Steady-State Unconfined Aquifer Simulation of the Gareh-Bygone Plain, Iran

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Abstract: The first step of aquifer parameter and dependent variable estimation based on hydraulic modeling is generally to choose the best steady-state condition for the set time period. In order to define the best estimated hydraulic conductivity and boundary condition for Gareh-Bygone Plain in arid southern Iran, ten different steady-state conditions were simulated and calibrated with limited field observations. The investigated area covers about 6000 ha with a floodwater spreading system that was established on about 2000 ha to artificially recharge the groundwater. The results showed a consistency over the 14-year simulation period with estimated hydraulic conductivity in a quite narrow range. This makes us believe that even if the modeling problem is to some extent over-parameterized the results appear quite robust. This is further strengthened by verification of the model results. Furthermore, the results showed that in the steady-state groundwater flow with no recharge from surface water, the system is mainly recharged by the fault which conducts water into the area from an upper sub-basin.

Keywords: Steady-state, Groundwater Modeling, MODFLOW, Hydraulic Conductivity, GBP.

INTRODUCTION

A way to increase scarce water resources in arid and semi-arid areas is to use artificial recharge of surface water to the groundwater. Due to very small fresh water resources in the Gareh-Bygone Plain (GBP) in arid southern Iran, a floodwater spreading system was established between 1983 and 1987. The main objective of the system is to improve groundwater quantity and quality. However, after 25 years of operation there has still not been any evaluation of the function of the recharge system or aquifer characteristics.

As for most groundwater systems, hydraulic conductivity, recharge, and other aquifer variables cannot be measured directly in an accurate way [1]. However, simulation of the groundwater system can be used to estimate aquifer parameters and define boundary conditions. The first step of aquifer parameter and dependent variable estimation based on hydrological modeling is generally to choose the best steady-state condition during a given time period. In general also, calibration periods for several different years should be carried out to estimate the hydraulic conductivity and boundary conditions of steady-state models.

Davis and DeWiest [2] stated that steady flow may be conceived of as a limit case of unsteady flow, as time goes to infinity, or else as the average of unsteady flow over a given time. One way to simplify the model calibration is steady-state modeling for estimating the aquifer parameters, and then the estimated parameters can be transferred to the transient or unsteady models [3]. A steady-state condition represents the system response to a specific set of boundary conditions with sources and sinks. Therefore, long term changes in the system can be evaluated by adapting the boundary conditions for the regional model to specific evaluation scenarios [4]. As a first step, a time interval in which the flow is in steady-state or in which there is no change in the hydraulic head for observation wells within the aquifer should be used. However, in reality this situation is not common. Thus, an approximate method could be to define the steady-state conditions in the groundwater system as minimum hydraulic head variation in observation wells for successive months.

The analytical solution for steady-state groundwater flow assumes that the mean hydraulic conductivity and hydraulic gradient are constant. In addition, mean recharge should be zero over the study area for the simulation period [5]. Furthermore, estimated precision of aquifer parameters and determination of boundary conditions are based on graphical matching of estimated vs. observed hydraulic head [6-9]. Due to uncertain data, parameter estimation from calibrated groundwater models is generally uncertain [10]. However, by using confidence intervals it is possible to choose the best estimation of parameters in the simulations.

The general objective of the present work is to apply and evaluate the above general methodology for the GBP groundwater system with limited field observations. Consequently, partial objectives are to find the best steady-state condition to estimate aquifer parameters such as hydraulic conductivity with defined boundary conditions in order to analyze model precision, reliability, uncertainty, and model
sensitivity. For this purpose, several different steady-state conditions were simulated without using well or tracer tests. In the first section below, we define study area and available observations. The following section describes the methodology utilized. Then follow the modeling results. We close with a discussion of practical results of the study.

DESCRIPTION OF STUDY AREA AND OBSERVATIONS

The Unconfined Gareh-Bygone Aquifer (GBA)

The study area is located 200 km southeast of Shiraz city, in southern Iran (28°34´N-28°41´N, 53°52´E-54°00´E at an altitude of 1140 m above mean see level; Fig. 1). According to the FAO climate classification, this region is extremely dry with a mean annual precipitation of 243 mm and a Class A Pan evaporation of about 3200 mm per year [11]. Moreover, the area is affected by the Mediterranean synoptic system with high temporal and spatial variation of precipitation.

