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Relations between the *S-N*, *ε-N* and d*a*/d*N*-Δ*K* Curves of Materials

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Abstract: The *S-N*, *ε-N* and d*a*/d*N*-Δ*K* curves are the basic data of fatigue property for materials, and they are used in the stress-life method, the local strain method and the LEFM (Linear elastic fracture mechanics) crack propagation method to predict fatigue life of structures, respectively. In the present paper, the relations between these fatigue curves and their probability of predicting each other were discussed through several fatigue-life models. The discussed subjects include (1) predicting the *S-N* curves from the d*a*/d*N*-Δ*K* curves of materials, (2) predicting the *S-N* curves from the *ε-N* curves, and (3) predicting the d*a*/d*N*-Δ*K* curves from the *ε-N* curves. It is shown that there are certain relations between the *S-N*, *ε-N* and d*a*/d*N*-Δ*K* curves of materials and it is predictable from one curve to another.

Keywords: d*a*/d*N*-Δ*K* curve, *ε-N* curve, *S-N* curve, relation.

1. INTRODUCTION

Since the concept of fatigue *S-N* curve, i.e. the stress-life curve, was put forward by Wöhler in the 1860s, various methods of fatigue-life predictions have been developed. Among the methods the stress-life method, the local strain method and the LEFM (Linear elastic fracture mechanics) crack propagation method have become the most classical ones. For the three methods, the *S-N*, *ε-N* (strain-life) and d*a*/d*N*-Δ*K* (crack propagation rate *vs*. stress intensity factor range) curves are their bases to predict fatigue-life, respectively. In order to obtain the *S-N*, *ε-N* and d*a*/d*N*-Δ*K* data for one material, three sets of tests, i.e. the *S-N*, *ε-N* and d*a*/d*N*-Δ*K* tests of standard specimens under constant amplitude loading, must be performed, respectively, which always needs much money, time and manpower.

In fact, the *S-N*, *ε-N* and d*a*/d*N*-Δ*K* curves are only the characterization of fatigue property for materials under different cases. Now let's suppose that there are certain relations between the three curves of materials and one curve can be predicted from another through the relations, then, the existing fatigue curve will educe two others, its value will double, and a great amount of experimental expense will be saved.

In the present paper, through several fatigue-life models we will discuss the relations between the *S-N*, *ε-N* and d*a*/d*N*-Δ*K* curves of materials, as well as their probability of predicting each other.

2. PREDICTING THE *S-N* CURVES FROM THE d*a*/d*N*-Δ*K* CURVES

From the viewpoint of fracture mechanics, fatigue crack emanates from the surface or near-surface defects of structure, and then evolves into 3D (three-dimensional) crack until the structure fails. In Reference [1] a 2D (two-dimensional) fracture mechanics based full-life model was put forward. In the model, one standard d*a*/d*N*-Δ*K* curve, i.e. the Pairs curve of material, is extrapolated toward small crack to obtain "the equivalent initial crack length a_0" (also called as "the equivalent initial defect a_0") of the material; further the a_0 is regarded as the inherent parameter of this material and is considered to be applicable to any other structure of same material and same environment; at last, with the a_0 as initial crack length, the total fatigue-life of other structures of same material and same environment can be predicted through the routine LEFM method. In Reference [1], with one experimental data-point as subject, the 2D fracture mechanics method, with the linearly extrapolated Pairs curve in logarithm coordinate, is used to calculate the fatigue life for crack propagating from a_0 to a_L, and the trial-and-error method is applied to determine the a_0 of material, i.e., a series of a_0 are tried one by one until the calculated life is equal with the experimental result. The idea of Reference [1] is inspirational, but the 2D fracture mechanics based model has vital defects. Firstly, the a_0 through the extrapolated Pairs curve is very difficult to be equivalent with the 3D initial defects of the actual 3D structures; secondly, the geometrical size effect [2] during 3D crack growth is ignored. In Reference [3, 4] we put forward a 3D fracture mechanics method to predict the *S-N* curves of material through the d*a*/d*N*-Δ*K* data. In the 3D fracture mechanics method the effective SIF (stress intensity factor) ΔK_{eff} for the crack closure model, i.e., $da/dN=C(\Delta K_{eff})$, takes the Newman's expression [5]. In the Newman's expression, the 3D constraint factor a_g is given by Reference [2].

