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Analyses of Inverse Kinematics, Statics and Workspace of a Novel 3RPS-3SPR Serial-Parallel Manipulator

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Abstract: A novel 3RPS-3SPR serial-parallel manipulator (S-PM) with 6 degree of freedoms (DOFs) is proposed in this paper. It includes a lower 3RPS parallel manipulator (PM) and an upper 3SPR PM. Its inverse kinematics, active forces and workspace are solved. First, the inverse displacement is solved in close form based on the geometrical and the dimensional constraints of this S-PM. Second, the 9×9 and 6×6 form inverse Jacobian matrices are derived and the active forces are solved using principle of virtual work. Third, the workspace of this S-PM is constructed by using CAD variation geometry approach.

Keywords: Serial-parallel manipulator, kinematics, statics, workspace.

INTRODUCTION

Recently, parallel manipulators (PMs) have attracted much attention due to their merits and industrial applications [1]. This kind of manipulators has been studied widely [2-7]. However, few investigations have been developed on S-PMs.

The S-PM has merits of both serial manipulator (SM) and PM. This kind of manipulators is characterized by its high stiffness compared with SM and large workspace compared with PM. In this aspect, Tanev [8] solved the forward and inverse position problems of a hybrid manipulator. Lu and Hu [9] solved active forces of a 2(3-SPR) S-PM by CAD variation geometry approach, and solved the forward kinematics of this S-PM [10] based on the analytical results of 3RPS PM. Jaime[11] studied the kinematics and dynamics of 2(3-RPS) manipulators by means of screw theory and the principle of virtual work. Zheng *et al.,* [12] studied an S-PM which possess of a pure translational and a pure rotational 3UPU PMs. O. Ibrahim and W. Khalil [13] proposed a method for the calculation of the inverse and direct dynamic models for S-PMs using recursive Newton Euler formalism. A. Ramadan [14] proposed a compact but yet economical two-fingered micro–nano hybrid manipulator hand. Liu [15] studied an S-PM formed by adding a 2-dof mechanism on Tricept.

In Gallardo's work [11], an improved 2(3SPR) S-PM with a more compact topology than the original one proposed by Lu [9] was investigated. The improved 2(3SPR) S-PM with compact topology can effectively diminish undesirable deflections and the presence of bending moments over the kinematical pairs which affecting the accuracy of this manipulator [11]. From this point of view, a novel S-PM which consists of a 3RPS PM [16-18] and its invertible structure 3SPR PM [19] adopting compact topology by using compound spherical joint [20] is investigated in this paper.

The forward kinematics of this kind of manipulators has been studied [8-12]. However, the inverse kinematics of this kind of manipulators is a difficult work and hasn't been attempted. Inverse kinematics is a common issue for kinematics analysis and plays an important role in control. This paper focuses on establishing the inverse displacement, inverse velocity and active forces of a 3RPS-3SPR S-PM based on the dimension and geometrical constraints. It is also a challenging work to solve the workspace of 3RPS-3SPR S-PM due to its complicated kinematics. The workspace of this S-PM is solved using CAD variation geometry approach in this paper.

1. CHARACTERISTICS OF 3RPS-3SPR S-PM

The 3RPS-3SPR S-PM with 6-DOF is consisted of a lower 3RPS PM and an upper 3SPR PM. The 3RPS and 3SPR PMs are connected serially. The lower 3RPS PM includes a moving platform m_b, a base B, and three extendable active limbs r_{bi} (i=1, 2, 3) with their linear actuators. The upper 3SPR PM includes a moving platform m, a base m_b which is simultaneously used as the moving platform of the lower PM, and three extendable driving limbs r_{ci} (i=1, 2, 3) with their linear actuators. m is a equilateral triangle $\Delta c_1c_2c_3$ with o as its center and $l_i=l$ as its side. B is an equilateral triangle $\Delta a_1a_2a_3$ with O as its center and $L_i=L$ as its side. m_b is a equilateral triangle $\Delta b_1b_2b_3$ with $d_i=d$ as its sides.

Three identical RPS limbs r_{bi} (i=1, 2, 3) of the lower 3RPS PM connect m_b to B by a spherical joint S on m_b at b_i, a driving limb with a prismatic joint P, and a revolute joint R

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on B at a_i, respectively. The R joint on B at a_i is parallel with the subtense of a_i. Three identical SPR limbs r_{ci} (i=1, 2, 3) of the upper 3SPR PM connect m to m_b by a revolute joint R on m at point c_i, a driving limb with a prismatic joint P, and a spherical joint S on m_b at b_i, respectively. The R joint on m at c_i is parallel with the subtense of c_i. Let $\{B\}$ be a coordinate O-XYZ with O as its origin fixed on B at O, $\{m\}$ be a coordinate o-xyz with o as its origin fixed on m at o. Let $\|$ and $\perp$ be parallel and perpendicular constraints, respectively. Some constraints ($X\|a_1a_3$, $Y\perp a_1a_3$, $Z\perp B$, $x\|c_1c_3$, $y\perp c_1c_3$, $z\perp m$) are satisfied, see Fig. (**1**).

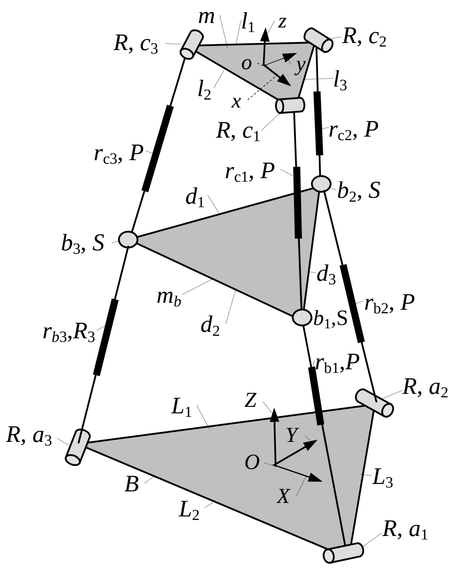

Fig. (1). 3RPS-3SPR S-PM.

