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Harmonic Estimation Algorithm for Power System Based on Improved MUISC and Linear Neural Networks

Sun Ligong*, Sun Xiangwen and Xiang Fei

Electrical Engineering College, Henan University of Science and Technology, Luoyang, Henan, 471023, P.R. China

Abstract: The paper proposes harmonic estimation algorithm for power system based on Multiple Signal Classification (MUSIC) and linear neural network because of the insufficiency of harmonic frequency estimation algorithm. The conventional MUSIC algorithm has the advantage of higher estimation accuracy, while the disadvantage is that the computational complexity is high and it cannot estimate the harmonic phase and amplitude. In the paper, a new harmonic estimation algorithm for power system is constructed with combining the MUSIC algorithm, the multistage Wiener filter (MSWF) and linear neural network. Theoretic analysis and simulation experiments show that the requirement to data is relatively low, and has good harmonic estimation accuracy and reliability.

Keywords: Harmonic estimation, MSWF, fast subspace decomposition, neural network

1. INTRODUCTION

For the 50Hz power system, the frequency spectrum of voltage and current is obtained by Fourier transformation, in which the spectrum component of integer multiple of 50Hz is called harmonic. It is well known that when the harmonic component of the grid is beyond the standard, the security and reliability of power system and electrical equipment is influenced seriously. Therefore the rapid detection method of harmonic becomes the research hot point, and a lot of achievements are obtained, such as fast Fourier transform (FFT) algorithm [1], singular value decomposition (SVD) algorithm [2], wavelet transform algorithm [3], artificial neural network (ANN) algorithm [4], multiple signal classification (MUSIC) algorithm [5], and so on.

Theoretically, the precise harmonic frequency, amplitude and phase can be obtained from the accurate spectrum of signal with the synchronous sampling of FFT algorithm. However, the actual power grid frequency is often fluctuated near 50Hz, which causes nonsynchronous sampling. In the condition of nonsynchronous sampling, the spectrum leakage and picket fence effect will be appeared, which leads to big errors in harmonic estimation. The results can be corrected by interpolated FFT, but with high load [6].

The methods of SVD [2], wavelet transform [3] and ANN improve the accuracy of harmonic detection, but with high complexity and more priori information, which cannot satisfy the needs of real-time harmonic detection.

MUSIC decomposes the signal feature space into signal subspace and its orthogonal noise subspace through the eigen decomposition of auto-correlation matrix of sampled signal. Pseudo space spectrum is constructed using the orthogonality of the signal subspace and the noise subspace.

Unknown parameters are estimated through searching the peak of the pseudo space spectrum [5]. Theoretically, it can restrain noise as much as possible owing to making use of the orthogonality of the signal subspace and the noise subspace, so it has high precision and good statistical properties. The algorithm has some disadvantages as follows, (1) high load; (2) only estimating spectrum frequency because the information about harmonic phase and amplitude is lost during the course of constructing pseudo space spectrum. The paper presents a new algorithm aiming at the shortcomings above, which achieves high-precision detection to harmonic frequency, amplitude ad phase with low computational complexity.

After analyzing the principle to MUSIC algorithm, it is not difficult to find that one of the main reason causing high computational complexity is that the algorithm needs to calculate the sample covariance matrix and carries out eigen decomposition. It is well known that for the eigen decomposition of N-dimension matrix the computational complexity is $O(N^3)$. The computational load is amazing when N is large. Another reason is that the algorithm needs to calculate point-by-point in the whole frequency domain and compare in order to find the peak of space spectrum. Meanwhile, if high estimation accuracy is required, the search step should not be too large; otherwise it will lead to the picket fence effect. So the improvements of the algorithm can be divided into the following three steps, (1) finding a rapid and effective method of subspace decomposition in order to reduce the computational load; (2) finding a fast search method to reduce the complexity of spectral peak searching; (3) making full use of the results of harmonic frequency estimation to construct estimation method of the amplitude and phase.

