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Different-lags Synchronization in Time-delay and Circuit Simulation of Fractional-order Chaotic System Based on Parameter Identification

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Abstract: This study constructs a novel four-dimensional fractional-order chaotic system. It verifies chaotic nonlinear dynamic behaviors and physical reliability by numerical simulation and hardware circuit design. For a class of parameter uncertainty fractional different-lags (τ_i) chaotic systems, the authors design a time-delayed (δ) synchronization controllers and parameter adaptive laws. It proves that the drive system and the response system tend to be synchronized and identified parameter when the control parameter matrix K satisfies the condition that $K-nE$ is a positive definite matrix. Simulation results show physical reliability of the fractional-order different-lags chaotic system and verify effectiveness of different-lags synchronization in time-delayed system method design.

Keywords: Fractional-order chaotic system, different-lags system synchronization, time-delay synchronization controllers, parameter identification.

1. INTRODUCTION

Chaos is ubiquitous nonlinear phenomena that it is a macro disorder and micro order in nature. Since the 1960s, Lorenz, the American meteorologist, stumbled across the first chaotic attractor from numerical weather changes experiments [1], Chaos theory has been gained tremendous and profound developments. Fractional-order calculus is the mathematics study of arbitrary order derivative, integral operator characteristics and applications; it is also an extension and promotion of integral-order calculus concepts. As the theory of fractional calculus widely application in a fluid mechanics of time-dependent, electro analytical chemistry, fractional model of animal nerves, modern signal analysis and processing, chaos phenomena in nonlinear regression model, molecular spectroscopy, fractional regression models and other areas [2], researches on the fractional calculus and fractional differential equations have become one of the hot issues in the field of applied mathematics and dynamics. Researchers discovered in the course of integer-order chaotic system, it exist richer dynamic behaviors when the system is fractional-order, it better able to accurately describe real-world dynamics and the system's actual physical phenomena using fractional calculus operator [3]. Therefore, the study of fractional-order chaotic system has extremely important theoretical values and practical significances.

Because of extreme sensitivity to initial conditions in the chaotic system, many scholars thought that it is impossible to achieve synchronization between two chaotic systems. Since Pecora and Carroll achieved synchronization within two chaotic systems by electronic circuits firstly in 1990 [4], synchronization of chaotic system has caused many scholars strongly concerns. Synchronization control methods have been proposed, such as complete synchronization [5-7], lag synchronization [8, 9], phase synchronization [10], anti-synchronization [11, 12], partial synchronization [13], generalization synchronization [5, 14], impulsive synchronization [15], projective synchronization [16, 17] and so on. Until now, synchronization of chaotic systems is still a hot research topic. In the hardware circuit, due to the effects of environment and other factors, component parameters will drift, small changes of system parameters can cause big changes in system performance. Therefore, it has important practical value to study on synchronization of chaotic systems with parameters uncertainty [6, 11, 12, 16, 17].

Strictly, the current status of any actual system is influenced by past state inevitably, that is the change rate of current state not only depends on the current state, but also the status of a past time, the system with this particular characteristic is called lag system [5, 8, 9, 15, 18-21]. It is clear that lag systems are existed in a wide physical world, such as, biophysics, lasers, electronic oscillators, nuclear reactors, neural networks, population dynamics and communication networks [19]. The lag systems have infinite-dimensional state space, they can produce more positive Lyapunov exponents than their dimensions, so simple structure lag systems also have very complex dynamical behaviors. Any signal transmission requires a certain time due to the limit of signal transmission speed, the response system usually delays the drive system. Therefore, research on time-delayed system has reality significance.

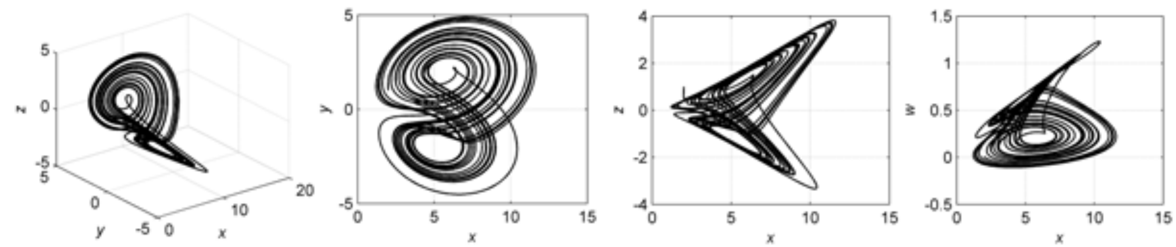


Fig. (1) Chaotic attractors of the system (1).

2. CONSTRUCTION AND CIRCUIT SIMULATION SYSTEM MODEL

2.1. Construction of the New Fractional-order Chaotic System

We construct a new four-dimensional chaotic system, its dynamic equation of state is:

$$\begin{cases} \dfrac{d^{q_1}x}{dt^{q_1}} = -ax + 3yz + y^2 \\ \dfrac{d^{q_2}y}{dt^{q_2}} = by - xz + 2w \\ \dfrac{d^{q_3}z}{dt^{q_3}} = -cz + xy \\ \dfrac{d^{q_4}w}{dt^{q_4}} = -dw + x - y - z \end{cases} \tag{1}$$

where x, y, z, w are state variables, a, b, c, d are parameters, q_i ($i = 1,2,3,4$) are the fractional-order dimension and $0 < q_i < 1$. For the case $a = 2, b = 3.65, c = 8, d = 12$, we calculate the Lyapunov exponents as: $L_1 = 0.7120$, $L_2 = 0.0041$, $L_3 = -7.0443$, $L_4 = -12.0218$, and its Lyapunov dimension is:

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 2.4736 \tag{2}$$

Therefore system (1) has a chaotic behavior. When $q_1 = q_2 = q_3 = q_4 = 0.95$, the phase diagram of the state variable trajectory as shown Fig. (**1**).