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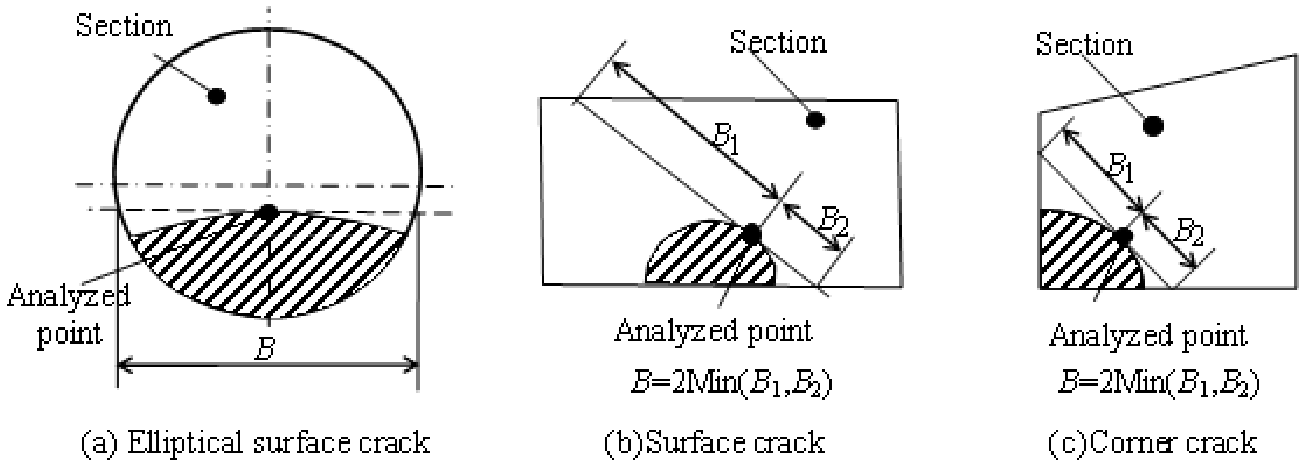


Fig. (1). Fatigue crack and the definition of equivalent thickness *B*.

$$a_g = \frac{1 + a_1 f(r_{po} / B)}{1 - 2\nu + b_1 f(r_{po} / B)} \qquad (1)$$

where $f(x) = x^{0.5} + 2x^2$; $r_{po} = \frac{\pi}{8}\left[\frac{K}{\sigma_{flow}}\right]^{1/2}$; K is the SIF of the investigated point of crack border; r_{po} is the plastic zone size; ν is the Poisson's ratio; σ_{flow} is the flow stress of material, namely, the average value of the yielding stress σ_s and the fracture stress σ_b; a_1=0.6378; b_1=0.5402; B is the equivalent thickness of specimen. For through-thickness cracked bodies, the B is the thickness of the specimen; for 3D cracked bodies, the B is defined as B=2min(B_1+B_2), where B_1 and B_2 are the distances from the analyzed point P on the crack border to the boundary of the cracked bodies along the tangential line of the crack front-line at P, as shown in Fig. (**1**). For elliptical surface crack in a round bar, the equivalent thickness B of the deepest point P is defined in Fig. (**1a**).

Apparently the constraint factor a_g of Eq. (1) is a function of the analyzed points on the 3D crack border, and the present crack closure model considers the effects of the specimen size and stress status.

By using the above closure model as well as one standard d*a*/d*N*-Δ*K* curve of material, the d*a*/d*N*-ΔK_{eff} curve can be obtained with the stress ratio, the specimen size and the stress status considered. According to the obtained d*a*/d*N*-ΔK_{eff} curve of material, the "cycle-by-cycle" method, the SIFs of 3D cracks published in the SIF handbooks [6] (or calculated by the finite element method [7]), and the crack shape evolvement given according to the theoretical method of Reference [8] or the analogous experiments [9], as well as the above 3D fracture mechanics method, we can predict the full-life of 3D structure, i.e. the fatigue life of crack propagating from the initial defect a_0 to the critical crack lengthen a_L in one direction of crack propagation, such as the propagation direction of the deepest point of 3D crack. Here the initial defect a_0 of material is given according to the following principles:

(1) Supposing that the initial defect of material is small semi-circular surface crack or fan-shaped corner crack. The radius of the cracks is a_0.