2. INVERSE DISPLACEMENT ANALYSIS

The position vectors $\boldsymbol{a}_i$ (i=1, 2, 3) of vertices a_i in $\{B\}$ can be expressed as follows

$$\boldsymbol{a}_1=\begin{bmatrix} a_{1x}\\ a_{1y}\\ a_{1z}\end{bmatrix}=\begin{bmatrix} qE/2\\ -E/2\\ 0\end{bmatrix},\ \boldsymbol{a}_2=\begin{bmatrix} a_{2x}\\ a_{2y}\\ a_{2z}\end{bmatrix}=\begin{bmatrix} 0\\ E\\ 0\end{bmatrix},$$
$$\boldsymbol{a}_3=\begin{bmatrix} a_{3x}\\ a_{3y}\\ a_{3z}\end{bmatrix}=\begin{bmatrix} -qE/2\\ -E/2\\ 0\end{bmatrix} \tag{1a}$$

The position vectors $\boldsymbol{c}_i$(i=1, 2, 3) of vertices c_i in $\{m\}$can be expressed as follows

$$^m\boldsymbol{c}_1=\begin{bmatrix} qC/2\\ -C/2\\ 0\end{bmatrix},\ ^m\boldsymbol{c}_2=\begin{bmatrix} 0\\ C\\ 0\end{bmatrix},\ ^m\boldsymbol{c}_3=\begin{bmatrix} -qC/2\\ -C/2\\ 0\end{bmatrix} \tag{1b}$$

The position vectors $\boldsymbol{c}_i$ of vertices c_i in $\{B\}$ can be expressed as follows

$$\boldsymbol{c}_i=\begin{bmatrix} c_{ix}\\ c_{iy}\\ c_{iz}\end{bmatrix}=\mathrm{R}\,^m\boldsymbol{c}_i+\boldsymbol{o} \tag{2}$$

$$\boldsymbol{o}=[\,X_o\quad Y_o\quad Z_o\,]^T,\ \mathrm{R}=[\,\mathbf{x}\quad \mathbf{y}\quad \mathbf{z}\,]$$
$$\boldsymbol{x}=[\,x_l\quad x_m\quad x_n\,]^T,\ \boldsymbol{y}=[\,y_l\quad y_m\quad y_n\,]^T,\ \boldsymbol{z}=[\,z_l\quad z_m\quad z_n\,]^T$$

Here X_o, Y_o and Z_o are three position coordinates of o, $\mathbf{R}$ is a rotation transformation matrix from $\{B\}$ to $\{m\}$, E denotes the distance from point O to a_i, C denotes the distance from point o to c_i and q=$3^{1/2}$.

Let α, β and λ be three Euler angles. Let φ be one of (α, β, λ), s_φ=sinφ, c_φ=cosφ. Using X-Y-X type Euler rotations, the rotation transformation matrix can be expressed as follows [16]:

$$\mathbf{R}=\begin{bmatrix} c_\beta & s_\beta s_\lambda & s_\beta c_\lambda\\ s_\alpha s_\beta & c_\alpha c_\lambda - s_\alpha c_\beta s_\lambda & -c_\alpha s_\lambda - s_\alpha c_\beta c_\lambda\\ -c_\alpha s_\beta & s_\alpha c_\lambda + c_\alpha c_\beta s_\lambda & -s_\alpha s_\lambda + c_\alpha c_\beta c_\lambda\end{bmatrix} \tag{3}$$

Let R_{i1} (i=1, 2, 3) be the revolute joints at a_i (i=1,2,3) on the lower PM, R_{i2} (i=1,2,3) be the revolute joints at c_i (i=1,2,3) on the upper PM. Then the unit vectors $\boldsymbol{R}_{ij}$ of R_{ij} (i=1, 2; j=1, 2, 3) in $\{B\}$ can be expressed as following

$$\mathbf{R}_{11}=\frac{1}{2}\begin{bmatrix}1 & q & 0\end{bmatrix}^T,\ \mathbf{R}_{12}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T,\ \mathbf{R}_{13}=\frac{1}{2}\begin{bmatrix}-1 & q & 0\end{bmatrix}^T \tag{4a}$$

$$\mathbf{R}_{21}=\mathrm{R}\,\mathbf{R}_{11}=\frac{1}{2}\begin{bmatrix} x_l+qy_l\\ x_m+qy_m\\ x_n+qy_n\end{bmatrix},\ \mathbf{R}_{22}=\mathrm{R}\,\mathbf{R}_{21}=\begin{bmatrix} x_l\\ x_m\\ x_n\end{bmatrix},$$
$$\mathbf{R}_{23}=\mathrm{R}\,\mathbf{R}_{31}=\frac{1}{2}\begin{bmatrix} -x_l+qy_l\\ -x_m+qy_m\\ -x_n+qy_n\end{bmatrix} \tag{4b}$$

In the lower and upper PMs, the geometrical constraints satisfy

$$R_{i1}\perp r_{bi},\ R_{i2}\perp r_{ci}\ (i\text{=1, 2, 3}) \tag{5a}$$

From Eq. (5a), it leads to

$$\mathbf{R}_{i1}\cdot(\boldsymbol{b}_i-\boldsymbol{a}_i)=0 \tag{5b}$$

$$\mathbf{R}_{i2}\cdot(\boldsymbol{b}_i-\boldsymbol{c}_i)=0 \tag{5c}$$

here, $\boldsymbol{b}_i$ denotes the position vectors of point b_i (i=1, 2, 3).

Let h_{ix}, h_{iy}, h_{iz} be three position coordinates of point h_i(i=1, 2, 3), where h comes form a, b, c.

