	-0.11455	9.81031×10^{-7}	9.73436×10^{-7}	9.69353×10^{-7}	0.11436

Table 3. The system sensitivity of Scheme 1.

	Matrix D of Scheme (1)		
∂V_1	0.65712 D_{11}	-5.70322×10^{-3}	0.95411 D_{12}
∂V_2	8.95447	-1.54966×10^{-3}	1.02558
∂V_3	0.98440	2.63313×10^{-3}	0.17726
$\partial \theta_1$	0.61517 D_{21}	-0.13862	0.17499 D_{22}
$\partial \theta_2$	0.46626	-0.21136	-0.26808
$\partial \theta_3$	0.21324	-2.68593×10^{-2}	2.47262×10^{-2}
$\partial \theta_4$	0.46284	-0.22456	-0.34674
	/ ∂VG5 (y_1)	/ ∂PG4 ($y_{2.1}$)	/ ∂VG4 ($y_{2.2}$)

Table 4. Variable constrained sensitivity of Scheme 1.

		Variable Constrained Sensitivity of Scheme 1	
y_1	∂V_{G5}	8.67908×10^{-3}	-1.45195
x_2	∂V_2	-7.72502×10^{-4}	0.89557
	∂V_3	1.11768×10^{-2}	-1.2520
	$\partial \theta_1$	-0.13328	-0.71821
	$\partial \theta_2$	-0.20732	-0.94508
	$\partial \theta_3$	-2.50085×10^{-2}	-0.28489
	$\partial \theta_4$	-0.22054	-1.01877
y_2		$/\,\partial P_{G4}$	$/\,\partial V_{G4}$

Table 5. Example solutions of power control.

Scheme	Operating Condition	Power Combination Adjustment Mode
6	V_3=1.05, P_{23}= 0.2	P_{G4} Adjust the line active power, V_{G5} voltage management
7	V_3=1.05, P_{23}=Q_{23}=0.2	P_{G4},V_{G4} Adjust the line active and reactive power, V_{G5} voltage management
8	V_3=1.05, P_{23}= 0.2, optimize reactive power	P_{G4} Adjust the line active power, V_{G4} optimize reactive power, V_{G5} voltage management
9	V_3=1.05, optimize active and reactive power	P_{G4} optimize active power, V_{G4} optimize reactive power, V_{G5} voltage management

Table 6. Calculation examples of power control.

Scheme	6	7	8	9
operating condition	V_3=1.05, P_{23}= 0.2	V_3=1.05, P_{23}= 0.2, Q_{23}= 0.2	V_3=1.05, P_{23}= 0.2, L_{VG}=0	V_3=1.05, L_{PG}=0, L_{VG}=0
V_1/ p.u	0.88679	0.91699	0.94101	0.94289
V_2/p.u	1.08191	1.11728	1.14622	1.14821
V_3/p.u	1.05	1.05	1.05	1.05
V_4/p.u	1.05	1.08493	1.11360	1.11551
V_5/p.u	1.04732	1.04095	1.03580	1.03557

Table 6. contd….

Scheme	6	7	8	9
θ_1/rad	-0.32009	-0.31583	-0.31334	-0.30723
θ_2/rad	-0.07064	-0.08155	-0.09052	-0.08229
θ_3/rad	-0.11881	-0.11906	-0.11937	-0.11818
θ_4/rad	-0.02523	-0.03886	-0.04987	-0.04127
P_{23}/p.u	0.2	0.2	0.2	0.22886
*Q_{23}/p.u	-0.22647	-0.11208	-0.01247	-0.01212
P_{loss}/p.u	0.11233	0.10648	0.10500	0.10492
$\partial P_L/\partial P_{G4}$	4.48478 $\times 10^{-3}$	-1.56166 $\times 10^{-4}$	-3.92074 $\times 10^{-3}$	-9.74024 $\times 10^{-9}$
$\partial P_L/\partial V_{G4}$	-0.23488	-0.10263	-9.8783 $\times 10^{-7}$	-9.87534 $\times 10^{-7}$

* Grounding capacitance branch is calculated in Q_{23}. The power transmission condition of the calculation case doesn't include the grounding capacitance branch.

Table **6** gives the calculation examples of power control and voltage management.

The examples of Table **5** and the results of Table **6** indicate that the mixed power flow model is formed by node power and branch power. A good differentiate coherence method can be an effective algorithm for power flow control and voltage management issues. As x_1 is definite value, and needs to revise y_1 constantly in the iteration, increasing the number of iterationsgives the extent of super-linear convergence of Newton method.

CONCLUSION

Association and coordination of Δx_1 and Δy_1 can solve the calculating coherence problem of tight constraints for x_1 . On the basis of the mixed power flow model formed by the power flow control and the solving iterative solution of the least variable patterns, which are similar to the normal power flow, its calculation scale, convergence and robustness are basically consistent with the conventional power flow algorithm, and it maintains the simplicity and efficiency required for calculations.

The constraint sensitivity of the variable may constitute the constraint sensitivity of the operating objective (The composite function of the variable) which is mainly concerned, and the constraint sensitivity of the operating objective to the control power can depict the adjustment direction and sensitivity. The composite function of the target (variable) sensitivity runs the target to control the power of constraint sensitivity which identifies the direction of regulation and sensitivity. The sensitivity from operating objective to control mode can make the effect of control mode definite, and for the flow control, it is required to provide a flexible choice of control methods and analysis.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

This paper belongs to the project of the "Innovation Project of Guangxi Graduate Education", No. GXU11T31069.

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Received: October 16, 2014 Revised: December 23, 2014 Accepted: December 31, 2014