Using the Reference Shrinkage Curve to Estimate the Corrected Crack Volume of a Soil Layer

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Abstract: To estimate the correct soil crack volume based on the shrinkage geometry factor, in addition to measurements with core samples, one also needs to know the shrinkage curve of the soil matrix without cracks. For such a curve one can use the soil reference shrinkage curve that is only stipulated by intraaggregate clay shrinkage without interaggregate cracking. The objective of this work is to illustrate such use of the reference shrinkage curve and, in addition, to argue the necessity to correct the usual estimates of the shrinkage geometry factor in order to correctly estimate the soil crack volume. The illustration is based on available data and reference shrinkage curves for eight soils.

INTRODUCTION

Knowledge of soil crack volume is important for many problems of soil hydrology. Bronswijk [1-4] suggested the estimation of the vertical-crack volume developing in a soil, based on his concept of the shrinkage geometry factor, \( r_s \). He was interested in field condition applications; that is, in the crack volume that develops in an infinite soil layer, but not in a sample. A corresponding known exact relation (neglecting horizontal cracks),

\[
(1 - \frac{\Delta V}{V}) = (1 - \frac{\Delta z}{z})^{r_s} \tag{1}
\]

links the variation \( \Delta V \) of an initial matrix volume, \( V \) in a soil layer with the variation \( \Delta z \) of an initial layer thickness, \( z \) at drying. The layer includes cracks along with soil matrix. The shrinkage geometry factor, \( r_s \) just determines partition of the matrix volume variation between cracks and layer thickness variation. Thus, the layer cracks enter Eq. (1) implicitly through the \( r_s \) value.

Usually for practical purposes, one deals with core sample measurements [5] instead of layer measurements. It is this replacement of a layer with a core (with coincidence between initial layer thickness, \( z \) and initial core height, \( z_c \)) that leads to the replacement of \( \Delta V/V \) and \( \Delta z \) in Eq. (1) with other values and that can lead to serious inaccuracy in the \( r_s \) estimates based on initially exact Eq. (1) [6, 7]. To obtain the correct \( r_s \) values and correct crack volume estimates, in addition to measurements with cores, one also needs to know the shrinkage curve \( \overline{V}(W) \) of the soil matrix without cracks [6, 7]. In general, \( \overline{V}(W) \) can be measured on sufficiently small samples and at sufficiently slow drying (for an example of such use of the measured aggregate shrinkage curve from [8], see [7]). However, the use of a physically modeled soil matrix shrinkage curve \( \overline{V}(W) \) is the most convenient for estimating the corrected shrinkage geometry factor and crack volume. In the case of pure clay one can use the model [9, 10] of a clay matrix shrinkage curve to estimate \( \overline{V}(W) \) (for an example of such use, see [6, 7]). Recently, Chertkov [11-13] suggested a model of the reference shrinkage curve. The model permits one to predict, for an aggregated soil with any clay content, its shrinkage curve which is only stipulated by clay shrinkage inside a soil matrix without cracking. That is, the reference shrinkage curve gives the soil matrix shrinkage curve, \( \overline{V}(W) \). The objective of this work is to illustrate the application of the soil reference shrinkage curve for estimating the corrected shrinkage geometry factor and soil crack volume using available data and results from [6, 7, 11-13]. In line with that we additionally argue and illustrate the necessity of correcting the shrinkage geometry factor and crack volume that are determined from Bronswijk’s approximation [5].

THEORETICAL BACKGROUND

In connection with \( \Delta V/V \) it is worth reiterating that, by definition, in exact Eq. (1) the \( V \) and \( \Delta V \) values only relate to the layer matrix. According to [5], in practice one uses in Eq. (1), instead of the \( \Delta V/V \) ratio for the soil matrix in the layer, the corresponding ratio for the relative volume variation of a core, \( \Delta V_c/V_c \). At a given water content the \( \Delta V/V \) ratio for the soil matrix in the layer coincides with a similar ratio for the soil matrix in the core, but not with \( \Delta V/V_c \). Indeed, the initial state of the core is usually identified with that of the water saturation, and one considers that initial cracks are lacking. Then, the initial core volume, \( V_c \) (as \( V \)) only includes the soil matrix. However, variation of the core volume, \( \Delta V_c \) at drying (unlike \( \Delta V \)) mostly includes a crack contribution along with a soil matrix contribution. For this reason \( \Delta V_c \) differs from \( \Delta V/V \). This difference increases with drying, introducing an essential inaccuracy in the \( r_s \) value [6, 7].