The paper organizes around the three problems mentioned above. Firstly, it discusses the principle of harmonic frequency estimation based on MUSIC algorithm. Secondly, it introduces the technology of fast subspace decomposition using MSWF recurrence. Thirdly, it describes estimation methods of harmonic numbers M. Fourthly, it introduces the strategy speeding up the spatial spectrum peak search using FFT. Fifthly, it discusses the estimation method for harmonic amplitude and phase using linear adaptive neural network. At last, numerical simulations verifies the harmonic estimation algorithm proposed in this paper.

2. SIGNAL MODEL

There are noise signals, 50Hz signals and harmonic signals in power systems, whose sampling signals of voltage and current signals can be expressed as,

$$x(n)=\sum_{i=1}^{M}A_i\cos(2\pi f_i n+\phi_i)+e(n) \tag{1}$$

where $x(n)$ is sampling signal, A_i, f_i and ϕ_i are the i^{th} harmonic amplitude, normalized frequency and initial phase, respectively, n is sampling time, $e(n)$ is white noise, and M is harmonic number.

Assume $\omega_i=2\pi f_i$, which is the i^{th} harmonic normalized angular frequency. Transform $x(n)$ into complex frequency as,

$$x(n)=\sum_{i=1}^{M}A_i e^{j(2\omega_i n+\phi_i)}+e(n) \tag{2}$$

Construct sampling matrix $X\in C^{K\times q}$ with K ($M<<K$) rows and q columns using N sampling points,

$$X=\begin{bmatrix}x_1(n)\\x_2(n)\\ \mathrm{M}\\x_q(n)\end{bmatrix}^T=\begin{bmatrix}x(1)&x(2)&\mathrm{M}&x(q)\\x(2)&x(3)&\mathrm{M}&x(q+1)\\ \mathrm{L}&\mathrm{L}&&\mathrm{L}\\x(K)&x(K+1)&\mathrm{M}&x(N)\end{bmatrix} \tag{3}$$

where, $x_i(n)=[x(i),x(i+1),\mathrm{L}\ \ x(i+K-1)]^T\in C^{K\times 1}$ is the i^{th} column of X.

Note

$$A=\begin{bmatrix}1&1&\mathrm{M}&1\\e^{j\omega_1}&e^{j\omega_2}&\mathrm{M}&e^{j\omega_M}\\ \mathrm{L}&\mathrm{L}&&\mathrm{L}\\e^{j(K-1)\omega_1}&e^{j(K-1)\omega_2}&\mathrm{M}&e^{j(K-1)\omega_M}\end{bmatrix}\in C^{K\times M}$$

$$S(n)=\begin{bmatrix}A_1e^{j(n\omega_1+\varphi_1)}&A_2e^{j(n\omega_2+\varphi_2)}&\mathrm{L}&A_Me^{j(n\omega_M+\varphi_M)}\end{bmatrix}^T\in C^{M\times 1}$$

$$E(n)=\begin{bmatrix}e(n)&e(n+1)&\mathrm{L}&e(n+M-1)\end{bmatrix}^T\in C^{M\times 1}$$

So formula (2) can be expressed as $X=AS+E$, the autocorrelation matrix of X is

$$R_x=APA^H+\sigma^2 I\in C^{K\times K}.$$

On the law of matrix theory, rank(R_x) $=M<K$, therefore R_x should have M non-zero eigenvalues, which are λ_1, λ_2, ..., λ_M according to the sequence from large to small. The corresponding eigenvectors V_1, V_2, ..., V_M constitute signal subspace $V_s=[V_1, V_2, ..., V_M]$. The rest K-M eigenvalues λ_i are the smallest and approximately equal, i.e., $\lambda_{M+1}=\lambda_{M+2}=\cdots=\lambda_{K_\circ}$ The corresponding eigenvectors V_{M+1}, V_{M+2}, ..., V_K constitute noise subspace $V_n=[V_{M+1}, V_{M+2}, ..., V_K]$. Then

$$R_x=V_s\Sigma_s V_s^H+\sigma_n^2 V_n V_n^H \tag{4}$$

3. BASIC PRINCIPLE OF HARMONIC FREQUENCY ESTIMATION BASED ON MUSIC ALGORITHM

To construct the signal direction vector,

$$a(\omega)=\begin{bmatrix}1&e^{j\omega}&\mathrm{L}&e^{j(K-1)\omega}\end{bmatrix} \tag{5}$$