The divergence of flow of the system (1) is

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{\omega}}{\partial \omega} = -a + b - c - d \tag{3}$$

When $a = 2, b = 3.65, c = 8, d = 12$, $-a + b - c - d = -18.35 < 0$. Therefore the system (1) is dissipative and its exponential rate is

$$\frac{dV}{dt} = e^{-(a-b+c+d)t} = e^{-18.35t} \tag{4}$$

That is, in the system (1), a volume element $V(0)$ is apparently contracted by the flow into a volume element $V(0)e^{-18.35t}$ in the time t. This means that each volume containing the trajectories of the system (1) shrinks to zero as $t \to \infty$ at an exponential rate $-(a - b + c + d)$. Therefore, all the orbits of the dynamical system (1) will be eventually confined to a special subset that has zero volume, and the asymptotic motion of system (1) will settle onto an attractor of the system.

2.2. Fractional-order Chaotic System Circuit Simulation

2.2.1. Definition of Fractional Calculus and Frequency-Domain Approximation

The fractional-order integer differential operator [17] can be expressed as follow:

$$_aD_t^q = \begin{cases} \dfrac{d^q}{dt^q}, & R(q) > 0, \\ 1, & R(q) = 0, \\ \int_a^t (d\tau)^{-q}, & R(q) < 0. \end{cases} \tag{5}$$

where q is the fractional-order and $R(q)$ is the real of q, and in this paper $0 < q < 1$. α and t are the upper and low limits of the integral operation.

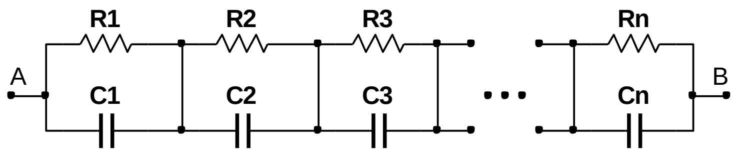
Many scholars propose several different definitions [3] in the development process of fractional calculus, and the most common is RL (Riemann-Liouville) fractional calculus definition, which is given by

$$\frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau \tag{6}$$

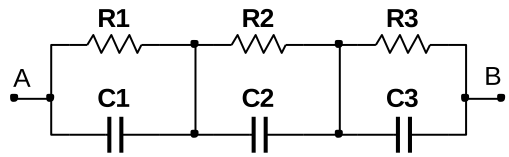
where $\Gamma(\cdot)$ is the gamma function and $n - 1 \le q < n$. Upon considering all the initial conditions to be zero, the Laplace transform of the RL fractional calculus is

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} \tag{7}$$

Thus, the fractional integer differential operator q can be represented by the transfer function $H(s) = 1/s^q$ in the frequency domain. Engineering often uses time domain and complex frequency domain conversion method to solve the fractional differential equations, Bode plot approximation



(a) $1/s^q$ chain circuit unit



(b) $1/s^{0.95}$ chain circuit unit

Fig. (2) Chain circuit unit.

method [22] based on the use of the frequency domain Charef *et al.* proposed, Ahmad et al. gave approximate transfer function $H(0)/s^q$ ($q = 0.1 - 0.9$, step 0.1, 2dB approximation error) in frequency domain expressions [23], Li and Chen gave approximate transfer function $H(0)/s^q$ ($q = 0.95$, 1dB approximation error) in frequency domain expressions [24].

2.2.2. Fractional-order Chain Circuit Unit

Fractional equivalent circuit of the complex frequency-domain can be achieved through chain circuit unit, A and B can be realized $q = 0.1 \sim 0.9$ (2dB approximation error, step 0.1) and $q = 0.95$ (1dB approximation error) of the approximate $1/s^q$ in Fig. (**2a**). According to circuit theory, chain complex frequency domain transfer function of equivalent circuit expression is given by

$$H(s) = \frac{R_1}{sR_1C_1+1} + \frac{R_2}{sR_2C_2+1} + \frac{R_3}{sR_3C_3+1} + \ldots + \frac{R_n}{sR_nC_n+1} \tag{8}$$

When $q = 0.95$, $H(s)$ approximate expression can be given as follow (9), and achieving $1/s^{0.95}$ chain circuit unit is shown in Fig. (**2b**).

$$H(s) = \frac{1}{s^{0.95}} \approx \frac{1.2831s^2 + 18.6004s + 2.0833}{s^3 + 18.4738s^2 + 2.6574s + 0.003} \tag{9}$$

When $q = 0.95$, $n = 3$, the transfer function is given by

$$H(s) = \frac{R_1}{sR_1C_1+1} + \frac{R_2}{sR_2C_2+1} + \frac{R_3}{sR_3C_3+1}$$

$$= \frac{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}{(s+1/R_1C_1)(s+1/R_2C_2)(s+1/R_3C_3)}$$

$$\times \left[s^2 + \frac{\left(\frac{C_2+C_3}{R_1} + \frac{C_1+C_3}{R_2} + \frac{C_1+C_2}{R_3}\right)s + \frac{R_1+R_2+R_3}{R_1R_2R_3}}{C_1C_2 + C_2C_3 + C_1C_3} \right] \tag{10}$$

It can be calculated the element parameter values by comparing (10) and (9) in Fig. (**2**) as follows:

$R_1 = 15.1k\Omega$, $R_2 = 1.51M\Omega$, $R_3 = 692.9M\Omega$, $C_1 = 3.616\mu F$, $C_2 = 4.602\mu F$, $C_3 = 1.267\mu F$.

2.2.3. The Circuit Simulation of the New Fractional-order Chaotic System

Due to the allowable voltage limitations of electronic components, circuit experiment is required reliably, therefore, the output signal of the system is reduced to half of its original size. In accordance with the system of equation (1), its design of fractional-order circuit diagram is shown in Fig. (**3a**). According to the system circuit schematic diagram and circuit basic theory, mathematical equations can be obtained for the system (11).