Moreover, according to [5], in practice one can use in Eq. (1), instead of the thickness variation, \( \Delta z \) of an unlimited layer, the corresponding variation of the core height, \( \Delta z_c \) (at
However, at a given water content $\Delta z_c$ differs from $\Delta z$ ($\Delta z_c < \Delta z$). This difference increases with drying, introducing an additional essential inaccuracy in the $r_s$ value [6, 7]. The two above inaccuracies, in part mutually compensate each other. Nevertheless, the resulting $r_s(W)$ dependence (after replacements in Eq. (1): $\Delta V/V \rightarrow \Delta V_c/V_c$ and $\Delta z \rightarrow \Delta z_c$) is essentially distorted compared to the true one [7].

In addition to the above application of Eq. (1) (with indicated replacements and inaccuracies) for description of a situation with layer drying [1-4], there is also another application. Equation (1) in formally the same view is also the exact one for core shrinkage and cracking. In this case $\Delta z = \Delta z_c$ (and $z = z_c$), but the $V$ and $\Delta V$ values again, by definition, only relate to the core matrix, and consequently again $\Delta V/V \neq \Delta V_c/V_c$ because $\Delta V_c$ can contain both crack and matrix contributions. For this reason the use in Eq. (1) for core shrinkage of $\Delta V_c/V_c$ instead of $\Delta V/V$ leads to inaccuracy in the $r_s$ value [6, 7]. Note that the $r_s$ value in Bronswijk’s approximation [5] is the same for core and layer. However, the true $r_s$ value for the core differs from the true $r_s$ value for the layer [7] and has another meaning. $r_s$ determines partition of the volume variation of the core matrix between core vertical shrinkage, core lateral shrinkage, and internal cracks. Thus, core cracks enter Eq. (1) implicitly through the $r_s$ value. Equation (1) for the core shrinkage is mostly used to characterize the shrinkage anisotropy of the core matrix. Core matrix shrinkage is isotropic (i.e., matrix deformations are the same for all directions) if $r_s = 3$ [1]. Available works discussing core matrix anisotropy at shrinkage (e.g., [14, 15]) including recent ones (e.g., [16, 17]), use $\Delta V_c/V_c$ for estimating $r_s$ instead of $\Delta V/V$, ignoring the inaccuracy that is introduced in the $r_s$ value (as a function of water content).

Chertkov [7] suggested an approach for estimating the corrected $r_s(W)$ and corresponding corrected specific crack volume, $\overline{V}_c'(W)$, as functions of water content $0 < W < W_h$ ($W_h$ is the maximum swelling point before shrinkage starts) for both cases of layer and core. In the frame of this approach the corrected $r_s(W)$ and $\overline{V}_c'(W)$ are expressed through four shrinkage curves (Fig. 1): the curve $\overline{V}_l'(W)$ of a soil layer with cracks in Bronswijk’s approximation (the initial layer is composed of contacting, but disconnected cubes); the curve $\overline{V}_l(W)$ of a real connected soil layer with cracks; the curve $\overline{V}_c'(W)$ of a soil core sample with cracks; and the curve $\overline{V}(W)$ of a soil matrix without cracks. Given $\overline{V}_l(W)$, $\overline{V}_l'(W)$, $\overline{V}_c'(W)$, and $\overline{V}(W)$ (including $\overline{V}_h = \overline{V}(W_h)$; Fig. 1), the shrinkage geometry factor in Bronswijk’s approximation, $r'_s (\Delta V/V \rightarrow \Delta V_c/V_c$ and $\Delta z \rightarrow \Delta z_c$; as noted above, in this

![Fig. 1](image-url)

**Fig. 1.** Scheme of the different shrinkage curves of an aggregated soil. 1-initial specific volume (without cracks) of a shrinking and cracking soil layer. 2-shrinkage curve $\overline{V}_l'(W)$ of a soil layer with developing cracks in Bronswijk’s approximation (disconnected layer). 3-corrected shrinkage curve $\overline{V}_l(W)$ of a (connected) soil layer with developing cracks. 4-shrinkage curve $\overline{V}_c'(W)$ of a soil core sample with developing cracks. 5-shrinkage curve $\overline{V}_c(W)$ of a soil matrix without cracks (the reference shrinkage curve). 6-the line that is parallel to the reference shrinkage curve in the basic shrinkage area. 7-1:1 line (saturation or pseudo saturation line). $\overline{V}_i$, $\overline{V}_s$, $\overline{V}_c$, and $\overline{V}_h$ designate values of corresponding specific volumes after oven drying. AB - the specific volume of layer subsidence in Bronswijk’s approximation at a given $W$; AC - the corrected specific volume of layer subsidence at a given $W$; BD - the specific volume of cracks in the soil layer in Bronswijk’s approximation at a given $W$; CE - the corrected specific volume of cracks in the soil layer at a given $W$; DE - the specific volume of cracks in the sample with free boundaries at a given $W$. 
approximation \( r \equiv r' \) is the same for layer and core sample) is calculated as (Fig. 1)

\[
r'(W) = \log(\hat{V}(W)/\hat{V}_h) / \log(\hat{V}(W)/\hat{V}_h)
\]

