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Using an Improved Joint Normal Transform Method for Modeling Stochastic Dependence in Power System

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Abstract: The dependence factors in power systems should be considered in stochastic power flow computation, so Joint Normal Transform (JNT), belonging to the copula function technology, is improved to model these dependences. Firstly, the procedure of traditional JNT method is introduced and the principle of correlation structure's remaining unchanged is analyzed combined with the properties of rank correlations when JNT method is utilized in dependence modelling. Then, an improved JNT sampling method is proposed to raise sampling efficiency by applying Orthogonal Transformation according to the characteristic that JNT method is based on Normal Distribution. Finally, a calculation example is designed to verify the feasibility of the proposed improved JNT sampling method.

Keywords: Dependence modeling, JNT, Monte Carlo Method, stochastic power flow.

1. INTRODUCTION

Since the constraints of the environment and resources to the energy industry are becoming more serious, the application of distributed new energy sources, such as wind power, is experiencing rapid development. However, the large-scale integration of these renewable energy systems with stochastic output has brought about uncertainty analysis problems in the operation and planning of power systems [1][2][3]. In system planning, quantifying the power generation uncertainty is necessary for evaluating the variation range of system power flow, which is central for the system dimensioning. In system operation, the uncertainty analysis can be considered as uncertainty forecast. The combination of system management with this uncertainty forecast is essential for the optimal operation of the power systems with high penetration of distributed renewable energy sources [4][5]. Therefore, as a tool for studying the uncertainty problems, stochastic power flow computation (SPFC) has drawn wide attention of academics [6][7][8]. While computing stochastic power flow, the dependence factors are always necessary to be taken into consideration for their influence on power systems' planning and operation, such as the strong correlation of similar loads in the same area and the wind speed and power of wind farms from near geographical location [9][10].

At present, many achievements have been obtained in the field of SPFC with the dependence of random input variables considered. In [6], a cumulant method of SPFC was proposed to process the dependence of input variables. In [11], incorporating three-point estimate method with the third-order polynomial normal transformation (TPNT) technique, a method was proposed to solve probabilistic power flow problems with non-normal dependent variables. In [12], inverse Nataf transformation was used to generate correlated wind speed samples to get the correlating wind farms' generations. However, both TPNT technique and Nataf transformation adopt product moment correlation coefficient (PMCC). The PMCC in multivariate actual domain is converted to the PMCC in multivariate normal domain by solving corresponding non-linear equations, which will necessarily occupy a certain period of calculation time.

Joint Normal Transform (JNT), one kind of copula function method, incorporates rank correlation coefficient (RCC), which keeps constant during the whole transformation process. JNT method can avoid the procedure of solving complicated non-linear equations to implement the PMCC transformation in TPNT and Nataf methods, so the sampling process of JNT is more efficient. Besides, this method can establish correlation models of random variables submitting to arbitrary distribution, which makes JNT achieve wide application in the research of relevance modeling in power systems [13].

Nevertheless, when JNT method is utilized to establish correlation models and obtain relevant sampling vectors, it is necessary to sample the variable vectors following multivariate normal distribution or the copula function corresponding with JNT method, which will definitely have negative influence on sampling speed and efficiency [13-15].

In this paper, the basic procedures of traditional JNT method were generally introduced and the reason for which the dependence structure can keep constant in the process of JNT modeling was analyzed based on the properties of RCC. Then, on the premise of maintaining the domain transformation concept, a one-dimensional normal distribution domain was added into the three multivariate domains of traditional JNT method and Orthogonal Transformation was applied to realize the conversion from

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one-dimensional normal distribution domain to multivariate normal domain, so the sampling work can be launched in one-dimensional normal distribution domain. That means sampling of multivariate distribution function can be replaced with repeated and independent sampling of univariate standard normal distribution function. For this reason, the algorithm complexity and sampling time were reduced, so the efficiency of improved JNT method achieved great promotion. This is the most important contribution of this paper. Finally, a calculation example is designed to verify the feasibility of the proposed improved JNT sampling method.