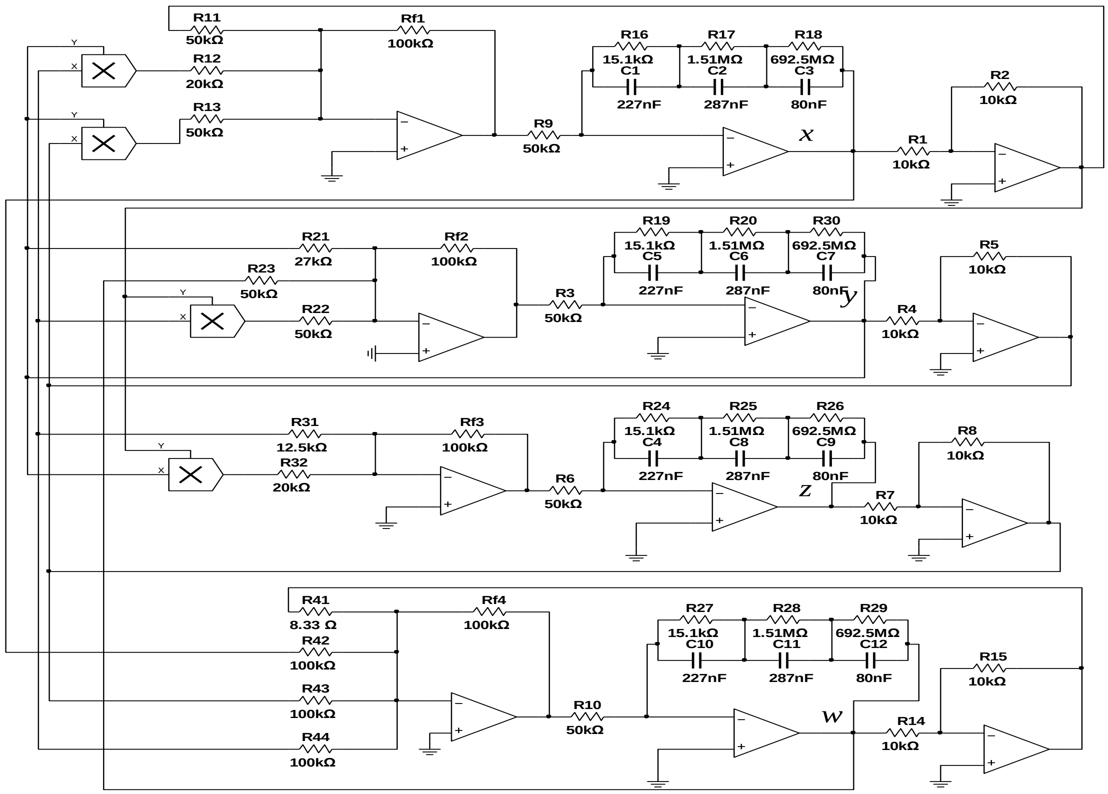
$$\begin{cases} \frac{d^{q_1}x}{dt^{q_1}} = -\frac{R_{f1}}{R_{11}R_9C_1}x + \frac{R_{f1}}{10R_{12}R_9C_1}yz + \frac{R_{f1}}{10R_{13}R_9C_1}y^2 \\ \frac{d^{q_2}y}{dt^{q_2}} = \frac{R_{f2}}{R_{21}R_3C_2}y - \frac{R_{f2}}{10R_{22}R_3C_2}xz + \frac{R_{f2}}{R_{23}R_3C_2}w \\ \frac{d^{q_3}z}{dt^{q_3}} = -\frac{R_{f3}}{R_{31}R_6C_3}z + \frac{R_{f3}}{10R_{32}R_6C_3}xy \\ \frac{d^{q_4}w}{dt^{q_4}} = -\frac{R_{f4}}{R_{41}R_{10}C_4}w + \frac{R_{f4}}{R_{42}R_{10}C_4}x - \frac{R_{f4}}{R_{43}R_{10}C_4}y - \frac{R_{f4}}{R_{44}R_{10}C_4}z \end{cases} \tag{11}$$

Comparing system (1) with (11) and we can obtain

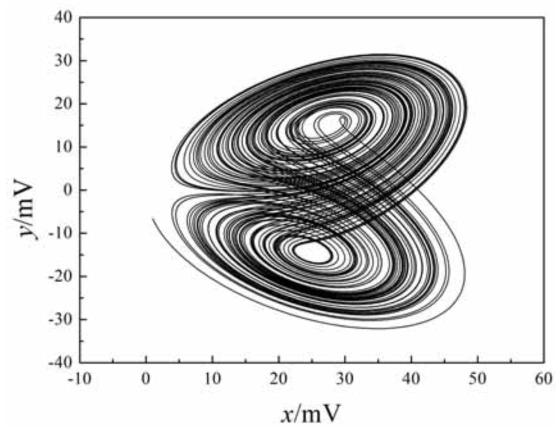
$$a = \frac{R_{f1}}{R_{11}R_9C_1},\ b = \frac{R_{f2}}{R_{21}R_3C_2},\ c = \frac{R_{f3}}{R_{31}R_6C_3},\ d = \frac{R_{f4}}{R_{41}R_{10}C_4},$$

$$6 = \frac{R_{f1}}{10R_{12}R_9C_1}, \quad 1 = \frac{R_{f4}}{R_{42}R_{10}C_4} = \frac{R_{f4}}{R_{43}R_{10}C_4} = \frac{R_{f4}}{R_{44}R_{10}C_4},$$

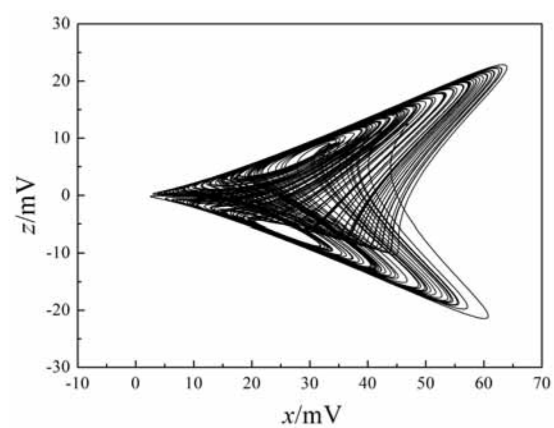
$$2 = \frac{R_{f1}}{10R_{13}R_9C_1} = \frac{R_{f2}}{10R_{22}R_3C_2} = \frac{R_{f3}}{10R_{32}R_6C_3} = \frac{R_{f2}}{R_{23}R_3C_2}.$$