(2)

the corrected shrinkage geometry factor of a soil core, \( r_{sM} \) is calculated as (Fig. 1)

\[
r_{sM}(W) = \log(\hat{V}(W)/\hat{V}_h) / \log(\hat{V}(W)/\hat{V}_h)
\]

(3)

and the corrected shrinkage geometry factor of a soil layer, \( r_s \) is calculated as (Fig. 1)

\[
r_s(W) = \log(\hat{V}(W)/\hat{V}_h) / \log(\hat{V}(W)/\hat{V}_h)
\]

(4)

The specific crack volume of the layer in Bronswijk’s approximation, \( \bar{V}_{crl}(W) \) is calculated as (Fig. 1)

\[
\bar{V}_{crl}(W) = \hat{V}_l(W) - \hat{V}(W)
\]

(5)

The specific crack volume of the core in Bronswijk’s approximation is \( \bar{V}_{crs}(W) = 0 \).

The corrected specific crack volume of the layer, \( \bar{V}_{crl}(W) \) is calculated as (Fig. 1)

\[
\bar{V}_{crl}(W) = \hat{V}_l(W) - \bar{V}(W)
\]

(6)

and the corrected specific crack volume of the core, \( \bar{V}_{crs}(W) \) is calculated as (Fig. 1)

\[
\bar{V}_{crs}(W) = \hat{V}_s(W) - \bar{V}(W)
\]

(7)

The shrinkage curves, \( \hat{V}_s(W) \), \( \hat{V}_l(W) \), and \( \bar{V}_l(W) \), that, in general, contain the crack volume contribution, are found from data on vertical and lateral core shrinkage [6, 7]. The shrinkage curve, \( V(W) \) of the soil matrix without cracks is of special interest. Prediction of the reference shrinkage curve that can be used as \( V(W) \) was considered in [11-13].

**MATERIALS AND METHODS**

A measured shrinkage curve usually contains a crack contribution and does not coincide with the reference shrinkage curve of the soil. To estimate the reference shrinkage curve one needs to know a number of input parameters [11-13]. Available data on such parameters are so far limited. Works [11, 13] showed that experimental shrinkage curves relating to samples from four horizons of two soils from [18] (the shrinkage of one of the soils was appreciably weaker) can be predicted as the reference shrinkage curves. That is, these soil core samples did not contain cracks at any water content. We use these eight predicted reference shrinkage curves as \( V(W) \) to illustrate the estimation of the corrected specific crack volume and corrected shrinkage geometry factor. Figs. (2 and 3) show two examples of the \( V(W) \) curve (these curves were denoted as \( Y(W) \) in [11, 13]). In addition, following [18] we assume that, for soils under consideration, the core matrix shrinkage is isotropic (we should take that into consideration because this assumption was

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Fig. (2). The example of estimates based on the reference shrinkage curve [11, 13] for ferruginous soil, horizon A [18]. Shown are the shrinkage curves of: the soil matrix (the reference shrinkage curve), \( V(W) \); the core sample, \( \hat{V}_s(W) \) (in the case under consideration \( V(W) = \hat{V}_s(W) \)); and the layer before (\( \bar{V}_{crl}(W) \)) and after (\( \bar{V}_{crs}(W) \)) correction. The arrows show the specific volume of layer subsidence before (\( \bar{V}_{sub}(W) \)) and after (\( \bar{V}_{sub}(W) \)) correction as well as the specific volume of cracks in the layer before (\( \bar{V}_{crl}(W) \)) and after (\( \bar{V}_{crs}(W) \)) correction. The inclined dash-dot line is a pseudo saturation one. The dotted line is parallel to the reference shrinkage curve (\( V(W) \)) in the basic shrinkage area.
essential for obtaining the experimental shrinkage curves. Finally, while estimating the corrected shrinkage geometry factors and crack volumes as functions of soil water content, we used the values of the core diameter, \( d = 5.5 \) cm and height, \( h = 3 \) cm in the oven-dried state from [18]. Thus, in the examples under consideration the core samples only contain the soil matrix. That is, \( V_s = V(z, w) = \frac{d z}{2} V h d m = (8) \). The oven-dried mass of a sample, \( m_d \) is found to be
\[
m_d = \frac{\pi d^2 h z}{4 V z}
\]
where \( V z = V(0) \). It follows from the isotropy of matrix shrinkage and the absence of cracks in samples that an initial core diameter, \( d_h \), current core diameter, \( d(W) \), and current core height, \( h(W) \) can be presented as
\[
d_h = (V h / V z)^{1/3} d z, \quad d(W) = (V(W)/V z)^{1/3} d z,
\]
and
\[
h(W) = (V(W)/V z)^{1/3} h z
\]