Because the direction vector corresponding to the signal component and the eigenvector of noise subspace are orthogonal, i.e., for harmonic component,

$$\begin{aligned}f(\omega)&=a^H(\omega)V_nV_n^Ha(\omega)\\&=a^H(\omega)(I-V_sV_s^H)a(\omega)\\&=0\end{aligned} \tag{6}$$

Then the harmonic frequency can be estimated through searching the peak of the space spectrum,

$$P_{MUSIC}=\frac{1}{f(\omega)}\qquad \omega=1,2,\cdots,M \tag{7}$$

4. FAST SUBSPACE DECOMPOSITION BASED ON MSWF

As mentioned before, when the harmonic is estimated by MUSIC algorithm, the eigen decomposition of sample autocorrelation matrix is required, which brings high computational complexity and is not conductive to real-time process. In order to reduce computational complexity, many researchers have done a lot of research and achieved many valuable results, such as Jacobi iteration, QR iteration, power method, Lanczos iteration and so on. These methods avoid eigen decomposition of the sample covariance matrix, which reduce the computational complexity to a certain extent, but still need to calculate the covariance matrix, which cause high computational complexity as usual.

Goldstein proposed MSWF in 1997, which was a reduced rand Wiener filter realization form and consisted of a decomposition filter and a composite filter. Accordingly, MSWF algorithm is divided into forward recursion and backward recursion algorithm [6, 7].

The basic steps of MSWF algorithm are given below:

1. Construct matrix X using N sample points according to formula (3). Definite training signal $d_0(k)=e_1^Tx_1(k+\tau)$ and observing signal $X_0=[x_2,x_3,\cdots,x_p]^T$.

2. M-times forward recursion:

For i=1, 2, …, M

$$h_i = E[d_{i-1}^*(k)X_{i-1}(k)] \Big/ \left\| E[d_{i-1}^*(k)X_{i-1}(k)] \right\|_2$$

$$d_i(k) = h_i^{\mathrm{H}} X_{i-1}(k)$$

$$X_i(k) = X_{i-1}(k) - h_i d_i(k)$$

Consider m-dimensional Krylov subspace $\kappa^m(B,f) = Span\{f, Bf, \cdots, B^{m-1}f\}$ which is consisted of arbitrary matrix $B \in C^{N\times N}$ and arbitrary vector $f \in C^{N\times 1}$. It is known by the nature of the Krylov subspace that

$$\kappa^m(R_x, f) = \kappa^m(R_x - \sigma_n^2 I_N, f) \tag{8}$$

Substitute formula (4) into (8), the following formula (9) can be obtained,

$$\begin{aligned} \kappa^m(R_x, f) &= \kappa^m(R_x - \sigma_n^2 I_N, f) \\ &= \kappa^m(V_s \Sigma_s V_s^H + \sigma_n^2 V_n V_n^H - \sigma_n^2 I_N, f) \\ &= \kappa^m(V_s(\Sigma_s - \sigma_n^2 I)V_s^H, f) \end{aligned} \tag{9}$$

Therefore if $f \in V_s$, $\kappa^m(R_x, f)$ is the signal subspace exactly. As long as solving a group of basis, V_s can be obtained. It has been proved that when $e(n)$ is white noise, $d_0(k) = e_1^T x_0(k+\tau) \in V_s$ [8]. So V_s can be replaced by $\kappa^m(R_x, d_0(k))$, avoiding eigen decomposition of R_x. Meanwhile, according to the MSWF theory, $\{h_1, h_2, \ldots, h_M\}$ generated by M-times forward recursion is a group of standard orthogonal basis, i.e.,

$$span\{h_1, h_2, \cdots h_M\} = \kappa^m(R_x, d_0(k)) = V_s \tag{10}$$

Therefore, the fast calculation of signal subspace V_s can be achieved using MSWF.