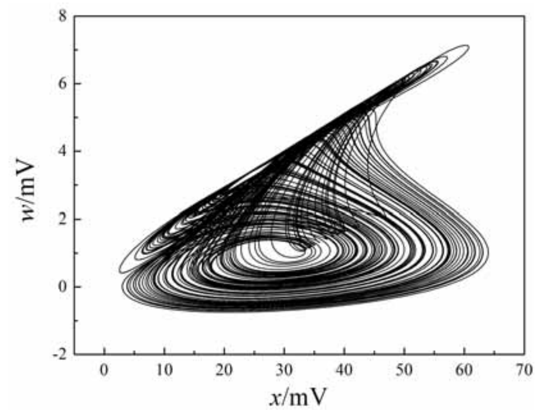
(a) Circuit diagram



(b) *x*-*y*



(c) *x*-*z*



(d) *x-w*

Fig. (3) Fractional-order chaotic circuit simulation and its phase portraits.

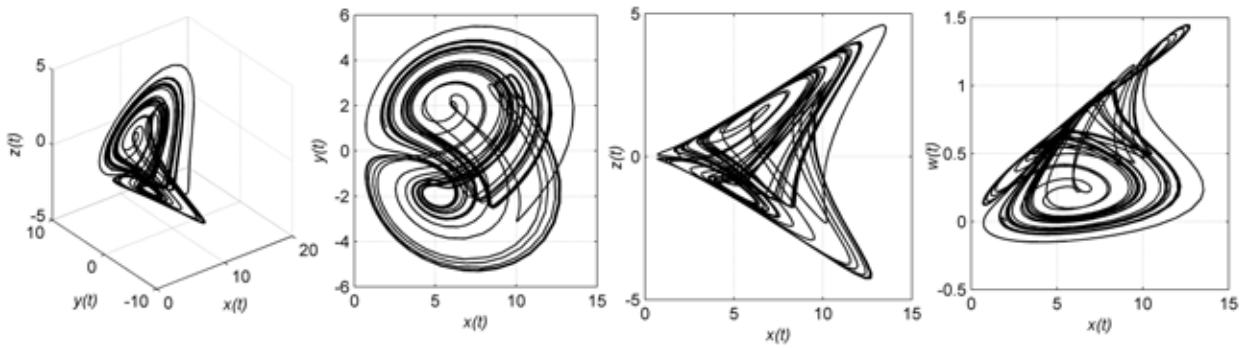


Fig. (4) Chaotic attractors of system (12).

Let $q_1 = q_2 = q_3 = q_4 = 0.95$, $C_1 = C_2 = C_3 = C_4 = 1\mu F$, $R_1 = R_2 = R_4 = R_5 = R_7 = R_8 = R_{14} = R_{15} = 10k\Omega$, $R_{41} = 8.33k\Omega$,

$R_3 = R_6 = R_9 = R_{10} = R_{11} = R_{13} = R_{22} = R_{23} = 50k\Omega$,

$R_{f1} = R_{f2} = R_{f3} = R_{f4} = R_{42} = R_{43} = R_{44} = 100k\Omega$

$R_{12} = R_{32} = 20k\Omega$, $R_{21} = 27k\Omega$, $R_{31} = 12.5k\Omega$, we obtain its phase portraits as shown in Fig. (**3b~d**) by using Multisim 10 software to simulate system (11). Comparing with Fig. (**1**), it can be seen that circuit simulation results and numerical calculations agree well, so the fractional-order chaotic system circuit can be implemented by physical.

3. FRACTIONAL DIFFERENT-LAGS CHAOTIC SYSTEM AND ITS ADAPTIVE TIME-DELAY SYNCHRONIZATION

3.1. Fractional-order Lags Chaotic System Model

Typically, fractional delay differential equation can be described as

$$ {}_aD_t^q X = F(X(t), X(t-\tau_i), t)p + f(X(t), X(t-\tau_i), t) \qquad (12)$$

where $X = (x_1(t), x_2(t), \cdots, x_n(t))^T \in R^n$ is the system state variables, $F: R^n \to R^{n\times m}$ is a linear function, $f: R^n \to R^n$ is a nonlinear function, $p \in R^m$ is a linear parameter vector of the system, $\tau_i > 0$ $(i = 1,2,\cdots,n)$ are the lags system constants.

The system (12) as the drive system, if it participates in synchronization controller $U(t)$, the system response is obtained as follow:

$$ {}_aD_t^q \tilde{X} = F(\tilde{X}(t), \tilde{X}(t-\tau_i), t)\tilde{p} + f(\tilde{X}(t), \tilde{X}(t-\tau_i), t) + U(t) \qquad (13)$$

where $\tilde{X} = (\tilde{x}_1(t), \tilde{x}_2(t), \cdots, \tilde{x}_n(t))^T \in R^n$ is the system state variables, $\tilde{p} \in R^m$ is a linear unknown parameter vector of the system, $U(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in R^n$ is the synchronization controller.