From Eq. (5b), it leads to

$$\begin{bmatrix}1 & q & 0\end{bmatrix}\begin{bmatrix}b_{1x}-a_{1x} & b_{1y}-a_{1y} & b_{1z}-a_{1z}\end{bmatrix}^T=0 \tag{6a}$$

$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}b_{2x}-a_{2x} & b_{2y}-a_{2y} & b_{2z}-a_{2z}\end{bmatrix}^T=0 \tag{6b}$$

$$\begin{bmatrix}-1 & q & 0\end{bmatrix}\begin{bmatrix}b_{3x}-a_{3x} & b_{3y}-a_{3y} & b_{3z}-a_{3z}\end{bmatrix}^T=0 \tag{6c}$$

From Eqs. (6a) to (6c), it leads to

$$b_{1x}+qb_{1y}=0\Rightarrow b_{1x}=-qb_{1y} \tag{7a}$$

$$b_{2x}=a_{2x}=0 \tag{7b}$$

$$b_{3x}-qb_{3y}=0\Rightarrow b_{3x}=qb_{3y} \tag{7c}$$

From (5c), it leads to

$$\left[x_l+qy_l \quad x_m+qy_m \quad x_n+qy_n\right]\left[b_{1x}-c_{1x} \quad b_{1y}-c_{1y} \quad b_{1z}-c_{1z}\right]^T=0 \tag{8a}$$

$$\left[x_l \quad x_m \quad x_n\right]\left[b_{2x}-c_{2x} \quad b_{2y}-c_{2y} \quad b_{2z}-c_{2z}\right]^T=0 \tag{8b}$$

$$\left[-x_l+qy_l \quad -x_m+qy_m \quad -x_n+qy_n\right]\left[b_{3x}-c_{3x} \quad b_{3y}-c_{3y} \quad b_{3z}-c_{3z}\right]^T=0 \tag{8c}$$

Expanding Eqs. (8a-c), it leads to

$$\begin{aligned}&-q(x_l+qy_l)b_{1y}+(x_m+qy_m)b_{1y}+(x_n+qy_n)b_{1z}\\&=(x_l+qy_l)c_{1x}+(x_m+qy_m)c_{1y}+(x_n+qy_n)c_{1z}\end{aligned} \tag{9a}$$

$$x_m b_{2y}+x_n b_{2z}=x_l c_{2x}+x_m c_{2y}+x_n c_{2z} \tag{9b}$$

$$\begin{aligned}&q(-x_l+qy_l)b_{3y}+(-x_m+qy_m)b_{3y}+(-x_n+qy_n)b_{3z}\\&=(-x_l+qy_l)c_{3x}+(-x_m+qy_m)c_{3y}+(-x_n+qy_n)c_{3z}\end{aligned} \tag{9c}$$

From Eqs. (9a-c), b_{1z}, b_{2z} and b_{3z} can be expressed as follows:

$$\begin{aligned}&b_{1z}=s_{11}+s_{12}b_{1y},\\&s_{11}=\frac{(x_l+qy_l)c_{1x}+(x_m+qy_m)c_{1y}+(x_n+qy_n)c_{1z}}{x_n+qy_n},\\&s_{12}=\frac{qx_l+3y_l-x_m-qy_m}{x_n+qy_n},\end{aligned} \tag{10a}$$

$$\begin{aligned}&b_{2z}=s_{21}+s_{22}b_{2y}\\&s_{21}=\frac{x_l c_{2x}+x_m c_{2y}+x_n c_{2z}}{x_n},\ s_{22}=\frac{-x_m}{x_n}\end{aligned} \tag{10b}$$

$$\begin{aligned}&b_{3z}=s_{31}+s_{32}b_{3y}\\&s_{31}=\frac{(-x_l+qy_l)c_{3x}-(x_m-qy_m)c_{3y}-(x_n-qy_n)c_{3z}}{-x_n+qy_n}\\&s_{32}=\frac{qx_l-3y_l+x_m-qy_m}{-x_n+qy_n}\end{aligned} \tag{10c}$$

From Eqs. (7a-c) and Eqs. (10a-c), the position vectors $\boldsymbol{b}_i$ (i=1, 2, 3) of vertices b_i can be expressed as follows:

$$\mathbf{b}_1=\begin{bmatrix}-qb_{1y}\\b_{1y}\\s_{11}+s_{12}b_{1y}\end{bmatrix},\mathbf{b}_2=\begin{bmatrix}0\\b_{2y}\\s_{21}+s_{22}b_{2y}\end{bmatrix},\mathbf{b}_3=\begin{bmatrix}qb_{3y}\\b_{3y}\\s_{31}+s_{32}b_{3y}\end{bmatrix} \tag{11}$$

From the dimension constraints of the m_b, it leads to

$$(b_{1x}-b_{2x})^2+(b_{1y}-b_{2y})^2+(b_{1z}-b_{2z})^2=d^2 \tag{12a}$$

$$(b_{3x}-b_{2x})^2+(b_{3y}-b_{2y})^2+(b_{3z}-b_{2z})^2=d^2 \tag{12b}$$

$$(b_{1x}-b_{3x})^2+(b_{1y}-b_{3y})^2+(b_{1z}-b_{3z})^2=d^2 \tag{12c}$$

By substituting Eq. (11) into (12a-b), it leads to

$$\begin{aligned}&p_{15}b_{1y}^2+p_{14}b_{2y}^2+p_{13}b_{1y}b_{2y}+p_{12}b_{2y}+p_{11}b_{1y}+p_{10}=0\\&p_{25}b_{3y}^2+p_{24}b_{2y}^2+p_{23}b_{2y}b_{3y}+p_{22}b_{3y}+p_{21}b_{2y}+p_{20}=0\\&p_{35}b_{3y}^2+p_{34}b_{1y}^2+p_{33}b_{1y}b_{3y}+p_{32}b_{3y}+p_{31}b_{1y}+p_{30}=0\end{aligned} \tag{13}$$

Where

$$\begin{aligned}&p_{15}=4+s_{12}^2,\ p_{14}=1+s_{22}^2,\ p_{13}=-2-2s_{12}s_{22},\\&p_{12}=-2s_{22}(s_{11}-s_{21}),\ p_{11}=2s_{12}(s_{11}-s_{21}),\ p_{10}=(s_{11}-s_{21})^2-d^2,\\&p_{25}=4+s_{32}^2,\ p_{24}=1+s_{22}^2,\ p_{23}=-2-2s_{32}s_{22},\ p_{22}=2s_{32}(s_{31}-s_{21}),\\&p_{21}=-2s_{22}(s_{31}-s_{21}),\ p_{20}=(s_{31}-s_{21})^2-d^2,\\&p_{35}=4+s_{32}^2,\ p_{34}=4+s_{12}^2,\ p_{33}=4-2s_{32}s_{12},\ p_{32}=2s_{32}(s_{31}-s_{11}),\\&p_{31}=-2s_{12}(s_{31}-s_{11}),\ p_{30}=(s_{31}-s_{11})^2-d^2,\end{aligned}$$