2. PMCC AND RCC

2.1. PMCC

PMCC is widely used to measure the linear correlation of two random variables. Supposing that there are two variables: X and Y, their pair data at n observation points are denoted as (X_1, Y_1), (X_2, Y_2), ……, (X_n, Y_n). According to these pair data, the values of their deviation from the two variables' expectations can be calculated, denoted by (x_1, y_1), (x_2, y_2), ……, (x_n, y_n), where $x_i = X_i - \overline{X}$, $y_i = Y_i - \overline{Y}$, i = 1, 2, 3, …, n.

Based on the products of pair data deviations, the PMCC can be calculated as the following equation (1):

$$\rho = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \quad (1)$$

where ρ stands for the PMCC of the data of the two variables X and Y [16][17].

2.2. RCC

The two random variables X and Y are described the same as the last section. In this section, both X and Y are sorted in ascending or descending order at the same time so that the rank set of X and Y are obtained and denoted by x and y. x_i stands for the rank of X_i in the data set X and the definition of y_i can be informed analogically. The RCC of X and Y can be calculated from the rank sets x and y by the equation (2):

$$\rho_r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}} \quad (2)$$

where ρ_r stands for the RCC of the data of the two variables X and Y [18][19].

2.3. Characteristics and Relationship of RCC and PMCC

PMCC is easy for computation and it can accurately represent the correlation of variables submitting to elliptical distribution. However, when this condition is not satisfied, the value of PMCC loses significance. PMCC can remain unchanged after linear transformation of random variables, while after nonlinear strictly increasing transformation, their PMCC will alter.

By contrast, RCC always exists and will not be influenced by the distribution function of variables, so after nonlinear strictly increasing transformation, RCC of random variables can keep invariant.

PMCC ρ and RCC ρ_r can convert to each other by equation (3):

$$\rho_r(X,Y) = \rho(F_X(X), F_Y(Y)) \quad (3)$$

where F_X and F_Y are separately the cumulative distribution functions (cdf) of X and Y.

Particularly, when both X and Y submit to uniform distribution, $\rho = \rho_r$; when both X and Y submit to normal distribution, the conversion equation of ρ and ρ_r are as follows:

$$\rho(X,Y) = 2\sin\left(\frac{\pi}{6}\rho_r(X,Y)\right) \quad (4)$$

The equation (4) is important in the dependence modeling process by means of JNT in this paper [13].

3. TRADITIONAL JNT METHOD

3.1. Procedures of Traditional JNT

The main purpose of dependence modeling is separating the influence of different marginal distributions from the dependence structure and copula functions are often used. JNT method, belonging to copulas, is corresponding to multivariate normal distribution and can construct a joint distribution function which unites one-dimensional variables following different marginal distributions. All the margins of this joint distribution are uniform distribution on the interval [0, 1].

Assuming that there are n relevant random variables to be modeled, the concrete steps of traditional JNT method are as follows:

1) Compute the RCC ρ_r of any two of n variables x_1, x_2, ……, x_n to form the RCC matrix R_r;
2) According to equation (4), convert the RCC matrix R_r to the PMCC matrix R;
3) Form the joint normal distribution function of JNT method based on R. Sample this distribution function directly and obtain n-dimensional standard normal vector $\vec{N}$ = (n_1, n_2, ……, n_n). Transform $\vec{N}$ to by one-dimensional standard normal cdf Φ. $\vec{U}$ = (u_1, u_2, ……, u_n) and each component follows uniform distribution on the interval [0, 1] [13].
4) On basis of the marginal distribution functions of $\vec{X}$ and their inverse functions, apply inverse transformation to u_1, u_2, ……, u_n to achieve the samples of x_1, x_2, ……, x_n [20].

3.2. Analysis of the Correlation in JNT Method

What need to be clarified first is that the dependence keeping unchanged in JNT method is the rank correlation of the original input variables.

Referring to the step 1 and 2 of traditional JNT method mentioned in section 3.1, the matrix R_r should be calculated first, and then equation (4) is used to transform R_r to R for the reason that the parameter set of multivariate normal distribution includes the PMCC matrix. Since the expectation of each component random variable is zero, the expression of multivariate normal distribution function can be determined and its components remain the primary rank correlation of input variables.