The GBP is a 6000 ha expanse with colluvial soils and old debris cones of low slope covered with moving sand [11]. The Agha Jari formation constitutes the major bedrock (red clay) on which the alluvium has been deposited. The thickness of the aquifer ranges from practically zero at the foothills to about 43 m at the center of the GBA. The upper 12 m alluvium contains fine sand and gravel. The deeper layers consist of medium and coarse sand, gravel, and stones of different size, up to 0.4 m in diameter.

A floodwater spreading system (FSS) to artificially recharge the groundwater was established in this area between 1983 and 1987 on about 2000 ha. This system diverts surface runoff from ephemeral rivers to the plain which then infiltrates and recharges groundwater and improves the vegetation cover [12]. The FSS is one of the best solutions for arid and semiarid areas where soil texture is coarse with high water holding capacity and infiltration rate is rather high. The system is intended to improve groundwater quantity, quality, and related farming of the area. Groundwater is the main source of fresh water in the area and inhabitants exploit groundwater by pumping from wells for drinking and irrigation purposes. Despite the artificial recharge by FSS, over-exploitation of groundwater has lead to a water table drop of about 8 m (in some areas more than 12 m) during the last 10 years. At present, the average groundwater depth in the GBA is about 30 m from ground surface.

OBSERVATIONS

In the GBA, hydraulic head has been recorded monthly since 1993 by the Fasa District Water Organization. Monthly observations from four wells located within the GBA during 14 years between 1993 and 2007 were used in this study for calibration. Monthly observations from two newer well during the period 2007-2009 were used for verification. A typical well log from the GBA is presented in Fig. (2).

Fig. (1). Location of study area in southern Iran.

Fig. (2). Typical geological log in one of the observation wells in the center the GBA (after Kowsar and Pakparvar, 2003).

MODEL DESIGN

A conceptual groundwater model represents the groundwater system in a simplified way [13]. Conceptual model design is usually divided into model construction and comparison with field measurements [14]. Unrealistic results are often due to over-simplification and/or lack of detailed observations. Hence, in groundwater modeling it is important that the conceptual model is well represented by a valid hydrogeologic system and well defined boundary conditions for the study area.

A widely used commercial groundwater flow model MODFLOW-2000 [15] within the GMS software was selected to simulate and define the parameter estimation and boundary conditions. The GBA with 6000 ha was discretized into uniform square grids with side length 250 m into a finite-difference three-dimensional model. Geological investigation prepared from well cutting showed that the GBA could be represented by only one layer of red clay bedrock boundary at the bottom.

Four observation wells were used to build the conceptual model for the fourteen-year period. Due to the limited number of observation wells and but homogeneity of the geologic formation in the GBA, the Thiessen method was used to find representative zones around each observation well (Z1, Z2, Z3, and Z4). The model was calibrated with observed hydraulic head at the observation wells in each zone. An optimization modeling technique using the PEST [16] software was used to determine best simulation of groundwater flow system and aquifer parameters.

STEADY-STATE GROUNDWATER CONDITIONS

Todd and Mays [17] stated that steady flow implies that no change occurs with time. This situation will occur when the hydraulic head variation for observed wells is at minimum. Graham and Neff [5] calculated the steady-state piezometric head by temporally averaging all head measurements available for the period 1980-1989 in their study area. After that, they conducted a two-dimensional steady-state simulation by MODFLOW using the estimated transmissivity field and incorporating the estimated areal recharge. Sonnenborg et al. [3] used calculated daily values of recharge from a root zone model and average groundwater abstraction in the calibration period, resulting in mean values as input to a steady-state model. Gedeon et al. [4], in a regional groundwater model study, considered two different steady-state cases under active and non-active pumping wells in the aquifer during the given time. The mean absolute error between calculated and observed hydraulic head levels was less than two meters.