(2) With one experimental data-point as subject, the above 3D fracture mechanics method is used to calculate its fatigue life for crack propagating from a_0 to a_L. During the calculation, a series of a_0 are tried one by one until the calculated life is equal with the experimental result.

Apparently the a_0 is relative with environment and material. Once the a_0 is obtained, it can be repeatedly used to predict the total fatigue life of other 3D structures for same environment, same material and same quality of material by the above 3D fracture mechanics method. In other words, we can predict the *S*-*N* curves through the d*a*/d*N*-Δ*K* curves of material.

By the above method and the d*a*/d*N*-Δ*K* curves of 30CrMnSiA steel and LC9Cgs aluminum [10], the *S*-*N* curves of the rotating-bending round-bars of the two metals are predicted respectively. The 30CrMnSiA and LC9Cgs bars have the diameters of 7.5mm and 20mm respectively, and both their stress ratio r = -1. The predicted *S*-*N* curves as well as the experimental results [11] are shown in Fig. (**2**). In Fig. (**2**) the experimental data points marked by the "↑" are used to determine the a_0 of two materials through the trial-and-error method. Here the obtained a_0 of 30CrMnSiA steel and LC9Cgs aluminum is 0.015mm and 0.03mm respectively. From Fig. (**2**), it is shown that the predicted *S*-*N* curves are close to the experimental results [11].

3. PREDICTING THE *S*-*N* CURVES FROM THE ε-*N* CURVES

Considering that the classical local strain method is only valid to low cycle fatigue, in References [12] a modified local strain method was put forward by us for the life calculation of high cycle fatigue. In fact, the modified method has the same processes as the classical one. Their only difference lies in that the ε-*N* curve used in the modified