Above is the preparation before domain transformation. The domain transformation happens twice and there are three domains in the JNT method, shown in Fig. (**1**).

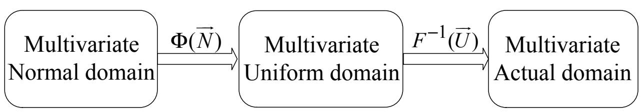

Fig. (1). Domain transformation of JNT method.

It can be observed that two domain transformations, normal distribution function $\Phi(\cdot)$ and the inverse functions of all components' margins $F^{-1}(\cdot)$, are both nonlinear strictly increasing transformations. According to the characteristics mentioned in section 2.3, RCC remains unchanged under these domain transformations. Hence, the final variables obtained in actual domain will maintain the same RCC as the variables in normal domain, which is also consistent with the RCC of primary input variables. That means the sampling series of all variables achieved can keep the rank correlation structure established at the beginning of JNT method.

4. IMPROVED JNT METHOD

When using traditional JNT method to model the dependence of variables, the problem of joint distribution sampling is unavoidable in both multivariate normal domain and uniform domain. The methods such as conditional density method and multi-dimensional acceptance-rejection method will be employed for multivariate distribution sampling. However, these methods will decrease the efficiency of JNT and occupy plenty of computing time. For example, conditional density method needs to calculate the conditional density function value before achieving a single sample for each component variable at every turn, which is uneconomical when sampling scale is large.

On the premise of preserving the domain transformation, a one-dimensional normal distribution domain was added into the three multivariate domains of traditional JNT method and Orthogonal Transformation was applied to realize the conversion from one-dimensional normal distribution domain to multivariate normal domain, so the sampling work can be launched in one-dimensional normal distribution domain. Therefore, the improved JNT method only need to repeat independent sampling for n times (n is the number of input variables) with the correlation structure neglected temporarily. Then, one linear and two nonlinear transformations are conducted on the aforementioned samples to obtain a sample group considering the variable correlation. The improved algorithm reduces the calculation time apparently. The concrete steps are as follows:

1) Calculate the RCC matrix R_r and convert R_r to the PMCC matrix R ccording to equation (4);

2) Every component's variance of standard normal distribution is equal to 1, so the values of corresponding elements of R and Σ are the same. Since multi-dimensional normal distribution function requires the covariance matrix being positive definite, if R is not positive definite, a method should be employed to repairing the violations of positive definiteness to form a positive definite matrix that is as close as possible to the original one [21]. Then cholesky decomposition method is utilized to decompose R into the product of A and A^{T}.

3) Repeat the independent sampling procedure of one-dimensional random variable for n times and compose a sampling vector = (η_1, η_2, ……, η_n).

   Transform to the n-dimensional sampling vector in multivariate normal domain by Orthogonal Transformation shown in equation (5):

$$\vec{N} = A\vec{\eta} \tag{5}$$

   where $\vec{N}$ = (n_1, n_2, ……, n_n).

4) Convert $\vec{N}$ to a sampling vector in multivariate uniform domain by $\Phi(\cdot)$:

$$\vec{U} = \Phi(\vec{N}) = (u_1, u_2, \cdots, u_n) \tag{6}$$

   $\Phi(\cdot)$ can be stored in the form of discrete data pairs. The function value between two neighbouring points can be approximately calculated by means of linear interpolation.

5) Apply inverse transformation to u_1, u_2, ……, u_n to achieve the samples of x_1, x_2, ……, x_n.

5. CASE STUDY

An example was designed based on IEEE-30 node network for the correctness validation of improved JNT method. The power injection of node 2, 7 and 12 was considered to have dependence in this chapter.