The partial-differential equation for groundwater flow used in MODFLOW is [15]:

$$\frac{\partial}{\partial x} [K_{xx} \frac{\partial h}{\partial x}] + \frac{\partial}{\partial y} [K_{xy} \frac{\partial h}{\partial y}] + \frac{\partial}{\partial z} [K_{zz} \frac{\partial h}{\partial z}] + W = S_0 \frac{\partial h}{\partial t}$$  \hspace{1cm} (1)

where $K_{xx}$, $K_{yy}$, and $K_{zz}$ are hydraulic conductivity along the $x$, $y$, and $z$ coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity ($L/T$); $h$ is the potentiometric head ($L$); $W$ is a volumetric flux per unit volume representing sources and/or sinks of water, with $W>0.0$ for flow out of the groundwater system, and $W>0.0$ for flow in ($T^{-1}$); $S_0$ is the specific storage of the porous material ($L^{-1}$); and $t$ is time ($T$). Equation (1) is solved using the finite-difference method in which the groundwater flow system is divided into a grid of cells. The finite-difference equation for a cell is [15]:

$$CR_{i,j,k} \left( h_{i,j,k}^n - h_{i,j,k}^{n+1} \right) + CR_{i,j,k} \left( h_{i,j,k+1}^n - h_{i,j,k}^n \right) + CC_{i,j,k} \left( h_{i,j,k-1}^n - h_{i,j,k}^n \right)$$

$$+ CV_{i,j,k} \left( h_{i+1,j,k}^n - h_{i,j,k}^n \right) + CV_{i,j,k} \left( h_{i-1,j,k}^n - h_{i,j,k}^n \right)$$

$$+ P_{i,j,k} h_{i,j,k}^n + Q_{i,j,k} = SS_{i,j,k} \left( DELR_{i,j,k} \times DELC_{i,j,k} \times THICK_{i,j,k} \right) \frac{h_{i,j,k}^n - h_{i,j,k}^{n+1}}{t^n - t^{n+1}}$$

where $h_{i,j,k}^n$ is head at cell $i,j,k$ at the time step $m$ ($L$); $CV$, $CR$, and $CC$ are hydraulic conductance, or branch conductance, between node $i,j,k$ and a neighboring node ($L^2/T$); $P_{i,j,k}$ and $Q_{i,j,k}$ are volumetric recharge and discharge at cell $i,j,k$ ($L^3/T$), respectively.
Fig. (4). Satellite image of the study area representing the upper catchment (Bisheh-Zard Sub Basin) of Gareh-Bygone Plain, floodwater spreading systems (FSS), observation wells (P1, P2, P3, P4 and P5, P6) and model boundary.

Fig. (5). Model domain, boundary conditions and observation wells location in the Gareh-Bygone Aquifer.
is the sum of coefficients of head from source and sink terms \((L^2/T)\); \(Q_{i,j,k}\) is the sum of constants from source and sink terms, with \(Q_{i,j,k} < 0.0\) for flow out of the groundwater system, and \(Q_{i,j,k} > 0.0\) for flow in \((L^2/T)\); \(SS_{i,j,k}\) is the specific storage \((L^3/L^2T)\); \(DELC_i\) is the cell width of row \(i\) in all rows \((L)\); \(THICK_{i,j,k}\) is the vertical thickness of cell \(i,j,k\) \((L)\); and \(t^n\) is the time at the time step \(m\) \((T)\). To designate hydraulic conductance between nodes, as opposed to hydraulic conductance within a cell, the subscript notation \(1/2\) is used.

Normally, for a steady-state period, the storage term and therefore, the right hand side of Eqs. (1) and (2) is set to zero. Based on these equations and the difference in hydraulic head in a given zone for successive time intervals \((dh > 0)\) the system might be unsteady. Observed \(dh\) (hydraulic head change) during the simulation period was not absolutely zero but very small. Consequently, it was assumed that \(dh = 0\) and this eliminates \(dh / dt\) from the equations. The assumption may result in a somewhat lower estimated conductance \((C)\) or hydraulic conductivity \((K)\) in the left hand side of the equations. Due to missing data for one or two observation wells during some time periods, only ten different steady-state periods were selected during the total observation period when the absolute difference between heads for successive months was approximately close to zero or less than 0.35 m (Fig. 3).