5. ESTIMATING THE NUMBER OF SIGNAL *M*

As mentioned before, the dimension of signal subspace M is required to obtain the signal subspace using MSWF. Apparently M cannot be obtained directly in the actual situation.

Theoretically, when the real covariance matrix R_x is known, as mentioned before, the dimension of signal subspace M is required to obtain the signal subspace using MSWF.

Apparently R_x cannot be obtained directly in the actual situation, and the approximate estimation $\widetilde{R_x}$ can just be obtained. So the number of harmonic M must be estimated by other effective methods. At present, the two common criterions are AIC criterion and MDL criterion used to estimate the dimension of signal subspace. In comparison, the AIC criterion is not the consistent estimation of M and over-estimation will appear with fewer samples. In contrast, MDL criterion is the consistent estimation, which is described below,

$$MDL(k) = -\ln\left(\frac{\prod_{i=k+1}^{M} \lambda_i^{\frac{1}{M-k}}}{\frac{1}{M-k}\sum_{i=k+1}^{M}\lambda_i} \right)^{(M-k)N} + \frac{k}{2}(2M-k)\ln N$$

where the k ($k \in [0, 1, \ldots, M$-1]) is the signal subspace dimension number M when the minimum value of the formula is obtained. Obviously, the eigenvalue of $\widetilde{R_x}$ is required, which needs to calculate $\widetilde{R_x}$ and carry out eigen decomposition. It can be proved that in the absence of eigenvalues of covariance matrix, the MDL criterion can be calculated by the following formula using MSWF recursive results,

$$MDL(k) = -\ln\left(\frac{\left(\prod_{i=k+1}^{M} \sigma_{d_i}^2\right)^{\frac{1}{M-k}}}{\frac{1}{M-k}\sum_{i=k+1}^{M}\sigma_{d_i}^2} \right)^{(M-k)N} + \frac{k}{2}(2M-k)\ln N$$

where $\sigma_{d_i}^2 = E(|d_i|^2)$. Thus the information generated by the MSWF recursion can be made full use of to obtain the estimation of signal subspace dimension number M.

6. PEAK SEARCHING

It is not difficult to find that $f(\omega)$ is a periodic function of ω, whose Fourier expansion is,

$$f(\omega) = \sum_{m=-\infty}^{\infty} F_m e^{jm\omega} \tag{11}$$

where $F_m = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(\omega)e^{-jm\omega}d\omega$. Truncate it to a length of 2M-1 point, then

$$f(\omega) \simeq \sum_{m=-M}^{M} F_m e^{jm\omega} = \sum_{m=-M}^{M} F_m z^m = P(z) \tag{12}$$

where $z = e^{j\omega}$. So the harmonic angular frequency can be estimated through solving the roots on the unit circle of equation $P(z) = 0$. The coefficient F_m can be approximated by $\hat{F}_m = \frac{1}{2\pi}\sum_{l=-(M-1)}^{M-1} f(l\Delta\omega)e^{-jml\Delta\omega}$, which is the FFT of $f(\omega)$, where $\Delta\omega = \frac{2\pi}{2L-1}$. As long as M is larger, the truncation error is smaller. But when M is larger, the computational load of solving roots of $P(z) = 0$ will increase. So zero point can be inserted into $P(z)$, and the new equation is,