3.2. Fractional Different-lags Chaotic System and its Circuit Simulation

3.2.1. Fractional-order Different-lags Chaotic System

According to (12), the dynamic lags equation of the system (1) as follow:

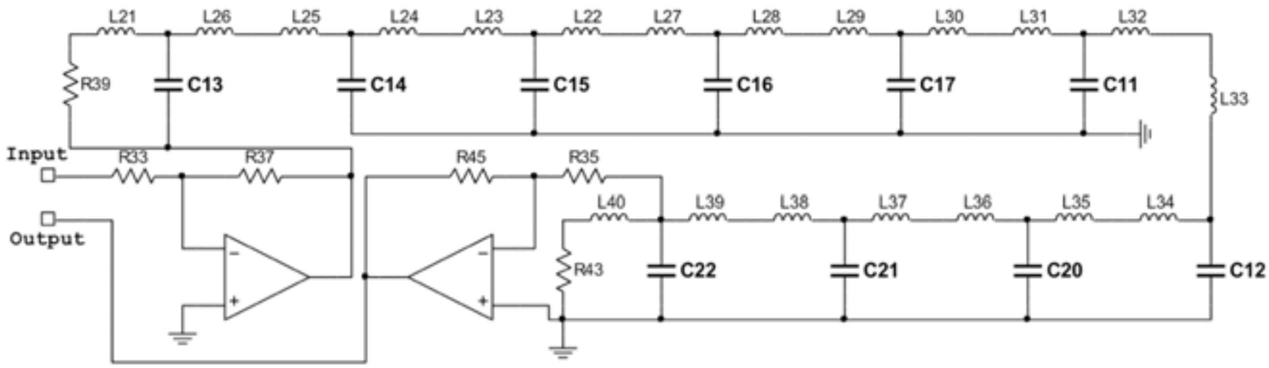


Fig. (5). Circuit implementation of the time delay unit.

$$\begin{cases} \dfrac{d^{q_1}x}{dt^{q_1}} = -ax(t-\tau_1) + 3y(t)z(t) + y^2(t) \\ \dfrac{d^{q_2}y}{dt^{q_2}} = by(t-\tau_2) - x(t)z(t) + 2w(t) \\ \dfrac{d^{q_3}z}{dt^{q_3}} = -cz(t-\tau_3) + x(t)y(t) \\ \dfrac{d^{q_4}w}{dt^{q_4}} = -dw(t-\tau_4) + x(t) - y(t) - z(t) \end{cases} \tag{14}$$

When $q_1 = q_2 = q_3 = q_4 = 0.95$, $a = 2$, $b = 3.65$, $c = 8$, $d = 12$, $\tau_1 = 0.03, \tau_2 = 0.02$, $\tau_3 = 0.01, \tau_4 = 0.08$, the largest Lyapunov exponent of the system (14) is 3.6532, clearly greater than the system (1), indicating that the system (14) has a more complex nonlinear dynamical behaviour. The phase diagram of the state variable trajectory as shown Fig. (**4**).

3.2.2. Circuit Implementation of the Time Delay Unit

A circuit implementation of the delay unit is shown in Fig. (**5**). This is a network of T-type LCL filters with matching resistors of the circuit unit. The time delay τ can be approximated by

$$\tau \approx n\sqrt{2LC} \tag{15}$$

where n is the number of the LCL filter and $n \ge 1$. Low-pass filter network is limited by signal frequency, the delay circuit unit cut-off frequency below 100 with smooth features. When the noise frequency and the signal frequency are near, single-stage filter fails to achieve the desired effect. It needs to use multiple filters to prevent noise interference, and located between the input and output $n = 10$ groups T-type filter, match port configuration resistors $R_{29} = R_{43} = 1k\Omega$ and passband characteristic impedance is constant, and $R_{45} = 22k\Omega$, $R_{33} = R_{35} = R_{37} = 10k\Omega$ are chosen in this circuit. When the values of the time delay $\tau_i (i = 1,2,3,4)$ are $\tau_1 = 0.03$, $\tau_2 = 0.02$, $\tau_3 = 0.01$, $\tau_4 = 0.08$, the value of capacitance and inductance of each time delay circuit unit can be calculated respectively 1mH, 4.5nF; 1mH, 2nF; 1mH, 0.5nF and 1mH, 32nF according to (15).

3.2.3. Fractional-order Different-lags Chaotic System Circuit Design and its Circuit Simulation

When $q_1 = q_2 = q_3 = q_4 = 0.95$, $a = 2$, $b = 3.65$, $c = 8$, $d = 12$, $\tau_1 = 0.03, \tau_2 = 0.02$, $\tau_3 = 0.01, \tau_4 = 0.08$, in accordance with the system of equations (14) design of fractional-order different-lags circuit diagram as shown in Fig. (**6a**), and the circuit simulation experimental results as shown in Fig. (**6**) b~d. Compared with Fig. (**5**), it can be seen that circuit simulation experimental results and numerical results agree well, so the fractional-order different-lags chaotic system circuit can be implemented on the physical.

3.3. Adaptive Synchronization in Time-delayed Design of the Fractional Different-lags System

Synchronization in time-delayed refers the state of the drive system after a fixed period of time δ to the state of the response system, drive system tends to become synchronized with the response system finally.