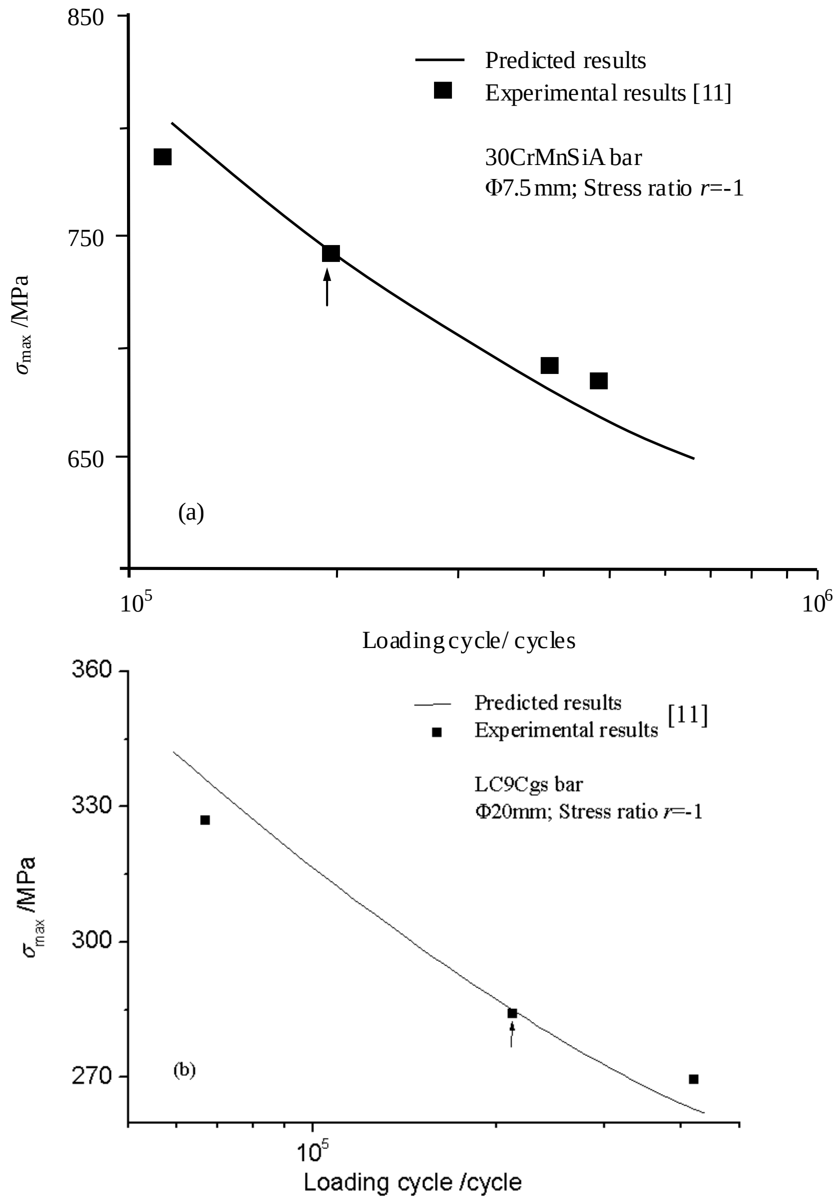


Fig.(2). The *S-N* curves of the rotating-bending round-bars.

method must have its elastic component modified according to the specimen size and surface quality. The followings are the modification procedures of the *ε-N* curve used in the modified local strain method.

In Fig. (**3**) Line1, i.e.; the AB line, is the elastic component of the *ε-N* curve for the classical local strain method. For high cycle fatigue, when the specimen size and surface quality are considered the fatigue-limit falls to the C point from the B point. Log(2*N*)=0 corresponds to the monotone load, and the specimen size and surface quality have no effect on the monotone load, so the A point has no change in position. Line2, i.e. the AC line, is the modified elastic component of the *ε-N* curve with the specimen size and surface quality considered. Considering that the specimen size and surface quality have only little effect on low cycles fatigue, the plastic component of the *ε-N* curve, i.e. Line 3, needs no modification. After the elastic component is modified, the *ε-N* curve changes into Line 5 from Line 4. The modified elastic component, i.e. Line 2, has the slope of

$$b' = \frac{\log(\sigma_{-1}\gamma_c\varepsilon_d\beta) - \log(\sigma'_f\gamma'_c)}{\log(2N_0)} \tag{2}$$

where σ'_f is the fatigue strength coefficient; γ'_c is the crystallite size coefficient for fracture strength; $\sigma_{-1}$is the fatigue-limit corresponding to the stress ratio r = -1; γ_c is the crystallite size coefficient for fatigue-limit; $\log(2N_0)\approx 7$; ε_d and β are the specimen size coefficient and the surface quality coefficient respectively, and the ε_d and β for some typical metal specimens are given in Reference [13, 14].

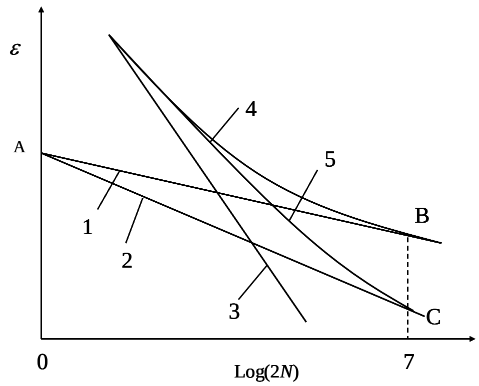


Fig.(3). The ε -N curves of materials.

The unmodified elastic component, i.e. Line 1, has the slope of,

$$b = \frac{\log(\sigma_{-1}\gamma_c) - \log(\sigma'_f \gamma'_c)}{\log(2N_0)} \tag{3}$$

Combining Eq. (2) and Eq. (3), the slope of Line 2

$$b' = b\frac{\log(\sigma_{-1}\gamma_c\varepsilon_d\beta) - \log(\sigma'_f \gamma'_c)}{\log(\sigma_{-1}\gamma_c) - \log(\sigma'_f \gamma'_c)} \tag{4}$$

After the modification, the Morrow equation for ε-N curves changes into,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f \gamma'_c - \sigma_m}{E}\left(2N\right)^{b'} + \varepsilon'\left(2N\right)^c \tag{5}$$

where E is the elastic module; ε' is the fatigue ductility coefficient; c is the fatigue ductility exponent; b is the fatigue strength exponent; b' is the modified fatigue strength exponent; σ_m is the average stress.