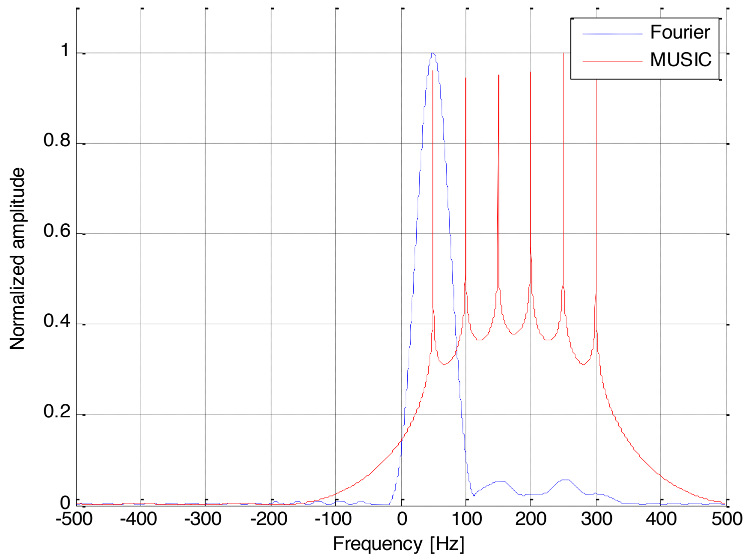

Fig. (1). Analysis Results of Experiment 1 time sequences.

$$P(\mathrm{z}) = \sum_{m=-j+1}^{j-1} F_m e^{jm\omega} = \sum_{m=-(M-1)}^{M-1} F_m e^{jm\omega} = \sum_{m=-(M-1)}^{M-1} F_m z^m \quad (13)$$

By solving the roots of equation (11) in unit circle, the approximate estimation of harmonic frequency can be achieved.

7. ESTIMATION OF HARMONIC AMPLITUDE AND PHASE

Rewrite formula (2) into

$x(n) = \sum_{i=1}^{M} e^{j2\omega_i n} A_i e^{j\phi_i} + e(n)$ (14) where $x(n)$ can be regarded as the out of linear function with m unknowns and $e^{j2\omega_i n}$ as variables. Obviously, if the coefficient $A_i e^{j\phi_i}$ of $e^{j2\omega_i n}$ can be solved, the harmonic amplitude A_i and phase ϕ_i can be achieved easily.

It is well known that artificial neural networks can approximate arbitrary function with high accuracy. Therefore a neural network can be constructed to approximate the function with $e^{j2\omega_i n}$ as input and $x(n)$ as output so as to solve $A_i e^{j\phi_i}$. Considering the linear relationship of $x(n)$ and $A_i e^{j\phi_i}$, in order to reduce computational load, the paper does not adopt BP or RBF neural network, instead of utilizing a linear network with single neuron to realize the method.

Input vector, weight vector and output are expressed as

$r(n) = [e^{j2\omega_1 n}, e^{j2\omega_2 n}, \cdots, e^{j2\omega_M n}]$, $W = [A_1, \cdots, A_M]^T$ and $\hat{x}(n) = r(n)W$, respectively. Formula (14) can be written as $x(n) = r(n)W + e(n)$. Weight vector W can be solved by minimizing $\left|\hat{x}(n) - x(n)\right|$, then harmonic amplitude A_i and phase ϕ_i can be achieved.

In summary, the algorithm for harmonic estimation presented by the paper is described below,

1. Solve h_i and d_i with MSWF forward recursion;
2. Estimate the dimension of signal subspace using modified MDL criterion, and construct sampling signal subspace using h_i;
3. Construct spatial spectrum P_{MUSIC} according to formula (6) and (7);
4. Use search strategy presented in section 6 to complete peak search and obtain estimation of harmonic frequency;
5. Construct linear neural network, and estimate weigh vector W. Calculate harmonic amplitude and phase using W.

8. SIMULATION EXPERIMENTS

Experiment 1: In Xi'an steel mill, harmonic parameters of a linear voltage V_{bc} is measured on 35kV bus, the phase is self-set. Simulation signal is as follows,

$$x(t) = 37.66\cos(2\pi \times 50t + \pi/4) + 0.933\cos(2\pi \times 100t + \pi/36)$$
$$+1.813\cos(2\pi \times 150t + \pi/18) + 0.885\cos(2\pi \times 200t + \pi/12)$$
$$+1.943\cos(2\pi \times 250t + \pi/9) + +0.97\cos(2\pi \times 300t + \pi/8)$$

In this experiment, the sampled frequency is 1000Hz, and the experimental time is 30ms. The sampling sequences are estimated by FFT and MUSIC respectively. The results are shown as Fig. (**1**).