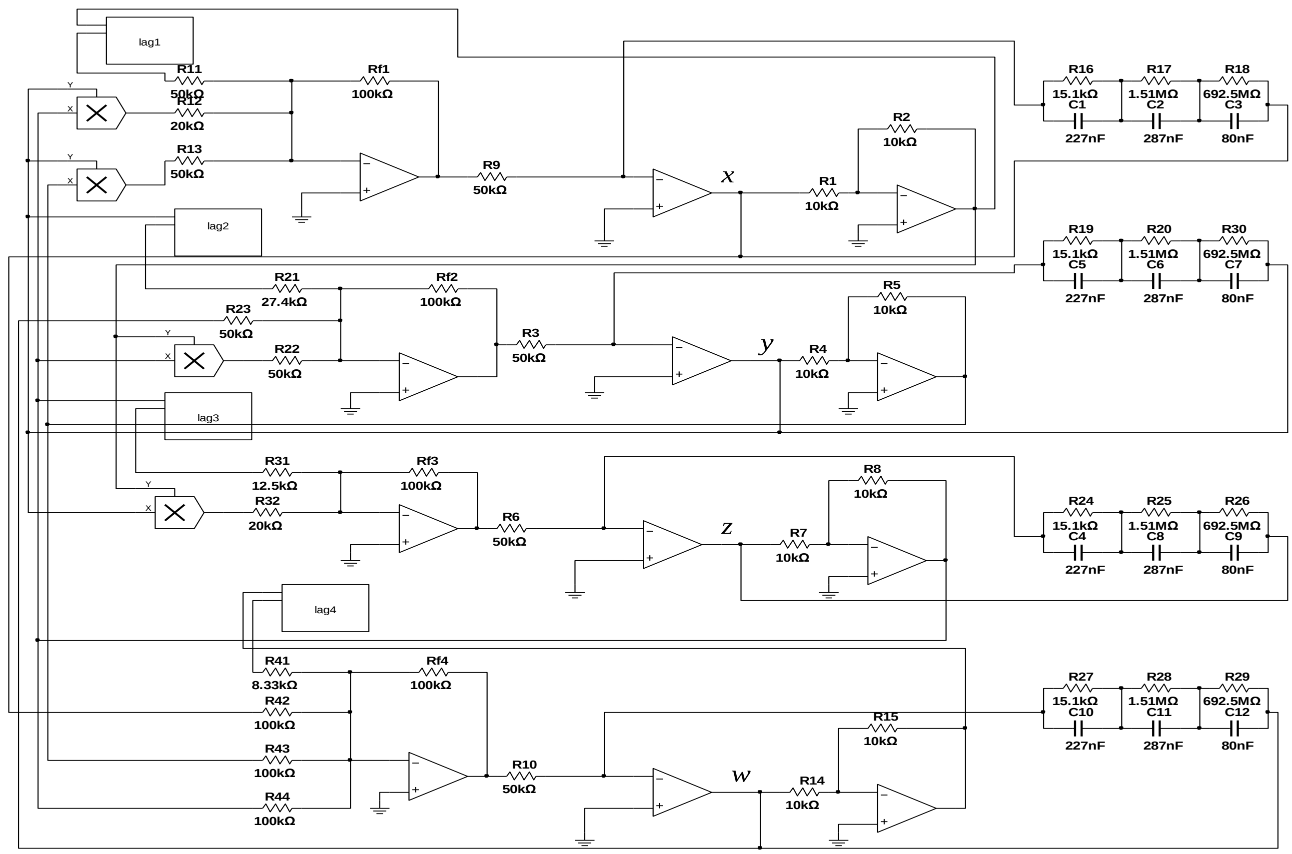
The system (14) corresponds to the fractional different-lags system of delay-time as follow:

$$ {}_aD_t^q X(t-\delta) = F(X(t-\delta), X(t-\tau_i-\delta), t-\delta)p$$
$$+ f(X(t-\delta), X(t-\tau_i-\delta), t-\delta) \tag{16}$$

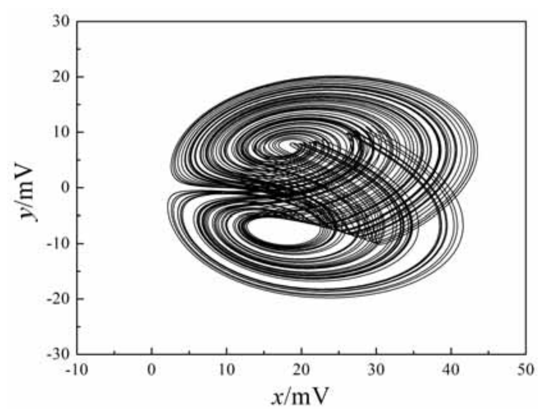
where $X(t-\delta) = (x_1(t-\delta), x_2(t-\delta), \cdots, x_n(t-\delta))^T$, $\delta > 0$ is delay-time constant.

Let $e(t) = \tilde{X} - X(t-\delta)$, $e_p(t) = \tilde{p}(t) - p$. Therefore the error equation can be written as

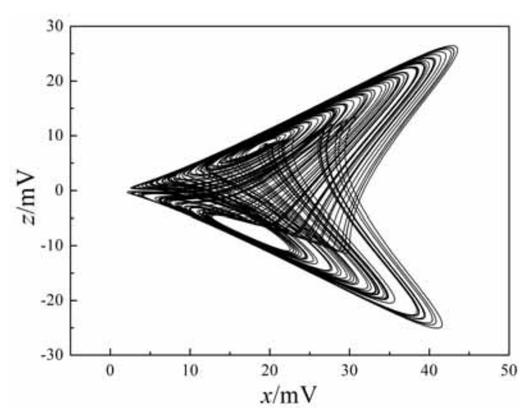
$$ {}_aD_t^q e(t) = F(\tilde{X}(t), \tilde{X}(t-\tau_i), t)\tilde{p}(t)$$
$$-F(X(t-\delta), X(t-\tau_i-\delta), t-\delta)p +$$
$$f(\tilde{X}(t), \tilde{X}(t-\tau_i), t)$$
$$-f(X(t-\delta), X(t-\tau_i-\delta), t-\delta) + U(t) \tag{17}$$