By using the modified method with the same steps as the classical local strain method [15], in addition to the standard ε-N curve of material, the specimen size coefficient ε_d and the surface quality coefficient β, the S-N curves for standard specimens of the material can be predicted.

Fig. (**4**) presents the experimental fatigue-life results for 3Cr13 and 2Cr11NiMolV stainless-steel plates (steam-

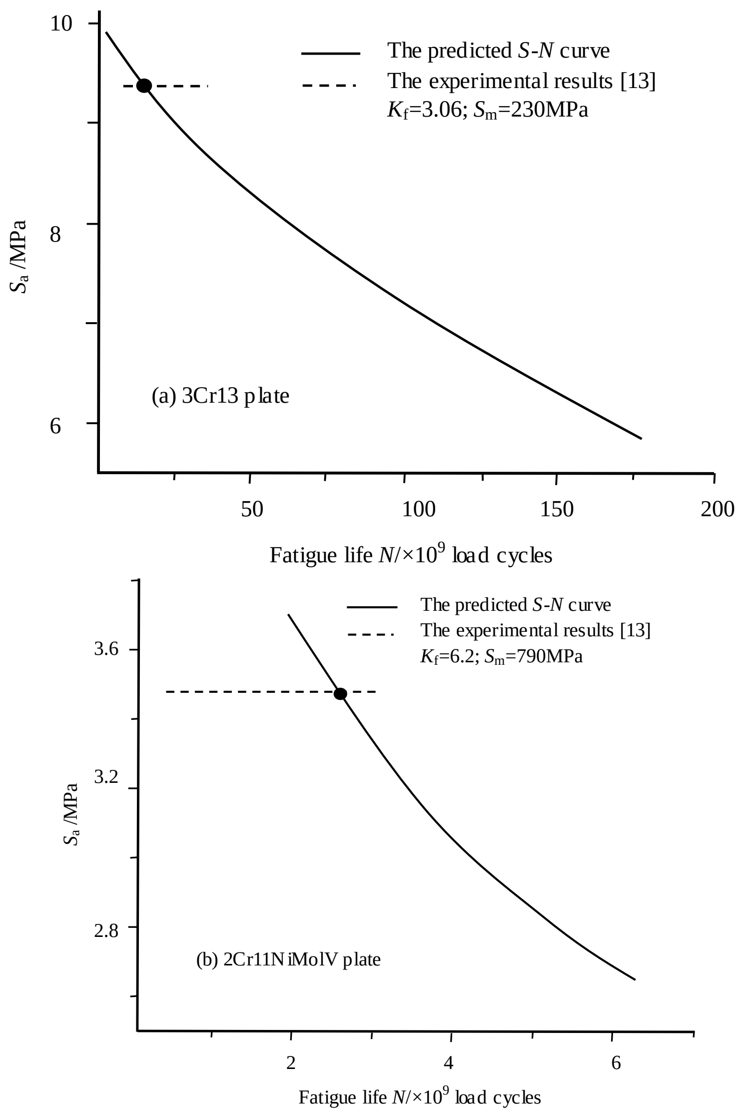


Fig. (4). The S-N curves for 3Cr13 and 2Cr11NiMolV plates.