5.1. Modeling of Margins and Dependence Structure

Before the verification, several adjustments were made to IEEE-30 node network. The generation power at node 2 was regarded as stochastic wind power. The original generation active power of node 2 in IEEE-30 network was set as the rated power of the wind farm. Meanwhile, it was assumed that the load power at node 2, 7 and 12 followed the same distribution law to fluctuate. The original load active power values of these nodes were set as the high load mean values. The settings were important in the next part of modeling margins.

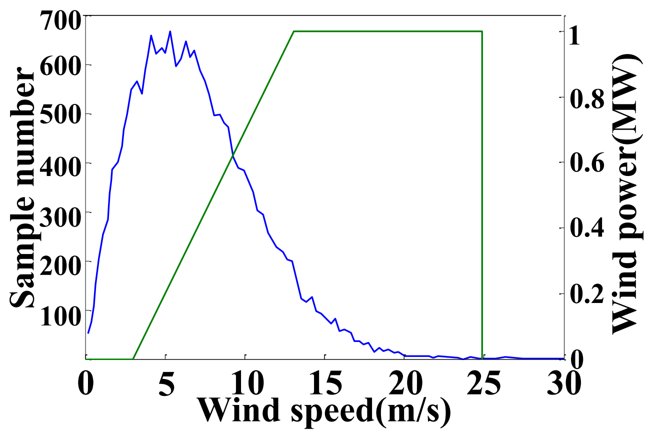

Fig. (2). Curves of typical pdf of wind speed and conversion characteristics of wind turbine.

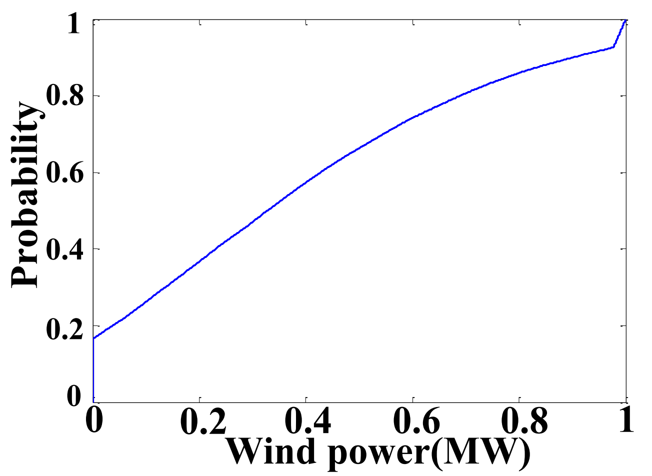

Fig. (3). Cdf curve of wind power.

In Fig. (**2**), the left curve reflected the typical wind speed distribution, which was derived from sampling data by statistical treatment. The right one was the typical simplified characteristic curve of power converter, representing the relationship of wind speed and the output power of a single wind turbine, as displayed in equation (7).

$$P_W = \begin{cases} 0 & v < v_{in}, v > v_{out} \\ \dfrac{v - v_{in}}{v_R - v_{in}} P_R & v_{in} \le v \le v_R \\ P_R & v_R \le v \le v_{out} \end{cases} \tag{7}$$

In the equation above, P_W was the actual output of wind turbine. Cut-in speed v_{in} = 3.00 m/s. Rated speed v_R = 13.13 m/s. Cut-out speed v_{out} = 24.86 m/s. The rated output P_R of a single wind turbine was 1MW.

The cdf curve of wind power could be derived from the curves shown in Fig. (**2**), which was delineated in Fig. (**3**). Hence, the actual margin of wind output power was achieved.

The probability density function (pdf) and cdf of load power at node 2 were obtained by counting the sampling data in Figs. (**4**, **5**). This curve shape was representative and also suitable for node 7 and 12. It was just necessary to conduct reduction in proportion of the high load mean value of each node.