From Eqs. (13), it leads to

$$k_{12}b_{1y}^2+k_{11}b_{1y}+k_{10}=0 \tag{14a}$$

$$k_{22}b_{3y}^2+k_{21}b_{3y}+k_{20}=0 \tag{14b}$$

$$k_{32}b_{3y}^2+k_{31}b_{3y}+k_{30}=0 \tag{14c}$$

here

$$\begin{aligned}&k_{12}=p_{15},\ k_{11}=p_{11}+p_{13}b_{2y},\\&k_{10}=p_{14}b_{2y}^2+p_{12}b_{2y}+p_{10}\\&k_{22}=p_{25},\ k_{21}=p_{23}b_{2y}+p_{22},\\&k_{20}=p_{24}b_{2y}^2+p_{21}b_{2y}+p_{20}\\&k_{32}=p_{35},\ k_{31}=p_{33}b_{1y}+p_{32},\\&k_{30}=p_{34}b_{1y}^2+p_{31}b_{1y}+p_{30}\end{aligned}$$

From Eq. (14a), it leads to

$$b_{3y}^2=\frac{k_{21}k_{30}-k_{31}k_{20}}{k_{31}k_{22}-k_{21}k_{32}} \tag{15a}$$

From Eq. (14b), it leads to

$$b_{3y}=\frac{k_{22}k_{30}-k_{32}k_{20}}{k_{32}k_{21}-k_{22}k_{31}} \tag{15b}$$

From Eqs. (15a-b), it leads to

$$\frac{k_{21}k_{30}-k_{31}k_{20}}{k_{31}k_{22}-k_{21}k_{32}}=\left(\frac{k_{22}k_{30}-k_{32}k_{20}}{k_{32}k_{21}-k_{22}k_{31}}\right)^2 \tag{16}$$

From Eq. (16), it leads to

$$t_{14}b_{1y}^4+t_{13}b_{1y}^3+t_{12}b_{1y}^2+t_{11}b_{1y}+t_{10}=0 \tag{17}$$

here

$$\begin{aligned}&t_{14}=k_{22}^2p_{34}^2,\ t_{13}=2k_{22}^2p_{34}p_{31}+k_{21}k_{22}p_{33}p_{34},\\&t_{12}=k_{22}^2(p_{31}^2+2p_{30}p_{34})-p_{34}(2k_{20}k_{22}k_{32}+k_{21}^2k_{32})+\\&k_{21}k_{22}(p_{33}p_{31}+p_{32}p_{34})\text{-}k_{20}k_{22}p_{33}^2,\\&t_{11}=2k_{22}^2p_{30}p_{31}-p_{31}(2k_{20}k_{22}k_{32}+k_{21}^2k_{32})+\\&k_{21}k_{22}(p_{33}p_{30}+p_{32}p_{31})\text{-}2k_{20}k_{22}p_{32}p_{33}+k_{20}k_{21}k_{32}p_{33},\\&t_{10}=k_{22}^2p_{30}^2-p_{30}(2k_{20}k_{22}k_{32}+k_{21}^2k_{32})+k_{21}k_{22}p_{32}p_{30}\text{-}\\&k_{20}k_{22}p_{32}^2+k_{20}k_{21}k_{32}p_{32}+k_{20}^2k_{32}^2,\end{aligned}$$

From Eqs. (14a and 17), it leads to

$$\mathrm{U}\begin{bmatrix} b_{1y}^5 \\ b_{1y}^4 \\ b_{1y}^3 \\ b_{1y}^2 \\ b_{1y} \\ 1 \end{bmatrix} = 0,\ \mathrm{U} = \begin{bmatrix} 0 & t_{14} & t_{13} & t_{12} & t_{11} & t_{10} \\ t_{14} & t_{13} & t_{12} & t_{11} & t_{10} & 0 \\ 0 & 0 & 0 & k_{12} & k_{11} & k_{10} \\ 0 & 0 & k_{12} & k_{11} & k_{10} & 0 \\ 0 & k_{12} & k_{11} & k_{10} & 0 & 0 \\ k_{12} & k_{11} & k_{10} & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

The necessary condition for Eq. (18) to have nontrivial solutions is

$$\left|\mathrm{U}\right| = 0 \tag{19}$$

Eq. (19) is a polynomial about b_{2y}. When X_o, Y_o, Z_o, α, β and λ are given, b_{2y} can be solved from Eq. (19), b_{1y} and b_{3y} can be solved from Eq. (14a-b) and the coordinates b_i(i=1, 2, 3) can be solved form Eq. (11). Then the inverse kinematics can be solved subsequently by the following equations.

$$r_{bi} = \left|\boldsymbol{a}_i - \boldsymbol{b}_i\right| \ (i=1, 2, 3) \tag{20a}$$

$$r_{ci} = \left|\boldsymbol{b}_i - \boldsymbol{c}_i\right| \ (i=1, 2, 3) \tag{20b}$$

3. INVERSE JACOBIAN ANALYSIS

The loop equation of $Oa_ib_ic_io$ (i=1, 2, 3) can be written as follows

$$\boldsymbol{Oa}_i + \boldsymbol{a}_i\boldsymbol{b}_i + \boldsymbol{b}_i\boldsymbol{c}_i = \boldsymbol{Oo} + \boldsymbol{oc}_i \tag{21}$$

By differentiating both sides of Eq. (21) with respect to time, it leads to

$$v_{rbi}\delta_{bi} + \omega_{rbi} \times \boldsymbol{r}_{bi} + v_{rci}\delta_{ci} + \omega_{rci} \times \boldsymbol{r}_{ci} = \boldsymbol{v} + \omega \times \boldsymbol{e}_i \tag{22}$$

Where $\boldsymbol{r}_{bi}$ and $\boldsymbol{r}_{ci}$ are the vectors, $\boldsymbol{\delta}_{bi}$ and $\boldsymbol{\delta}_{ci}$ are the unit vectors of r_{bi} and r_{ci}, respectively. v_{rbi} and v_{rci}(i=1,2,3) are the velocities, $\boldsymbol{\omega}_{rbi}$ and $\boldsymbol{\omega}_{rci}$ are the angular velocity vectors of r_{bi} and r_{ci}, respectively. $\boldsymbol{v}$ denotes the velocity of point o, $\boldsymbol{\omega}$ denotes the angular velocity of m and $\boldsymbol{e}_i$ denotes the vectors form $\boldsymbol{o}$ to $\boldsymbol{c}_i$.