turbine blades) under constant amplitude fatigue loading [13]. The 3Cr13 plate has the fatigue notch coefficient K_f = 3.06, the average nominal-stress S_m=230Mpa, and the nominal-stress amplitude S_a=9.73MPa; the 2Cr11NiMolV plate has K_f = 6.2, S_m=790Mpa, and S_a=3.47MPa. The experimental fatigue-life for the 3Cr13 plates distributes in the range from 8.8×10^9 to 35.3×10^9 load cycles, and that for 2Cr11NiMolV plate in the range from 0.32×10^9 to 3.2×10^9 load cycles. Their strain-fatigue parameters are as followings: E=210Gpa, ε' =0.381, c =-0.5791, σ_{-1} =301MPa, σ'_f =930MPa, b = -0.0743, cyclic strength coefficient K'=1325MPa, cyclic strain hardening exponent n'=0.099, and γ_c =1 for 3Cr13 steel; E=210Gpa, ε'=0.5561, c=-0.664, σ_{-1}=461MPa, σ'_f = 1249MPa, b=-0.0656, K'=1056MPa, n'=0.124, and γ_c = 1 for 2Cr11NiMolV steel. All the parameters come from Reference [16].

By using the present modified local-strain-method with $\varepsilon_d = \gamma'_c$ =1 and β=0.5 [11], Fig. (**4**) gives the predicted *S-N* curves for the two stainless-steel plates. From Fig. (**4**), it is found that the corresponding life-results, marked by solid circles in the predicted *S-N* curves, are close to the experimental ones [13].

4. PREDICTING THE D*a*/d*N*-Δ*K* CURVES FROM THE *ε-N* CURVES

From the view of low cycle fatigue, the fatigue crack propagation can be considered to be the process of the material at crack-tip continually fracturing under high-strain fatigue loading. In Reference [17] the strain-fatigue damage was introduced into crack-tip, and basing the idea the fatigue crack propagation was described. However, the model includes too many experimental parameters and is very inconvenient to analyze crack propagation. In Reference [18-20], we put forward a fatigue crack propagation model basing the concept of "crack-tip damage zone". In the model, it is assumed that:

(1) Within the plastic zone ω of crack-tip there is one small zone x^*, called as "crack-tip damage zone". In the damage zone, stress and strain have little gradient, see Fig. (**5**).

(2) The material in the zone x^* undergoes cyclic strain; when the life reaches N cycles, the material of the zone x^* cracks and the crack grows x^*.

(3) Outside the damage zone x^*, the stress and strain have very large gradient and attenuate very quickly, so the damage is relatively small and can be ignored.

(4) The stress and strain of the damage zone x^* can be respectively characterized with those at the midpoint of the zone, i.e. the position of 0.5 x^*.

Reference [18] gave the expressions of the stress σ and ε strain of the damage zone, i.e. the stress and strain at the position of $0.5x^*$, for power hardening material.

$$\sigma(0.5x^*) = \sigma_s\left(\frac{K^2}{0.5\pi(1+n)\sigma_s^2 x^*}\right)^{\frac{n}{1+n}}$$
$$\varepsilon(0.5x^*) = \varepsilon_s\left(\frac{K^2}{0.5\pi(1+n)\sigma_s^2 x^*}\right)^{\frac{1}{1+n}} \tag{6}$$

where σ_s is the yielding stress; ε_s (=σ_s/E) is the yielding strain; K is the SIF of the crack; n is the hardening exponential. Under cyclic loading, according to Eq. (6) the maximum stress σ_{max} and strain ε_{max} of the damage zone have the following relation:

$$\sigma_{max}\varepsilon_{max} = \frac{K_{max}^2}{0.5\pi E(1+n)x^*} \tag{7}$$

where K_{max} is the SIF corresponding to the maximal loading.

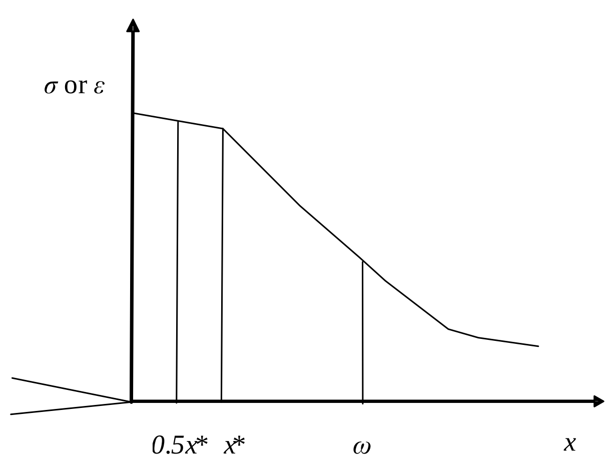


Fig.(5). The stress and strain distribution of crack-tip.