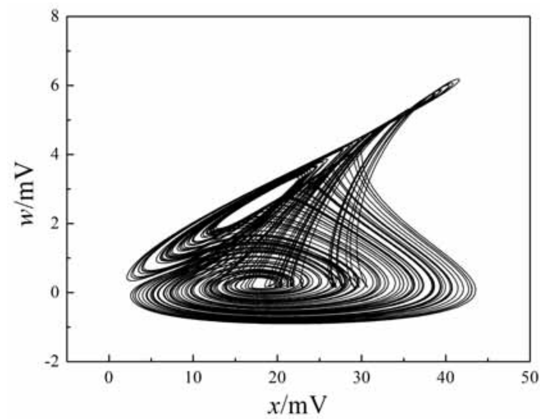
(a) Circuit diagram



(b) *x-y*

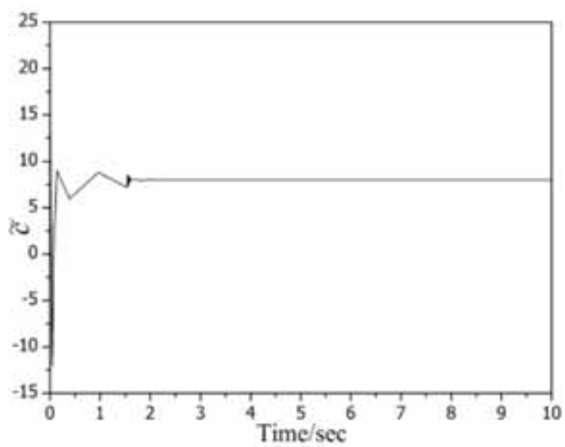


(c) *x-z*

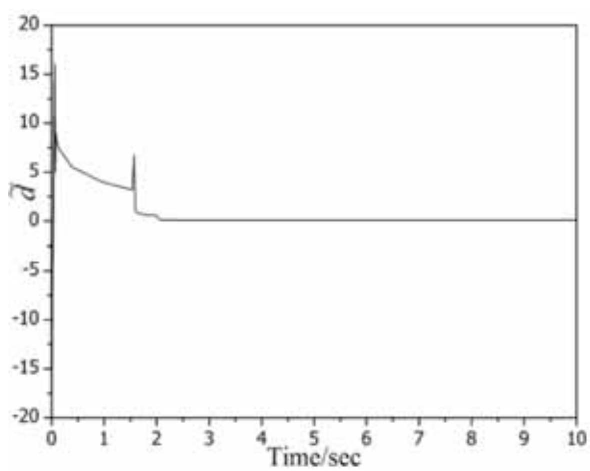


(d) *x*-*w*

Fig. (6) Different-lags chaotic system circuit simulation and its phase portraits.



Ⅰ The system error equations curves



Ⅱ The identification of the unknown parameters curves

Fig. (7) Adaptive synchronization simulation result in time-delayed design of the fractional different-lags system.

Theorem 1. If the synchronization controllers are designed as

$$U(t) = -Ke(t) + f(X(t-\delta), X(t-\tau_i-\delta), t-\delta) - f(\tilde{X}(t), \tilde{X}(t-\tau_i), t)$$
$$+(F(X(t-\delta), X(t-\tau_i-\delta), t-\delta) - F(\tilde{X}(t), \tilde{X}(t-\tau_i), t))\tilde{p}(t) \quad (18)$$

and the unknown parameter $\tilde{p}$ meets the parameter adaptive laws as

$$ {}_aD_t^q e_p(t) = -F^T(x(t-\delta), x(t-\tau_i-\delta), t-\delta)e(t) \tag{19}$$

where $K=[k_1,k_2,\cdots,k_i]^T$ is a positive vector of the controllers. If $K-nE$ is a positive definite matrix, the response of the system (13) and the drive system (16) tend to be synchronized, that is $\lim_{t\to\infty} e(t)=0$, where E is a identity matrix, n is the dimension of the system (14).

Proof. Substituting (18) into (17) yields

$$ {}_aD_t^q e(t) = -Ke(t) + F(X(t-\delta), X(t-\tau_i-\delta), t-\delta)e_p(t) \tag{20}$$

Construct the following Lyapunov-krasovskii functional:

$$ V(t) = \frac{1}{2}e^T(t)e(t) + \frac{1}{2}e_p^T(t)e_p(t) + \sum_{i=1}^{n}\int_{-\tau_i-\delta}^{0} e^T(t+\theta)e(t+\theta)d\theta \tag{21}$$

Its time derivative is

$$\begin{aligned}
{}_aD_t^q V(t) &= e^T(t)\dot{e}(t) + e_p^T(t)\dot{e}_p(t) + ne^T(t)e(t) \\
&- \sum_{i=1}^{n} e^T(t-\tau_i-\delta)e(t-\tau_i-\delta) \\
&= e^T(t)F(X(t-\delta), X(t-\tau_i-\delta), t-\delta)e_p + e_p^T(t)\dot{e}_p(t) \\
&- e^T(t)(K-nE)e(t) - \sum_{i=1}^{n} e^T(t-\tau_i-\delta)e(t-\tau_i-\delta) \\
&= e_p^T(t)(F^T(X(t-\delta), X(t-\tau_i-\delta), t-\delta)e(t) + \dot{e}_p(t)) \\
&- e^T(t)(K-nE)e(t) - \sum_{i=1}^{n} e^T(t-\tau_i-\delta)e(t-\tau_i-\delta)
\end{aligned} \tag{22}$$

Substituting (19) into (22) yields

$$ {}_aD_t^q V(t) = -e^T(t)(K-nE)e(t) - \sum_{i=1}^{n} e^T(t-\tau_i-\delta)e(t-\tau_i-\delta) \tag{23}$$

Since $K-nE$ is a positive definite matrix, thus ${}_aD_t^q V(t) \le 0$, obviously $V(t)\ge 0$, so $\lim_{t\to\infty} e(t)=0$. Therefore the response of the system (13) and the drive system (16) tend to be synchronized.

3.4. Numerical Simulation

According to (16), the drive system the system (14) with delay-time δ is

$$\begin{cases}
\frac{d^{q_1}x}{dt^{q_1}} = -ax(t-\tau_1-\delta) + 3y(t-\delta)z(t-\delta) + y^2(t-\delta) \\
\frac{d^{q_2}y}{dt^{q_2}} = by(t-\tau_2-\delta) - x(t-\delta)z(t-\delta) + 2w(t-\delta) \\
\frac{d^{q_3}z}{dt^{q_3}} = -cz(t-\tau_3-\delta) + x(t-\delta)y(t-\delta) \\
\frac{d^{q_4}w}{dt^{q_4}} = -dw(t-\tau_4-\delta) + x(t-\delta) - y(t-\delta) - z(t-\delta)
\end{cases} \tag{24}$$