Based on the geometrical constraints of the upper manipulator, it leads to

$$\boldsymbol{r}_{ci} \cdot \boldsymbol{R}_{i2} = 0 \tag{23a}$$

Since r_{bi} (i=1, 2, 3) rotate with R_{i1}(i=1,2,3), it leads to

$$\boldsymbol{\omega}_{rbi} = \omega_{rbi}\boldsymbol{R}_{i1} \tag{23b}$$

here, ω_{rbi} are the angular velocities of $\boldsymbol{R}_{i1}$.

Differentiating both sides of Eq. (23a) with respect to time, it leads to

$$(\boldsymbol{v}_{rci}\boldsymbol{\delta}_{ci} + \boldsymbol{\omega}_{rci} \times \boldsymbol{r}_{ci}) \cdot \boldsymbol{R}_{i2} + \boldsymbol{r}_{ci} \cdot (\omega \times \boldsymbol{R}_{i2}) = 0 \tag{24a}$$

$$(\boldsymbol{\omega}_{rci} \times \boldsymbol{r}_{ci}) \cdot \boldsymbol{R}_{i2} = -\boldsymbol{r}_{ci} \cdot (\omega \times \boldsymbol{R}_{i2}) \tag{24b}$$

Dot-multiplying Eq. (22) with $\boldsymbol{R}_{i2}$ at both sides and by means of (23a and 24b), it leads to

$$v_{rbi}\delta_{bi} \cdot \boldsymbol{R}_{i2} + \omega_{rbi}(\boldsymbol{R}_{i1} \times \boldsymbol{r}_{bi}) \cdot \boldsymbol{R}_{i2} - (\boldsymbol{R}_{i2} \times \boldsymbol{r}_{ci}) \cdot \omega = (\boldsymbol{v} + \omega \times \boldsymbol{e}_i) \cdot \boldsymbol{R}_{i2} \tag{25a}$$

From Eq. (25a), it leads to

$$v_{rbi}(\delta_{bi} \cdot \boldsymbol{R}_{i2}) + \omega_{rbi}[(\boldsymbol{R}_{i1} \times \boldsymbol{r}_{bi}) \cdot \boldsymbol{R}_{i2}] = \begin{bmatrix} \boldsymbol{R}_{i2}^T & [(\boldsymbol{e}_i - \boldsymbol{r}_{ci}) \times \boldsymbol{R}_{i2}]^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \omega \end{bmatrix} \tag{25b}$$

Dot-multiplying Eq.(22) with $\boldsymbol{\delta}_{ci}$ at both sides, it leads to

$$\boldsymbol{v}_{rbi}\boldsymbol{\delta}_{bi} \cdot \boldsymbol{\delta}_{ci} + (\boldsymbol{\omega}_{rbi} \times \boldsymbol{r}_{bi}) \cdot \boldsymbol{\delta}_{ci} + \boldsymbol{v}_{rci} = (\boldsymbol{v} + \ddot{\boldsymbol{a}} \times \boldsymbol{e}_i) \cdot \boldsymbol{\delta}_{ci} \tag{26a}$$

Substituting Eq. (23b) into Eq. (26*a*), it leads to

$$\begin{aligned} &v_{rbi}\delta_{bi} \cdot \delta_{ci} + \omega_{rbi}(\boldsymbol{R}_{i1} \times \boldsymbol{r}_{bi}) \cdot \delta_{ci} + v_{rci} = (\boldsymbol{v} + \omega \times \boldsymbol{e}_i) \cdot \delta_{ci} \\ &= \begin{bmatrix} \delta_{ci}^T & (\boldsymbol{e}_i \times \delta_{ci})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \omega \end{bmatrix} \end{aligned} \tag{26b}$$

The velocity of b_i can be expressed as following

$$\dot{\boldsymbol{b}}_i = \boldsymbol{v}_{rbi}\boldsymbol{\delta}_{bi} + \boldsymbol{\omega}_{rbi} \times r_{bi} = \boldsymbol{v}_{rbi}\boldsymbol{\delta}_{bi} + \boldsymbol{\omega}_{rbi}(\boldsymbol{R}_{i1} \times r_{bi}) \tag{27}$$

Since $b_1b_2b_3$ is an equilateral triangle, the dimension constrained equations can be expressed as follows:

$$(\boldsymbol{b}_i - \boldsymbol{b}_j) \cdot (\boldsymbol{b}_i - \boldsymbol{b}_j) = \boldsymbol{d}^2, \boldsymbol{i} = 1,2,3;\ \boldsymbol{j} = 1,2,3;\ \boldsymbol{i} \neq \boldsymbol{j} \tag{28a}$$

Differentiating both sides of Eq. (28a) with respect to time yields

$$(\boldsymbol{b}_i - \boldsymbol{b}_j) \cdot (\boldsymbol{b}_i - \boldsymbol{b}_j) = 0, \boldsymbol{i} = 1,2,3;\ \boldsymbol{j} = 1,2,3;\ \boldsymbol{i} \neq \boldsymbol{j} \tag{28b}$$

From Eq. (28b), it leads to

$$(\dot{\boldsymbol{b}}_2 - \dot{\boldsymbol{b}}_1) \cdot (\boldsymbol{b}_2 - \boldsymbol{b}_1) = 0 \tag{28c}$$

$$(\dot{\boldsymbol{b}}_3 - \dot{\boldsymbol{b}}_1) \cdot (\boldsymbol{b}_3 - \boldsymbol{b}_1) = 0 \tag{28d}$$

$$(\dot{\boldsymbol{b}}_2 - \dot{\boldsymbol{b}}_3) \cdot (\boldsymbol{b}_2 - \boldsymbol{b}_3) = 0 \tag{28e}$$