Supposing that material complies with the Ramberge-Osgood relationship, then

$$\varepsilon_{\max} = \frac{\sigma_{\max}}{E} + \left(\frac{\sigma_{\max}}{K'}\right)^{1/n} \tag{8}$$

For the center cracked plate, when the crack propagates x^* the released energy $U = \frac{K_{\max}^2}{E} x^*$ and the stress work of the damage zone $\bar{U} = \int \frac{1}{2}\sigma 2v(x)\,dx$. The U and $\bar{U}$ are equal in value, i.e.,

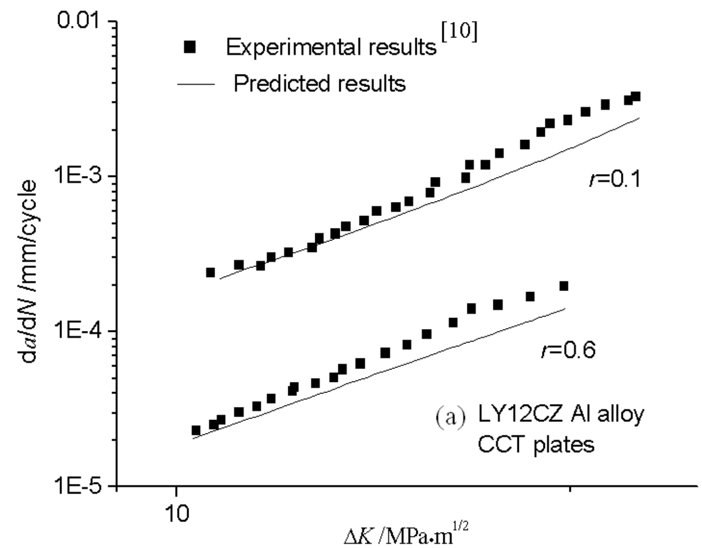
$$\frac{K_{\max}^2}{E} x^* = \int \frac{1}{2}\sigma 2v(x)\,dx \tag{9}$$

where $v(x)$ is the displacement vertical to the crack surface, and

$$v(x) = 4K_{\max} / E\sqrt{\frac{x^*-x}{2\pi}} \quad [21]$$

According to Eq. (7-9), the size x^*, the maximum stress $\sigma_{\max}$ and strain $\varepsilon_{\max}$ of the damage zone under $K_{\max}$ can be worked out.

Under constant amplitude fatigue loading, according to Eq. (7, 8), the stress amplitude $\Delta\sigma$ and the strain amplitude $\Delta\varepsilon$ have the following relations:



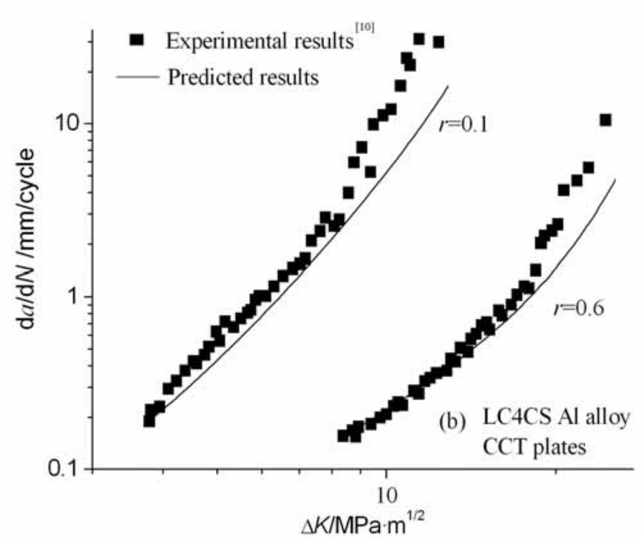


Fig. (6). The da/dN-ΔK curves of LY12CZ and LC4CS aluminum plates.

$$\Delta\sigma\Delta\varepsilon = \frac{K_{eff}^2}{0.5\pi E(1+n')x^*} \tag{10}$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{K''}\right)^{1/n'} \tag{11}$$

where n' is the cyclic hardening exponential; K'' is the cyclic hardening coefficient; the effective SIF range $\Delta K_{eff} = U'\Delta K$, and the closure factor $U' = (0.55+0.35r+0.1r^2)$ [22].