The shrinkage curve \( V h(W) \) of a soil layer with cracks in Bronswijk’s approximation is found to be
\[
V h(W) = \frac{\pi d^2 h(W)}{4 m_d} \]

The shrinkage curve \( V h(W) \) of a connected soil layer with cracks is found to be [7]
\[
V h(W) = V h(W) \chi^2(W)
\]
where at the crack lack in samples
\[
\chi(W) = (1+d(W)/d_h)/2
\]

Then, different variants of the specific crack volume \( V crl, V crl ', \) and shrinkage geometry factor \( r_s, r_{sM}, \) and \( r_s \) are found from Eq. (2)-(7).

RESULTS AND DISCUSSION
The major results for the eight soils from [18] are presented in Fig. (2-7). Figs. (2 and 3) show the specific volumes for ferruginous soil, horizon A and ferrallitic soil, horizon A, respectively, as representatives of the eight soils: the estimates of \( V s = \hat{V} \) (see Eq. (8)), \( \hat{V}_1 \) (Eq. (11)), and (corrected) \( \hat{V}_1 \) (Eq. (12) and (13)). In addition, crack volumes \( \hat{V}_c r l, \) and (corrected) \( \hat{V}_c r l \) (Eq. (6)) are indicated by two-ended arrows. In Fig. (2 and 3) one can see the appreciable difference between the layer volume in Bronswijk’s approximation \( \hat{V}_1 \), the corrected layer volume \( \hat{V}_1 \), and the soil matrix volume \( \hat{V}_s \).

Figs. (4 and 5), as examples, show the specific crack volume in the layer before (\( \hat{V}_c r l ; \) Bronswijk’s approximation; Eq. (5)) and after (\( \hat{V}_c r l ; \) Eq. (6)) correction for ferruginous...
soil, horizons B1 and AB (Fig. 4) and for ferralic soil, horizons A and B2 (Fig. 5). Note that in the case under consideration due to Eq. (7) and (8) the crack volume of cores, $V_{crs} = 0$. One can see that in all cases the corrected crack volume, $V_{crl}$ is approximately half as much compared to $V_{crl}$ in all the $0 < W < W_h$ range. Even at small crack volume as in the case of ferruginous soil, horizon A (see Fig. 2), when the maximum of $V_{crl} = 1.15 \times 10^{-3}$ dm$^3$ kg$^{-1}$ and $V_{crl}' = 2.3 \times 10^{-3}$ dm$^3$ kg$^{-1}$, the difference between $V_{crl}'$ and $V_{crl}$ is crucially important for correct and dependable estimation of soil hydraulic properties.

It follows from Eq. (2), (3) and (8) as well as from the isotropy of the (core) matrix shrinkage that in the case under consideration

$$r_s(W) = r_{sM}(W) = 3$$

(14)

Note that $r_s$ gives the shrinkage geometry factor for both a soil layer and soil core in Bronswijk’s approximation. Figs.
The Open Mechanics Journal, 2008, Volume 2

V.Y. Chertkov

Fig. (6). Estimates of the corrected shrinkage geometry factor of a soil layer, $r_s$ based on the reference shrinkage curves [11,13] for ferruginous soil [18]: 1-horizon A; 2-horizon B1; 3-horizon B2; 4-horizon AB.

Fig. (7). Estimates of the corrected shrinkage geometry factor of a soil layer, $r_s$ based on the reference shrinkage curves [11, 13] for a ferralitic soil [18]: 1-horizon A; 2-horizon B2; 3-horizon AB.