**BOUNDARY CONDITIONS**

Three different types of boundary conditions were defined for the study area, namely 1) no-flow boundary, 2) general head boundary, and 3) changing-head or time-variant specified-head boundary. No-flow boundary was assigned to the northern part of the model area at the Gar Mountain range located in the northern part of GBP (Figs. 4 and 5). This mountain range is a part of upper catchment (Bisheh-Zard sub basin), which was created during the Mio-Pliocene period in the Agha Jari formation. Floodwater is generated from this catchment during flood periods to the GBP and is recharging the groundwater by the floodwater spreading system. In addition, the area along Shur River of Jahrom, which is located at the western border of GBP, was defined as no-flow boundary.

The water exchange through the general head boundary was calculated from [18 and 19]:

\[
Q_i = C_i (h_b - h) \tag{3}
\]

where \(C_i\) is the conductance of the boundary, \(h_b\) is the hydraulic head at the boundary cell, and \(h\) is the hydraulic head at the aquifer cell adjacent to the boundary. Observed hydraulic head close to the northwestern border of GBA was used as a general head boundary condition (Fig. 5). According to the topography and bed rock map of GBP the general trend for groundwater flow is from north to southwest. Based on this, time-variant specified-head or changing-head boundary was defined along the south and southwest border of GBA (discharge area of groundwater; Fig. 5). In addition, according to observed elevation head in pumping wells located in the northeast of GBA (adjacent aquifer), a changing-head boundary was assigned to this short border to recharge the GBA.

Due to the large hydraulic head at observation well number one (P1) as compared to the other observation wells (Fig. 6) it was assumed that there is a direct connection between P1 and an external source. After analyzing satellite images and aerial photographs of the study area (Fig. 4), the existing geological map was modified with a fault affecting the hydraulic head at P1. This fault passes through the upper catchment and splits the GBA (Figs. 4 and 5), which creates a hydraulic groundwater connection between the upper catchment and the GBA. Kresic [18] mentions that faults may have one of the following three roles: 1) conduit for the groundwater flow, 2) storage of groundwater due to increased porosity within the fault, and 3) barriers to groundwater flow. According to role number 1 and observed hydraulic head at P1, the fault was defined as constant head within the aquifer. Consequently, it appears that it has an important recharging function.

To define the best boundary conditions for each simulation period the elevation head at the fault was maintained based on observed hydraulic head to reach a best model fit. In all cases, the results show that the fault is the main source of recharge to the GBA during steady-state conditions. The elevation head at the fault was different for different years and the model was calibrated accordingly.

According to existing information, 85 pumping wells have been dug in GBP during the last 20 years. There is, however, no control of the groundwater abstraction and the farmers pump up as much groundwater they need. Due to the numerous pumping wells located in the southern and southwestern part (more than 50 pumping wells) of the GBP to withdraw groundwater for irrigation, especially observation wells P3 and P4 have been affected by a large drawdown in groundwater level during the latest years. Because of the over-exploitation of groundwater, the elevation head has been gradually lowered every year for this border. Consequently, the changing-head boundary was determined manually during each calibration period to achieve a best model fit and hydraulic conductivity estimation.

**PRECISION AND UNCERTAINTY OF PARAMETER ESTIMATION**

The linear method using PEST [16] software was used for model calibration and optimization. The purpose of PEST is to assist in data interpretation and model calibration. PEST adjusts model parameters until the fit between model estimation and observations is optimized in the weighted least squares sense. This method computes individual or simultaneous objective functions and confidence intervals which are used to evaluate uncertainty in parameter estimates. [7].