From (25b), it leads to

$$v_{rb1}(\delta_{b1} \cdot \boldsymbol{R}_{12}) + \omega_{rb1}[(\boldsymbol{R}_{11} \times \boldsymbol{r}_{b1}) \cdot \boldsymbol{R}_{12}] = \begin{bmatrix} \boldsymbol{R}_{12}^T & [(\boldsymbol{e}_1 - \boldsymbol{r}_{c1}) \times \boldsymbol{R}_{12}]^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \omega \end{bmatrix} \tag{29a}$$

$$\begin{aligned} &\boldsymbol{v}_{rb2}(\boldsymbol{\delta}_{b2} \cdot \boldsymbol{R}_{22}) + \boldsymbol{\omega}_{rb2}[(\boldsymbol{R}_{21} \times \boldsymbol{r}_{b2}) \cdot \boldsymbol{R}_{22}] \\ &= \begin{bmatrix} \boldsymbol{R}_{22}^T & [(\boldsymbol{e}_2 - \boldsymbol{r}_{c2}) \times \boldsymbol{R}_{22}]^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} \end{aligned} \tag{29b}$$

$$\begin{aligned} &\boldsymbol{v}_{rb3}(\boldsymbol{\delta}_{b3} \cdot \boldsymbol{R}_{32}) + \boldsymbol{\omega}_{rb3}[(\boldsymbol{R}_{31} \times \boldsymbol{r}_{b3}) \cdot \boldsymbol{R}_{32}] \\ &= \begin{bmatrix} \boldsymbol{R}_{32}^T & [(\boldsymbol{e}_3 - \boldsymbol{r}_{c3}) \times \boldsymbol{R}_{32}]^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} \end{aligned} \tag{29c}$$

From Eq. (26b), it leads to

$$\begin{aligned} &\boldsymbol{v}_{rb1}\boldsymbol{\delta}_{b1} \cdot \boldsymbol{\delta}_{c1} + \boldsymbol{\omega}_{rb1}(\boldsymbol{R}_{11} \times \boldsymbol{r}_{b1}) \cdot \boldsymbol{\delta}_{c1} + \boldsymbol{v}_{rc1} \\ &= (\boldsymbol{v} + \omega \times \boldsymbol{e}_1) \cdot \delta_{c1} = \begin{bmatrix} \boldsymbol{\delta}_{c1}^T & (\boldsymbol{e}_1 \times \boldsymbol{\delta}_{c1})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} \end{aligned} \tag{29d}$$

$$\begin{aligned} &\boldsymbol{v}_{rb2}\boldsymbol{\delta}_{b2} \cdot \boldsymbol{\delta}_{c2} + \boldsymbol{\omega}_{rb2}(\boldsymbol{R}_{21} \times \boldsymbol{r}_{b2}) \cdot \boldsymbol{\delta}_{c2} + \boldsymbol{v}_{rc2} \\ &= (\boldsymbol{v} + \omega \times \boldsymbol{e}_2) \cdot \delta_{c2} = \begin{bmatrix} \boldsymbol{\delta}_{c2}^T & (\boldsymbol{e}_2 \times \boldsymbol{\delta}_{c2})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} \end{aligned} \tag{29e}$$

$$\begin{aligned} &\boldsymbol{v}_{rb3}\boldsymbol{\delta}_{b3} \cdot \boldsymbol{\delta}_{c3} + \boldsymbol{\omega}_{rb3}(\boldsymbol{R}_{31} \times \boldsymbol{r}_{b3}) \cdot \boldsymbol{\delta}_{c3} + \boldsymbol{v}_{rc3} \\ &= (\boldsymbol{v} + \omega \times \boldsymbol{e}_i) \cdot \delta_{c3} = \begin{bmatrix} \boldsymbol{\delta}_{c3}^T & (\boldsymbol{e}_3 \times \boldsymbol{\delta}_{c3})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} \end{aligned} \tag{29f}$$

Substituting Eq. (27) into Eqs. (28c-e), yield

$$[\boldsymbol{v}_{rb1}\delta_{b1}+\omega_{rb1}(\boldsymbol{R}_{11}\times\boldsymbol{r}_{b1})-\boldsymbol{v}_{rb2}\delta_{b2}-\omega_{rb2}(\boldsymbol{R}_{21}\times\boldsymbol{r}_{b2})]\cdot(\boldsymbol{b}_1-\boldsymbol{b}_2)=0 \tag{29g}$$

$$[\boldsymbol{v}_{rb3}\delta_{b3}+\omega_{rb3}(\boldsymbol{R}_{31}\times\boldsymbol{r}_{b3})-\boldsymbol{v}_{rb1}\delta_{b1}-\omega_{rb1}(\boldsymbol{R}_{11}\times\boldsymbol{r}_{b1})]\cdot(\boldsymbol{b}_3-\boldsymbol{b}_1)=0 \tag{29h}$$

$$[\boldsymbol{v}_{rb2}\delta_{b2}+\omega_{rb2}(\boldsymbol{R}_{21}\times\boldsymbol{r}_{b2})-\boldsymbol{v}_{rb3}\delta_{b3}-\omega_{rb3}(\boldsymbol{R}_{31}\times\boldsymbol{r}_{b3})]\cdot(\boldsymbol{b}_2-\boldsymbol{b}_3)=0 \tag{29i}$$

Where

$$\delta_{m1}=\frac{\boldsymbol{b}_1-\boldsymbol{b}_2}{|\boldsymbol{b}_1-\boldsymbol{b}_2|},\delta_{m2}=\frac{\boldsymbol{b}_3-\boldsymbol{b}_1}{|\boldsymbol{b}_3-\boldsymbol{b}_1|},\delta_{m3}=\frac{\boldsymbol{b}_2-\boldsymbol{b}_3}{|\boldsymbol{b}_2-\boldsymbol{b}_3|}$$

Eqs. (29a-i) can be expressed in matrix form as

$$\mathrm{J}_\alpha\, v_m=\mathrm{J}_\lambda\, v_s \tag{30a}$$

$\mathbf{v}_{\mathrm{m}}=[v_{rb1}\ v_{rb2}\ v_{rb3}\ v_{rc1}\ v_{rc2}\ v_{rc3}\ \omega_{rb1}\ \omega_{rb2}\ \omega_{rb3}]^{\mathrm{T}}$,