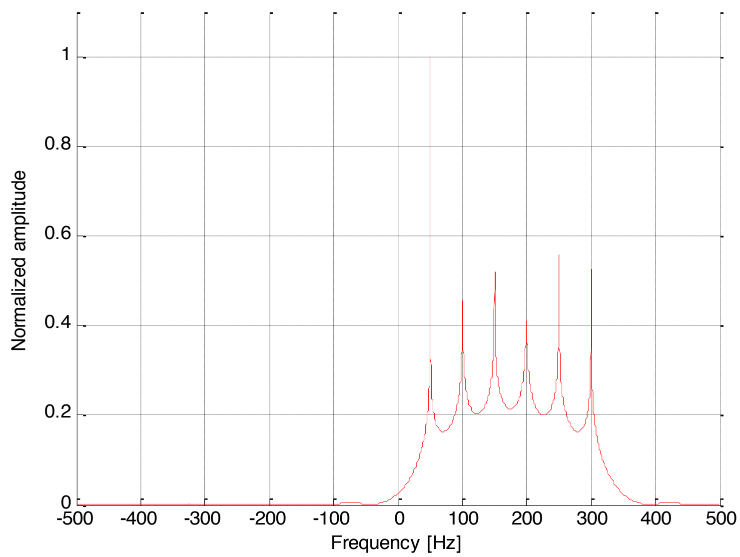

Fig. (2). Analysis Result of Experiment 2 time sequence.

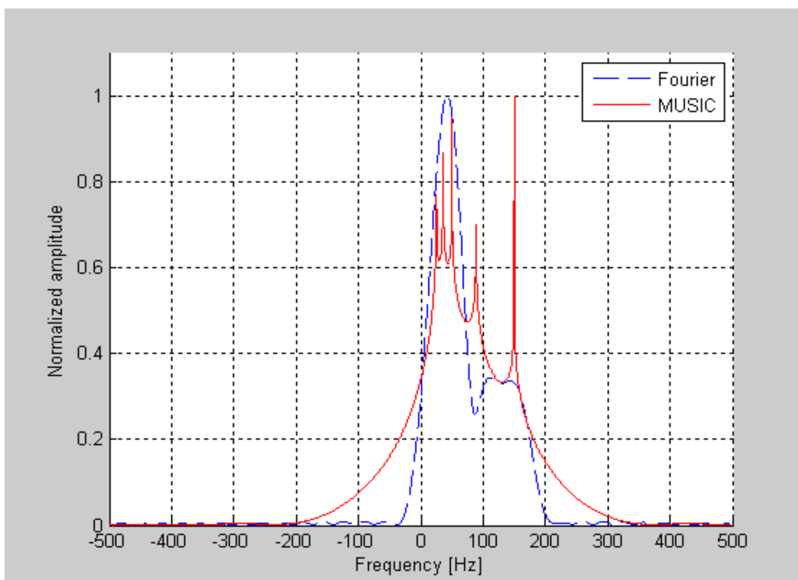

Fig. (3). Analysis Results of Experiment 3 time sequences.

From the experimental results, even in the absence of noise, and under the condition of that the sampling frequency is integer multiple of the fundamental frequency, when the sampling points are less, the FFT cannot detect the 2-times, 4-times and 6-times harmonic.

Experiment 2: simulation signal is the one of Experiment 1 adding white noise, whose SNR is 30 and sampling frequency is as usual, which is to verify the property under white noise. Simulation result is shown as Fig. (**2**).

Comparison of Fig. (**1**) and Fig. (**2**) shows that, under the condition of SNR decreased significantly, although the peak size changed obviously, the peak position did not change with the change of SNR, which indicated the algorithm had good noise immunity. It is very important in practical applications. In order to illustrate the problem deeply, the experiment is repeated 100 times with continuously changing SNR, and the covariance of each harmonic is calculated, so that to verify the statistical performance. Detailed data are listed as Table **1**.

Table **1**. shows that the algorithm is not sensitive to the change of SNR, where SNR is greater than 20, the covariance of each harmonic frequency has reduced to 0.