According to the Manson-Coffin equation for the ε-N curves,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f' - \sigma_m}{E}(2N)^b + \varepsilon'(2N)^c \tag{12}$$

The load-cyclic number N at the time of the damage zone x^* cracking, as well as the crack propagation ratio $da/dN=x^*/N$ under the present ΔK, can be obtained.

Repeatedly using Eq. (7)-Eq. (12), we will obtain the d*a*/d*N* corresponding to different Δ*K*, *i.e*. the d*a*/d*N*-Δ*K* curves of the material. Fig. (**6**) shows the experimental d*a*/d*N*-Δ*K* data [10] for LY12CZ and LC4CS aluminum plates under constant amplitude fatigue loading. All the aluminum plates are the standard CCT specimens, and the stress ratio *r* takes 0.1 and 0.6. The *ε*-*N* data (namely the parameters of Eq. (12)) of LY12CZ and LC4CS aluminum come from Reference [11].

According to Eq. (7)-Eq. (12), we predict the d*a*/d*N*-Δ*K* curves of the above LY12CZ and LC4CS aluminum plates. The predicted results are shown in Fig. (**6**). From Fig. (**6**), it is found that the predicted d*a*/d*N*-Δ*K* curves from the *ε*-*N* curves [11] are close to the experimental results [10].

Note: Here the plane stress assumption is taken, so the method is only suitable to the sheet materials.

5. CONCLUSIONS

Through several fatigue-life models the relations between the *S*-*N*, *ε*-*N* and d*a*/d*N*-Δ*K* curves of materials, as well as their probability of predicting each other, are discussed. It is found that it is possible to predict the *S*-*N* curves from the d*a*/d*N*-Δ*K* curves of material, to predict the *S*-*N* curves from the *ε*-*N* curves, and to predict the d*a*/d*N*-Δ*K* curves from the *ε*-*N* curves. The work in the present paper is very valuable for extending the available fatigue properties database and significant for unitizing the fatigue theory and the fracture theory.

NOMENCLATURE

a	=	Crack length
a_0	=	Initial crack length
a_L	=	Critical crack length
a_g	=	3D constraint factor
b	=	Fatigue strength exponent
b'	=	Modified fatigue strength exponent
B	=	Specimen equivalent thickness
β	=	Surface quality coefficient
c	=	Fatigue ductility exponent
ε	=	Strain
$\Delta\varepsilon$	=	Strain amplitude
ε_d	=	Specimen size coefficient
E	=	Elastic module
ε'	=	Fatigue ductility coefficient
ε_{max}	=	Maximum strain
ε_s	=	Yielding strain
K	=	Stress intensity factor (SIF)
K_f	=	Fatigue notch coefficient
K_{max}	=	Maximal SIF
K'	=	Cyclic strength coefficient
K''	=	Cyclic hardening coefficient
ΔK	=	SIF amplitude
ΔK_{eff}	=	Effective SIF amplitude
n	=	Hardening exponential
N	=	Fatigue life (loading cycle)
n'	=	Cyclic strain hardening exponent
r_{po}	=	Is the plastic zone size
S,σ	=	Stress
r	=	Stress ratio
γ_c	=	Crystallite size coefficient
γ_c'	=	Crystallite size coefficient
S_m	=	Average nominal-stress
S_a	=	Nominal-stress amplitude
U	=	Released energy
$\bar{U}$	=	Stress work
U	=	Closure factor
v	=	Displacement vertical to the crack surface
ν	=	Poisson's ratio
ω	=	Plastic zone
x^*	=	Crack-tip damage zone
σ_{flow}	=	Flow stress of material
σ_s	=	Yielding stress
σ_b	=	Fracture stress
σ'_f	=	Fatigue strength coefficient
σ_{-1}	=	Fatigue-limit
σ_m	=	Average stress

σ_{max} = Maximum stress

$\Delta\sigma$ = Stress amplitude

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