To define optimally estimated aquifer parameters and boundary conditions an objective function and confidence intervals were defined. Hill and Tiedeman [7] state that parameter estimates are more precise if the variance of objective function and confidence intervals is small. The objective function represents the fit to observations used in model calibration, which involves determining the residuals or weighted residuals. Weighted residuals have the advantage of including effects of error and are investigated to infer the true errors [7]. The objective function used is the following formula:
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\[ S(b) = \sum_{i=1}^{ND+NPR} w_i \{ y_i - y'(b) \}^2 = \sum_{i=1}^{ND+NPR} w_i e_i^2 \]  

(4)

where \( b \) is a vector containing values of each of the \( NP \) parameters being estimated, \( NP \) is the number of estimated parameters, \( ND \) is the number of observations, \( NPR \) is the number of prior information values, \( w_i \) is the weight for the \( i \)th contribution to the objective function, \( y_i \) the \( i \)th observation or prior information value being matched by the regression, \( y'(b) \) is the simulated equivalent, defined as the simulated value that corresponds to \( y_i \) and \( e_i \) is the weighted residual, equal to \( y_i - y'(b) \).

An individual linear confidence interval modified from Hill and Tiedeman [7] is defined as:

\[ b_j \pm t(n,1.0 - \alpha/2)s_{b_j} \]  

(5)

where \( t(n,1.0 - \alpha/2) \) is the Student \( t \)-statistic for \( n \) degrees of freedom and a significance level of \( \alpha \), \( n \) is the degrees of freedom, and \( s_{b_j} \) is the standard deviation of the \( j \)th parameter.

In the present study, the statistical distribution at the 95% confidence limits was used for the estimation of horizontal hydraulic conductivity within the four sub-zones (Z1, Z2, Z3, and Z4). For each simulation period the boundary conditions, inflow, and outflow were calibrated and optimized to define the best boundary condition and hydraulic conductivity.

MODEL VERIFICATION

Four observation wells were used to simulate the groundwater system in the study area during ten different steady-state time intervals and the model was calibrated based on observed hydraulic head at the observation wells for the 14-year period between 1993 and 2007. For verification of the model, two more observation wells with more recent observational data during 2008 were used. Consequently, the model was calibrated for 4 well observations and then verified using the observations from the new 2 wells during 2008.

RESULTS AND DISCUSSION

The result of steady-state simulation shows that the trend of groundwater flow below the fault in GBA is generally from north to south/southwest. The steady-state groundwater flow is to a great extent influenced by the water that comes from the upper catchment (Bisheh-Zard sub basin) via the fault (Fig. 7). The GBA is a small part of the main aquifer named Shib-Kooh Aquifer (SKA). According to the iso-potential groundwater map of SKA, the groundwater flow trend in the entire GBA is mainly from north to south and southwest. Nevertheless, the model results showed that the hydraulic head at the fault is higher than hydraulic head at both sides of the fault (north and south). For this reason, the fault influences and recharges both the northern and the southern part of GBA and the groundwater flow direction in the upper part of the fault in GBA is from northeast to the southwest (Fig. 7).

The results also showed that there is no significant change in the groundwater level in zone 1 and zone 2 (area around P1 and P2) during the simulated period. This is because of the vicinity of these zones to the fault as affected by the direct recharge. In addition, no pumping wells affected zone 2 during the investigated period (Fig. 7). On the other hand, augmentation of groundwater resources due to FSS has increased the irrigated area of farm fields eight-fold to 1193 ha. Consequently, the number of pumping wells in the area affected by the FSS has increased by over 85 wells since 1996, 10 times the number in 1983 and this situation has much more affected the groundwater resources in the central to the south and southwest part of the GBP. Due to the intensive pumping in zone 3 and zone 4, the groundwater level has dropped significantly here (Fig. 7c and d).
As mentioned above, ten different steady-state simulation periods were modeled in which the variation of groundwater level ($dh$) was negligible. These ten steady-state periods were optimized regarding horizontal hydraulic conductivity. The residual between simulated and observed hydraulic head value for all cases was close to zero (Fig. 8a). The main control for the simulation results was the hydraulic head at the fault and secondly the hydraulic head at the changing head boundary assigned at the south/southeast part of the GBA. The almost perfect fit between simulations and observations is partly due to fitting a model with many parameters to an experimental area with rather few observations. Even so, the simulation results should not be underestimated. The results show a consistency over the 14-year period with estimated hydraulic conductivity in a quite narrow range (Fig. 9). This makes us believe that even if the modeling problem is to some extent over-parameterized the results appear quite robust. This is further strengthened by the verification results shown below (Fig. 8b).