$\mathbf{v}_{\mathrm{s}}=[v_x\ v_y\ v_z\ \omega_x\ \omega_y\ \omega_z\ 0\ 0\ 0]^{\mathrm{T}}$.

$$\mathbf{J}_\alpha=\begin{bmatrix}
\delta_{b1}\cdot R_{12} & 0 & 0 & 0 & 0 & 0 & (R_{11}\times r_{b1})\cdot R_{12} & 0 & 0\\
0 & \delta_{b2}\cdot R_{22} & 0 & 0 & 0 & 0 & 0 & (R_{21}\times r_{b2})\cdot R_{22} & 0\\
0 & 0 & \delta_{b3}\cdot R_{32} & 0 & 0 & 0 & 0 & 0 & (R_{31}\times r_{b3})\cdot R_{32}\\
\delta_{b1}\cdot\delta_{c1} & 0 & 0 & 1 & 0 & 0 & (R_{11}\times r_{b1})\cdot\delta_{c1} & 0 & 0\\
0 & \delta_{b2}\cdot\delta_{c2} & 0 & 0 & 1 & 0 & 0 & (R_{21}\times r_{b2})\cdot\delta_{c2} & 0\\
0 & 0 & \delta_{b3}\cdot\delta_{c3} & 0 & 0 & 1 & 0 & 0 & (R_{31}\times r_{b3})\cdot\delta_{c3}\\
\delta_{m1}\cdot\delta_{b1} & -\delta_{m1}\cdot\delta_{b2} & 0 & 0 & 0 & 0 & \delta_{m1}\cdot(R_{11}\times r_{b1}) & -\delta_{m1}\cdot(R_{21}\times r_{b2}) & 0\\
-\delta_{m2}\cdot\delta_{b1} & 0 & \delta_{m2}\cdot\delta_{b3} & 0 & 0 & 0 & -\delta_{m2}\cdot(R_{11}\times r_{b1}) & 0 & \delta_{m2}\cdot(R_{31}\times r_{b3})\\
0 & \delta_{m3}\cdot\delta_{b2} & -\delta_{m3}\cdot\delta_{b3} & 0 & 0 & 0 & 0 & \delta_{m3}\cdot(R_{21}\times r_{b2}) & -\delta_{m3}\cdot(R_{31}\times r_{b3})
\end{bmatrix}$$

$$\mathbf{J}_\lambda=\begin{bmatrix}
R_{12}^T & [(e_1-r_{b1})\times R_{12}]^T & 0_{1\times3}\\
R_{22}^T & [(e_2-r_{b2})\times R_{22}]^T & 0_{1\times3}\\
R_{32}^T & [(e_3-r_{b3})\times R_{32}]^T & 0_{1\times3}\\
\delta_{c1}^T & (e_1\times\delta_{c1})^T & 0_{1\times3}\\
\delta_{c2}^T & (e_2\times\delta_{c2})^T & 0_{1\times3}\\
\delta_{c3}^T & (e_3\times\delta_{c3})^T & 0_{1\times3}\\
0_{1\times3} & 0_{1\times3} & 0_{1\times3}\\
0_{1\times3} & 0_{1\times3} & 0_{1\times3}\\
0_{1\times3} & 0_{1\times3} & 0_{1\times3}
\end{bmatrix},$$

From Eq. (30a), it leads to

$$v_m=\mathbf{J}v_s=\mathbf{J}_\alpha^{-1}\mathbf{J}_\lambda v_s \tag{30b}$$

Eq. (30b) has nine rows and nine columns. From the first six rows and the first six columns, it leads to

$$v_r=\mathbf{J}_f\begin{bmatrix} v\\ \omega\end{bmatrix} \tag{31}$$

here, $\boldsymbol{v}_r=[v_{rb1}\ v_{rb2}\ v_{rb3}\ v_{rc1}\ v_{rc2}\ v_{rc3}]^T$, $\mathbf{J}_{\mathrm{f}}$ is the inverse Jacobian matrix.

4. DRIVING FORCES ANALYSIS

Let f_{rbi} and f_{rci} (i=1, 2, 3) be the active forces along limbs r_{bi} and r_{ci} (i=1, 2, 3), respectively. Let $\boldsymbol{F}$ and $\boldsymbol{T}$ be a central force and a central torque applied onto m at o. Based on the principle of virtual work, it leads to

$$\boldsymbol{f}_r^T\boldsymbol{v}_r+\begin{bmatrix}\boldsymbol{F}^T & \boldsymbol{T}^T\end{bmatrix}\begin{bmatrix}\boldsymbol{v}\\ \boldsymbol{\omega}\end{bmatrix}=0 \tag{32}$$

From Eq. (32), it leads to

$$\boldsymbol{f}_r=-(\mathbf{J}_f^{-1})^T\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{T}\end{bmatrix} \tag{33}$$

Where, $\boldsymbol{f}_{\mathbf{r}}=[f_{\mathrm{rb1}}\ f_{\mathrm{rb2}}\ f_{\mathrm{rb3}}\ f_{\mathrm{rc1}}\ f_{\mathrm{rc2}}\ f_{\mathrm{rc3}}]^{\mathrm{T}}$, $\boldsymbol{F}=[F_{\mathrm{x}}\ F_{\mathrm{y}}\ F_{\mathrm{z}}]^{\mathrm{T}}$, $\boldsymbol{T}=[T_{\mathrm{x}}\ T_{\mathrm{y}}\ T_{\mathrm{z}}]^{\mathrm{T}}$.

5. NUMERICAL EXAMPLE

Set E=120/q, C=60/q, d=80 cm. $\boldsymbol{F}$=[-20, -30, -30]$^{\mathrm{T}}$ kN, T=[-30 -30, 100] kN·cm. The pose parameters of the moving platform are given as X_{o}=68.89, Y_{o}=49.63, Z_{o}=225.3 cm, α= -11.08, β = 32.97, γ = -12.88º. The velocity parameters of moving platform are given as v_x=1, v_y=2, v_z=3cm/s, ω_x=1, ω_y=2, ω_z=3 º/s, the position vectors of b_i (i=1, 2, 3) are solved as Table **1**, **2** and **3**.