And according to (13), its response system is

$$\begin{cases}
\frac{d^{q_1}\tilde{x}}{dt^{q_1}} = -\tilde{a}\tilde{x}(t-\tau_1) + 3\tilde{y}(t)\tilde{z}(t) + \tilde{y}^2(t) + u_1(t) \\
\frac{d^{q_2}\tilde{y}}{dt^{q_2}} = \tilde{b}\tilde{y}(t-\tau_2) - \tilde{x}(t)\tilde{z}(t) + 2\tilde{w}(t) + u_2(t) \\
\frac{d^{q_3}\tilde{z}}{dt^{q_3}} = -\tilde{c}\tilde{z}(t-\tau_3) + \tilde{x}(t)\tilde{y}(t) + u_3(t) \\
\frac{d^{q_4}\tilde{w}}{dt^{q_4}} = -\tilde{d}\tilde{w}(t-\tau_4) + \tilde{x}(t) - \tilde{y}(t) - \tilde{z}(t) + u_4(t)
\end{cases} \tag{25}$$

where, $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and $\tilde{w}$ are the system state variables, $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ are unknown parameters of the system, $\tau_i > 0 (i=1,2,3,4.)$ are the lags system constants, $u_i(t)(i=1,2,3,4)$ are the synchronization controllers.

Let the system error equations are

$$\begin{cases}
\frac{d^{q_1}e_1(t)}{dt^{q_1}} = \tilde{x}(t) - x(t-\delta) \\
\frac{d^{q_2}e_2(t)}{dt^{q_2}} = \tilde{y}(t) - y(t-\delta) \\
\frac{d^{q_3}e_3(t)}{dt^{q_3}} = \tilde{z}(t) - z(t-\delta) \\
\frac{d^{q_4}e_4(t)}{dt^{q_4}} = \tilde{w}(t) - w(t-\delta)
\end{cases} \tag{26}$$

According to Theorem 1, the synchronization in time-delayed controllers are designed as

$$\begin{cases}
u_1(t) = -k_1e_1(t) + 3y(t-\delta)z(t-\delta) - 3\tilde{y}(t)\tilde{z}(t) \\
+ y^2(t-\delta) - \tilde{y}^2(t) + \tilde{a}(\tilde{x}(t-\tau_1) - x(t-\tau_1-\delta)) \\
u_2(t) = -k_2e_2(t) - x(t-\delta)z(t-\delta) + \tilde{x}(t)\tilde{z}(t) \\
+ 2w(t-\delta) - 2\tilde{w}(t) + \tilde{b}(y(t-\tau_2-\delta) - \tilde{y}(t-\tau_2)) \\
u_3(t) = -k_3e_3(t) + x(t-\delta)y(t-\delta) - \tilde{x}(t)\tilde{y}(t) \\
+ \tilde{c}(\tilde{z}(t-\tau_3) - z(t-\tau_3-\delta)) \\
u_4(t) = -k_4e_4(t) + x(t-\delta) - \tilde{x}(t) - y(t-\delta) + \tilde{y}(t) \\
- z(t-\delta) + \tilde{z}(t) + \tilde{d}(\tilde{w}(t-\tau_4) - w(t-\tau_4-\delta))
\end{cases} \tag{27}$$

And the unknown parameters adaptive laws as

$$\begin{cases}
\frac{d^{q_1}\tilde{a}(t)}{dt^{q_1}} = -x(t-\tau_1-\delta)e_1(t) + a \\
\frac{d^{q_2}\tilde{b}(t)}{dt^{q_2}} = y(t-\tau_2-\delta)e_2(t) + b \\
\frac{d^{q_3}\tilde{c}(t)}{dt^{q_3}} = -z(t-\tau_3-\delta)e_3(t) + c \\
\frac{d^{q_4}\tilde{d}(t)}{dt^{q_4}} = -w(t-\tau_4-\delta)e_4(t) + d
\end{cases} \tag{28}$$

where $K=[k_1,k_2,k_3,k_4]^T$ is a positive vector of the controllers. Therefore the response system (25) and the drive system (24) tends to be synchronized, that is, $\lim_{t\to\infty} e_i(t)=0 (i=1,2,3,4)$. When $q_1=q_2=q_3=q_4=0.95$, $a=2$, $b=3.65$, $c=8$, $d=12$, $\tau_1=0.03, \tau_2=0.02$,

$\tau_3 = 0.01, \tau_4 = 0.08$, $\delta = 2$. The initial values for the drive system (24) and the response system (25) are given as $[-4,-2,2,1]$ and $[1,1,-3,-1]$. The initial values for the unknown parameters adaptive laws and the parameters of the controllers K are $[-1,-2,-3,-4]$ and $[5,5,5,5]^T$. Fig. (**7**) (I) shows the system error equations, and Fig. (**7**) (II) shows the identification of the unknown parameters.

Simulation results show that the proposed method in accordance with the design of synchronous controllers and adaptive laws can make the response system (25) and the drive system (24) tend to be synchronized, it demonstrates that the design method is effective. Further analysis showed that reaches the required to synchronize the system time with the initial value of the parameter, the delay time, and control the size of the parameter are closely related, due to space limitations, it isn't detailed discussion in the paper.

CONCLUSION

A novel four-dimensional fractional-order chaotic system is constructed. Numerical simulation and circuit experiment show that there exist chaotic behaviors and demonstrate that the new chaotic system can be implemented on the physical. For a class of parameter uncertainty fractional different-lags chaotic systems, this paper designs synchronization in time-delayed controllers and adaptive laws, and it proved that the drive system and the response system tend to be synchronized, when the control parameter matrix K satisfies certain conditions. The numerical simulations show that it is universality and effectiveness of different-lags synchronization in time-delayed system method design.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos: 11062002, 61363076), and the Key Science and Technology Project of Jiangxi Provincial Department of Education (Grant Nos: GJJ13435, GJJ14465), the National Natural Science Foundation of Jiangxi Province (Grant No: 2014BAB207020), and the Innovation Fund for Graduate Students in Jiangxi Province (Grant No: YC2013-S197).

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Received: December 15, 2014 Revised: January 04, 2015 Accepted: February 25, 2015