Table 1. Detection Results of Harmonic Frequency of Experiment 2.

Harmonic Frequency (Hz)	50	100	150	200	250	300
SNR=10	0	2.599879	0.581629	8.673336	0.031385	0.019835
SNR=20	0	0	0	0	0	0
SNR=30	0	0	0	0	0	0
SNR=40	0	0	0	0	0	0
SNR=50	0	0	0	0	0	0
SNR=60	0	1.01E-26	7.42E-27	9.97E-26	2.97E-26	1.32E-26
SNR=70	0	1.01E-26	7.42E-27	9.97E-26	2.97E-26	1.32E-26
SNR=80	0	1.01E-26	7.42E-27	9.97E-26	2.97E-26	1.32E-26

Table 2. Detection Results of Harmonic Frequency of Experiment 3

True Value (Hz)	25	35.85	50	88.6	150
MUSIC	25.16	35.81	50	88.71	150
Relative Error(%)	0.64	0.1157	0	0.1241	0
FFT	undetected	undetected	43.23	108.7	149.7
Relative Error(%)	100	100	13.54	22.69	0.2

Table 3. Detection Results of Harmonic Amplitude and Phase

Amplitude(V)		Phase(°)	
True value	Estimation	True value	Estimation
0.3	0.3014	70	69.9945
0.7	0.7013	80	79.9978
1	1.0000	30	30.0015
0.5	0.4976	90	90.0027
0.4	0.4031	40	39.9931

Experiment 3: Simulation signal is as follows,

$$x(t) = 0.3\cos\left(2\pi \times 25t + 70^{\circ}\right) + 0.7\cos\left(2\pi \times 35.85t + 80^{\circ}\right)$$
$$+ 1.0\cos\left(2\pi \times 50t + 30^{\circ}\right) + 0.5\cos\left(2\pi \times 88.60t + 90^{\circ}\right)$$
$$+0.4\cos\left(2\pi \times 150t + 40^{\circ}\right)$$

which includes three inter harmonic frequency components whose frequency is 25Hz, 35.85Hz and 88.6Hz and two integer harmonic frequency components whose frequency is 50Hz and 150Hz without noise. This experiment is to verify the estimation ability to inter harmonic.

In this experiment, sampling frequency is f_S=1000Hz, the experimental time is 30ms, sampling points is N=30. The sampling sequences are estimated by FFT and MUSIC respectively. The results are shown as Fig. (**3**).

As everyone knows, the frequency resolution of FFT algorithm mainly depends on the sampling frequency and the amount of data. In this experiment, f_s/N=33.3Hz, the harmonic signal cannot be detected accurately because of the fewer sampling points and lower resolution. Even the peak appeared in the spectrum, the peak number and the corresponding frequency cannot be contained with the number of harmonic and frequency. Detailed data are listed as Table **2**.

It is shown in Table **2**. that under the same condition the algorithm presented by the paper can detect and estimate every harmonic signal accurately, which illustrates the spectrum resolution of MUSIC algorithm is far superior to the FFT algorithm. On the basis of this, by constructing a linear neural network, harmonic amplitude and phase estimation can be achieved. The results are shown as Table **3**.

From the simulation, the algorithm realized the estimation of harmonic amplitude and phase with high accuracy, where the error is less than 1%.

CONCLUSION

MUSIC algorithm is one of the most effective methods for frequency, time-delay, direction-of-arrival estimation with high accuracy, which is suitable for the field of harmonic analysis in power system. But because of the requirement of eigen decomposition of auto-correlation matrix of sampling signals and searching for spatial spectrum peak in whole frequency domain, the computational load is high without detecting the harmonic amplitude and phase. The paper combines fast subspace decomposition based MSWF, MUSIC algorithm and adaptive artificial neural network, which reduce the computational complexity largely without influence of accuracy. Simulation results show that the algorithm can realize harmonic estimation with high resolution and anti-noise effect. And the algorithm does not need synchronous sampling without the problem of spectrum leakage, which is suitable for harmonic analysis in power system and has the superiority to Fourier analysis.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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