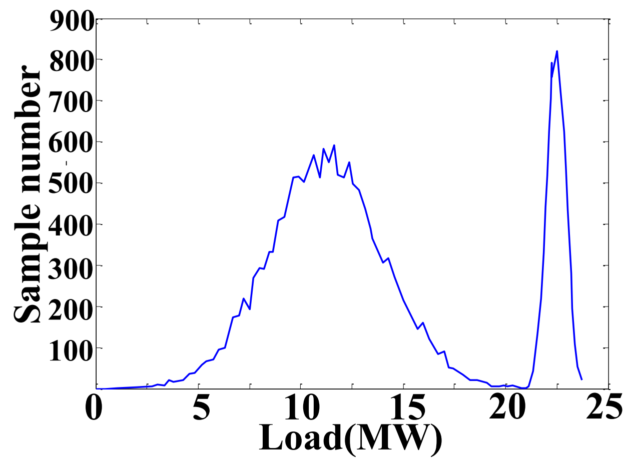

Fig. (4). Pdf curves of load at node 5.

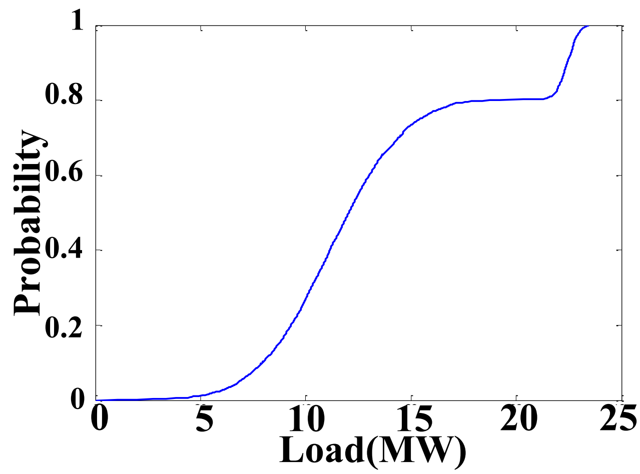

Fig. (5). Cdf curves of load at node 5.

At the same time, all the wind turbines at node 2 were integrated and represented by a single variable following the margin shown in Fig. (**3**).

The power factor of wind farm output at node 2 and the load at 2, 7 and 12 could be calculated according the proportions of original active and reactive power values at corresponding nodes in IEEE-30 network.

So far, the margins of all the random variables were determined.

According to the historical time series of the load power and wind output, equation (2) was employed to compute their RCC, which was approximately 0.191. Referring to literature [18], the RCC of the load in the same local area was 0.582. Based on the analysis above, the RCC matrix R_r of the wind power and load at node 2, 7 and 12 was constructed as below:

$$R_r = \begin{pmatrix} 1 & 0.191 & 0.191 & 0.191 \\ 0.191 & 1 & 0.582 & 0.582 \\ 0.191 & 0.582 & 1 & 0.582 \\ 0.191 & 0.582 & 0.582 & 1 \end{pmatrix} \quad (8)$$

Thus the correlation for these variables model was established.

5.2. Verification of JNT Method

Utilizing the improved JNT method mentioned in section 4.1, 10000 samples were obtained for each of the four variables respectively corresponding to the wind power and load at node 2, 7 and 12. By processing these samples, the proposed improved JNT method in this paper was verified in the aspects of margins and correlation structures separately.

5.2.1. Verification of Margin Modeling

By analyzing the samples with statistical method, the cdf curves of a single wind turbine's output and active load power were achieved. Each of these two curves was drawn with the statistical result curve of its corresponding historical time series in the same picture, so that the improved JNT samples could be compared with the original actual distribution derived from the time series to verify the correctness, shown in Figs. (**6**, **7**).

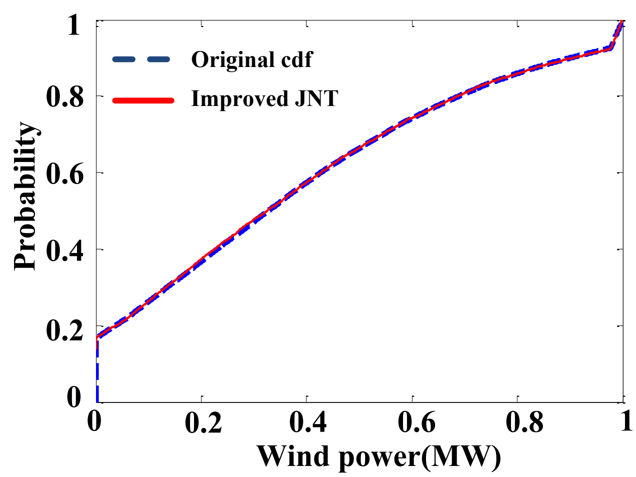

Fig. (6). Comparison of the Cdf curves of wind power at node 2.