Fig. (7). Simulated head above mean sea level (m) and groundwater flow direction for the experimental area for (a) 1993.06.15; (b) 1996.04.15; (c) 2001.04.15; (d) 2004.08.15.

Fig. (8). Observed head values (m) versus simulated head values (m), (a) for ten steady-states calibrated model and; (b) for verified model.
all time periods (steady-state) are quite narrow and estimated horizontal hydraulic conductivity for all periods and sub-zones varied between 0.0003 and 0.3 m/day. Based on the resulting confidence intervals it appears that results are statistically robust.

Consequently, the estimated hydraulic conductivity appears to be robustly estimated. Fig. (10), shows the empirical probability distribution for the estimated hydraulic conductivities and probability of $K$'s. The general visual appearance shows that the hydraulic conductivities are close to a gamma probability distribution.

The summary results of model calibration for ten steady periods and verification for observational data during 2008 are shown in Table 1. The results show that the $K$'s in both calibration and verification are close. The residual and objective functions are somewhat larger for the verification period. On the other hand, the estimated hydraulic conductivities are quite close.

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**Table 1. Calibration Result for Ten Steady-State Modeled Period Between 1993 and 2007 and Verification Result for 2008. In Model Verification, Two More Observation Wells were Used**

<table>
<thead>
<tr>
<th>Estimated value</th>
<th>Mean (m/day)</th>
<th>Max (m/day)</th>
<th>Min (m/day)</th>
<th>S.D (m/day)</th>
<th>Objective function (m)</th>
<th>Residual (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\text{Calibrated}}$</td>
<td>0.0880</td>
<td>0.1010</td>
<td>0.0700</td>
<td>0.0100</td>
<td>0.00001</td>
<td>0.0002</td>
</tr>
<tr>
<td>$K_{\text{Verified}}$</td>
<td>0.0840</td>
<td>0.2980</td>
<td>0.0002</td>
<td>0.1430</td>
<td>0.0480</td>
<td>0.0400</td>
</tr>
</tbody>
</table>
SENSITIVITY ANALYSIS

Knopman and Voss [20] note that the spatial and temporal variability of sensitivity has a significant impact on parameter estimation and sampling design. The sensitivity indicates how much a simulated value would change if a parameter value were changed [7]. Sensitivity analysis is useful to illustrate the importance of observations to parameter estimations. Therefore, the observation with highest sensitivity might be more affected by the estimated parameter during the simulation period.

In general, the sensitivity displays an increasing trend with time that may be connected to the increasingly lower groundwater level during the last few years (Fig. 11). As mentioned above, due to over-exploitation of groundwater, especially during the last few years, the groundwater level has dropped drastically. The model in general is becoming more sensitive to observations at P1 and the conditions at the fault. This is logical due to the importance of the fault for recharge of groundwater to the entire area. A major finding of this paper is that not only the recharging FSS is important for the area but also water through the fault.

CONCLUSIONS

One of the disadvantages of steady-state modeling is the limited estimation of aquifer parameters such as specific yield, recharge rate, etc. Even so the present study has shown that robust parameter estimation and boundary condition can be defined in steady-state groundwater model in unconfined aquifers with rather high precision and reliability. Also, the steady-state groundwater model can be a part of unsteady-state modeling provided that estimated parameters are transferred to the unsteady simulation. Here, it is possible to use either 2D or 3D conceptual model in the case of steady-state modeling.

The results showed that in the steady-state groundwater flow with no recharge from surface water, the system is mainly recharged by the fault which conducts water into the area from upper sub-basin. Consequently, the general groundwater flow direction is from north to south and southeast. The range of values of hydraulic conductivity for the alluvial fan varies from 0.001 to 1 m/day [21] and the average estimated hydraulic conductivity resulting from modeling for the study area is about 0.1 m/day.

The results of parameter estimation and boundary condition shows that can be applied for the Gareh-Bygone Aquifer to better understand the groundwater flow system and these findings can be transferred to a transient model to estimate another aquifer parameters and groundwater behavior in this area.

CONFLICT OF INTEREST

The author confirms that this article content has no conflicts of interest.

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