(6 and 7) show estimates of the corrected shrinkage geometry factor, $r_s$, for the soil layer (Eq. (4)) of the four ferruginous soils and three ferralitic soils, respectively ($r_s(W)$ for horizons B1 and B2 of ferralitic soil were very close). One can see in Fig. (6 and 7) two major features of the corrected $r_s$. The former one is the essential difference between $r_s=3$ (Bronswijk’s approximation) and corrected $r_s$ for all eight soils. The latter is that the initial $r_s$ value is equal to 1.5 and $r_s$ grows with drying in a small range close to 1.5. We will show that the latter feature is the direct consequence of isotropic core matrix shrinkage without cracking for the soils and core samples under consideration. In the case of core shrinkage without cracks and isotropic core matrix shrinkage (assumption of [18]), the specific volume of the soil matrix, $\bar{V}$ (the reference shrinkage curve) and core diameter, $d$ are connected as

$$\bar{V}(W)/\bar{V}_h = (d(W)/d_h)^3$$

(15)

By definition, the current specific layer volume in Bronswijk’s approximation, $V_1'$, is connected with the current core diameter, $d$ (or height $h$, cf. Eq. (11)) as

$$\frac{V_1'(W)}{\bar{V}_h} = d(W)/d_h$$

(16)

By definition of the corrected shrinkage geometry factor of soil layer, $r_s(W)$, one can write (cf. Eq. (4))

$$\bar{V}(W)/\bar{V}_h = (\bar{V}_1(W)/\bar{V}_h)^r$$

(17)
where \( \tilde{V}_l \) is the corrected specific volume of a soil layer with cracks. Finally, \( \tilde{V}'_l \) and \( \tilde{V}'_l \) in the case of core shrinkage without cracks and isotropic core matrix shrinkage, are connected as (cf. Eq. (12) and (13))

\[
\tilde{V}'_l(W) = \tilde{V}'_l(W)(1 + d(W)/d_h)^2/4
\]

(18)

Combining Eq. (15)-(18) one obtains the following relation

\[
(\tilde{V}_l(W)/\tilde{V}_h)^3 = (\tilde{V}'_l(W)/\tilde{V}'_h)^s + (1 + \tilde{V}_l(W)/\tilde{V}_h)^{2r(W)} - r(W)
\]

(19)

Denoting in Eq. (19) \( \tilde{V}'_l(W)/\tilde{V}_h \equiv (1 - x) \) one can rewrite Eq. (19) as

\[
(1 - x)^3 = (1 - x)^s \cdot (2 - x)^s \cdot 4^s
\]

(20)

At the shrinkage start \( W_0 \rightarrow W_h \) (see Fig. 2 and 3), \( r_s(W) \rightarrow r_s(W_h) \equiv r_{sh}, \ \tilde{V}'_l(W) \rightarrow \tilde{V}_h \) (see Fig. 2 and 3), then

\[
x = \frac{1 - \tilde{V}'_l(W)/\tilde{V}_h}{\tilde{V}_h} \rightarrow 0, \ \text{and Eq. (20) gives}
\]

\[
1-3x(1-r_{sh})x^2 = 1-2 r_{sh} x
\]

(21)

That is, the corrected shrinkage geometry factor of a soil layer at the shrinkage start, \( r_{sh} \equiv 1.5 \).

It is worth emphasizing that the above results reflect the specifics of data from [18] (isotropic shrinkage of core matrix without cracks at any water content). In general, \( \tilde{V}'_s(W) > \tilde{V}'(W) \) unlike in Fig. (2 and 3) at the expense of core cracks (see Fig. 1). Moreover, in the general case \( r_s(W) \neq r_{sh}(W) \neq 3 \) unlike in Eq. (14). In addition, in the general case the corrected \( r_s(W) \) dependence, unlike in Fig. 6 and 7, can have the maximum with a water content decrease and can start from a value that differs from 1.5 (see Fig. (8) from [7]). However, we aimed at illustrating how the reference shrinkage curve of a soil can be used to estimate the corrected values of the shrinkage geometry factor and crack volume at soil shrinkage. The above results give corresponding examples and clearly show the essential differences between \( \tilde{V}'_l \), \( \tilde{V}'_l \), and \( \tilde{V}'(W) \) (Fig. 2 and 3), \( \tilde{V}_l(r_{crl}) \) and \( \tilde{V}_l(r_{crl}) \) (Fig. 4 and 5) as well as between the \( r_s(W) \) dependencies in Fig. 6 and 7 and \( r_s(W) \equiv r_{sh}(W) = 3 \) (Eq. (14)). These differences, in turn, indicate the necessity to introduce corrections into \( r_s \) values and soil crack volume estimates from Bronswijk’s approximation.

CONCLUSION

In addition to [6, 7] this work argues and illustrates the necessity of introducing corrections into the shrinkage geometry factor and crack volume compared to values that are determined from Bronswijk’s approximation. In line with that, the work illustrates the use and role of the reference shrinkage curve [11-13] for obtaining the corrected values of the shrinkage geometry factor and soil crack volume. We believe that this promising application of the soil reference shrinkage curve will be expanded with the accumulation of corresponding data.

REFERENCES