From these two pictures, the improved JNT cdf curve nearly coincided with the original cdf, which means that these samples could represent the variable margins precisely. Hence, the improved JNT method is feasible for margin modeling.

5.2.2. Verification of Dependence Structure

RCC matrix was calculated with the aforementioned samples to confirm the validity of improved JNT method in the aspect of dependence modeling, which was shown below.

$$R_r' = \begin{pmatrix} 1 & 0.1904 & 0.1943 & 0.1865 \\ 0.1904 & 1 & 0.5726 & 0.5634 \\ 0.1943 & 0.5726 & 1 & 0.5592 \\ 0.1865 & 0.5634 & 0.5592 & 1 \end{pmatrix} \quad (9)$$

For the purpose of comparing the RCC matrixes calculated separately before and after sampling intuitively, R_r' in equation (9) was subtracted from R_r in equation (8). The difference was changed into absolute value displayed in equation (10):

$$\left|R_r' - R_r\right| = \begin{pmatrix} 0 & 0.0006 & 0.0033 & 0.0045 \\ 0.0006 & 0 & 0.0094 & 0.0186 \\ 0.0033 & 0.0094 & 0 & 0.0228 \\ 0.0045 & 0.0186 & 0.0228 & 0 \end{pmatrix} \quad (10)$$

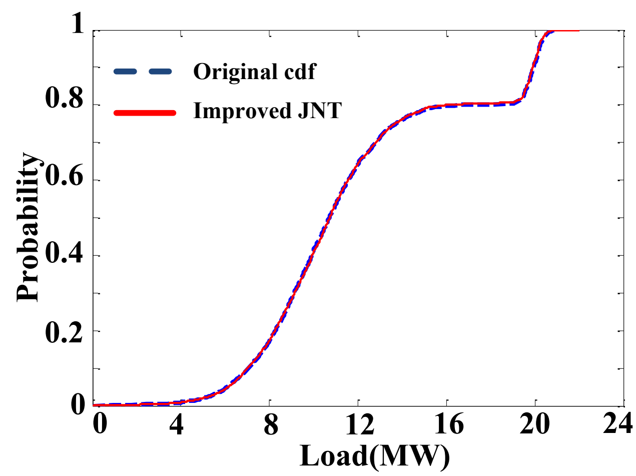

Fig. (7). Comparison of the Cdf curves of load at node 2.

According to equation (10), the value difference of the corresponding elements in R_r and R_r' was negligibly small, whose maximum relative error was about 3.92% (0.0228/0.582 ≈ 3.92%). Therefore, when considering the randomness in sampling and engineering practicability, the samples obtained by improved JNT method can deoxidize the correlation model derived from the historical time series of each input variable accurately.

In conclusion, improved JNT method is practical and valid in the modeling of both margins and dependence structures.

CONCLUSION

In this paper, an improved JNT method was introduced into the dependence modeling work of distributed renewable sources and load. the reason for which the dependence structure can maintain unchanged in the process of JNT modeling was analyzed based on the properties of RCC that it keeps constant under nonlinear strictly increasing transformation. According to characteristics of normal distribution, the sampling procedures of traditional JNT method were improved. Orthogonal Transformation was applied so that sampling of multivariate distribution function can be replaced with repeated and independent sampling of univariate standard normal distribution function. For this reason, the algorithm complexity and sampling time were reduced. The samples of improved JNT method were processed by statistical method to acquire the margins and dependence structures of variables. Finally, a calculation example is designed to verify the feasibility and accuracy of the proposed improved JNT sampling method. The proposed method can be applied in the planning and operation of power systems to provide technical support in the research of reliability analysis after clean energy's integration and so on.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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