Table 1. The Coordinates of b_1

b_1	1	2	3	4
X (cm)	50.667	45.141	39.998	21.167
Y (cm)	-29.253	-26.06	-23.093	-12.22
Z (cm)	115.64	116.73	117.74	121.44
	5	6	7	8
X (cm)	-90.26+167.89i	-51.398	-40.555	-90.26-167.89i
Y (cm)	52.111- 96.931i	29.674	23.415	52.111+ 96.931i
Z (cm)	143.37- 33.038i	135.72	133.59	143.37+ 33.038i

Table 2. The Coordinates of b_2

b_2 (cm)	1	2	3	4
X	0	0	0	0
Y	32.471	39.947	46.19	64.583
Z	120.44	118.98	117.75	114.15
b_2 (cm)	5	6	7	8
X	0	0	0	0
Y	-138.95- 172.52i	-31.569	-45.511	-138.95+ 172.52i
Z	154.01+ 33.785i	132.98	135.71	154.01- 33.785i

Table 3. The Coordinates of b_3

b_3 (cm)	1	2	3	4
X	64.528	65.347	-37.528	63.356
Y	37.255	37.728	-21.667	36.579
Z	73.394	72.88	137.43	74.13
b_3	5	6	7	8
X	-184.96+13.944i	-74.269	29.45	-184.96- 13.944i
Y	-106.79+8.0503i	-42.879	17.003	-106.79- 8.0503i
Z	229.93 + 8.75i	160.48	95.40	229.93+8.75i

Table 4. Inverse Solutions of 3RPS-3SPR PM

(cm)	1	2	3	4	5	6
r_{b1}	116.1401	117.9807	119.9824	129.4582	186.9944	176.9975
r_{b2}	125.9397	122.5390	119.9962	114.2482	166.8976	177.7500
r_{b3}	161.4403	162.0518	139.8558	160.5745	161.3236	140.6064
r_{c1}	125.1317	124.9137	125.0120	127.8178	167.8407	159.6376
r_{c2}	122.7075	121.0418	120.0042	118.8893	152.6122	161.9011
r_{c3}	175.3520	175.9541	150.1267	174.4991	168.4908	154.4953

Table 5. Velocity and Statics at the Reasonable Pose

v_{rb1} (cm/s)	v_{rb2}(cm/s)	v_{rb3}(cm/s)	v_{rc1}(cm/s)	v_{rc2}(cm/s)	v_{rc3}(cm/s)
-3.9609	11.9576	-31.7202	9.1684	-9.9490	35.6654
f_{rb1} (kN)	f_{rb2}(kN)	f_{rb3}(kN)	f_{rc1}(kN)	f_{rc2}(kN)	f_{rc3}(kN)
-48.9508	-104.8954	78.6797	-22.1896	-81.8418	79.9695

From the results, it can be seen that there are eight solutions of the position vectors of b_i (i=1, 2, 3), this means that the inverse displacement of the 3RPS-3SPR PM has eight solutions. However, not all of them are real solutions. The real solutions are used to solve inverse displacement as follows:

By using the CAD variation geometry approach proposed in reference [9], a simulation mechanism of this PM can be created and the forward displacement solutions can be derived subsequently. By comparing, it is known that the simulation solution coincides with the third solution of analytical result in Table **4**. Then the velocity and statics are calculated at this pose as following (See Table **5**)

WORKSPACE ANALYSIS

The workspace of this S-PM can be created using CAD variation geometry approach [9]. When given the maximum

Table 6. The Construction Processes of Sub-Workspace

r_{b1}	r_{b2}	r_{b3}	r_{c1}	r_{c2}	r_{c3}	
$r_{b\min}\sim r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{b\max}$	$r_{c\max}$	$r_{c\max}$	$r_{c\max}$	S_{t1}
$r_{b\min}\sim r_{b\max}$	$r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{c\max}$	$r_{c\max}$	$r_{c\max}$	S_{t2}
$r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{c\max}$	$r_{c\max}$	$r_{c\max}$	S_{t3}
$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{b\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	$r_{c\max}$	S_{c1}
$r_{b\min}$	$r_{b\min}$	$r_{b\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	S_{c2}
$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{b\max}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	S_{c3}
$r_{b\max}$	$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	S_{c4}
$r_{b\max}$	$r_{b\min}$	$r_{b\min}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}\sim r_{c\max}$	S_{c5}
$r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{c\max}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	S_{c6}
$r_{b\min}\sim r_{b\max}$	$r_{b\max}$	$r_{b\min}$	$r_{c\max}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	S_{c7}
$r_{b\min}$	$r_{b\max}$	$r_{b\min}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	S_{c8}
$r_{b\min}$	$r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	$r_{c\max}$	S_{c9}
$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	$r_{c\min}$	S_{m1}
$r_{b\min}\sim r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{c\max}$	$r_{c\max}$	$r_{c\min}$	S_{m2}
$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{b\min}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}$	S_{m3}
$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{b\min}$	$r_{c\max}$	$r_{c\min}$	$r_{c\min}\sim r_{c\max}$	S_{m4}
$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{c\max}$	$r_{c\min}$	$r_{c\max}$	S_{m5}
$r_{b\min}$	$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}$	$r_{c\max}$	S_{m6}
$r_{b\min}$	$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{c\min}$	$r_{c\min}\sim r_{c\max}$	$r_{c\max}$	S_{m7}
$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{b\min}\sim r_{b\max}$	$r_{c\min}$	$r_{c\max}$	$r_{c\max}$	S_{m8}
$r_{b\min}$	$r_{b\min}\sim r_{b\max}$	$r_{b\min}$	$r_{c\min}$	$r_{c\max}$	$r_{c\min}\sim r_{c\max}$	S_{m9}
$r_{b\min}$	$r_{b\min}$	$r_{b\min}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}$	$r_{c\min}\sim r_{c\max}$	S_{l1}
$r_{b\min}$	$r_{b\min}$	$r_{b\min}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}$	S_{l2}
$r_{b\min}$	$r_{b\min}$	$r_{b\min}$	$r_{c\min}$	$r_{c\min}\sim r_{c\max}$	$r_{c\min}\sim r_{c\max}$	S_{l3}

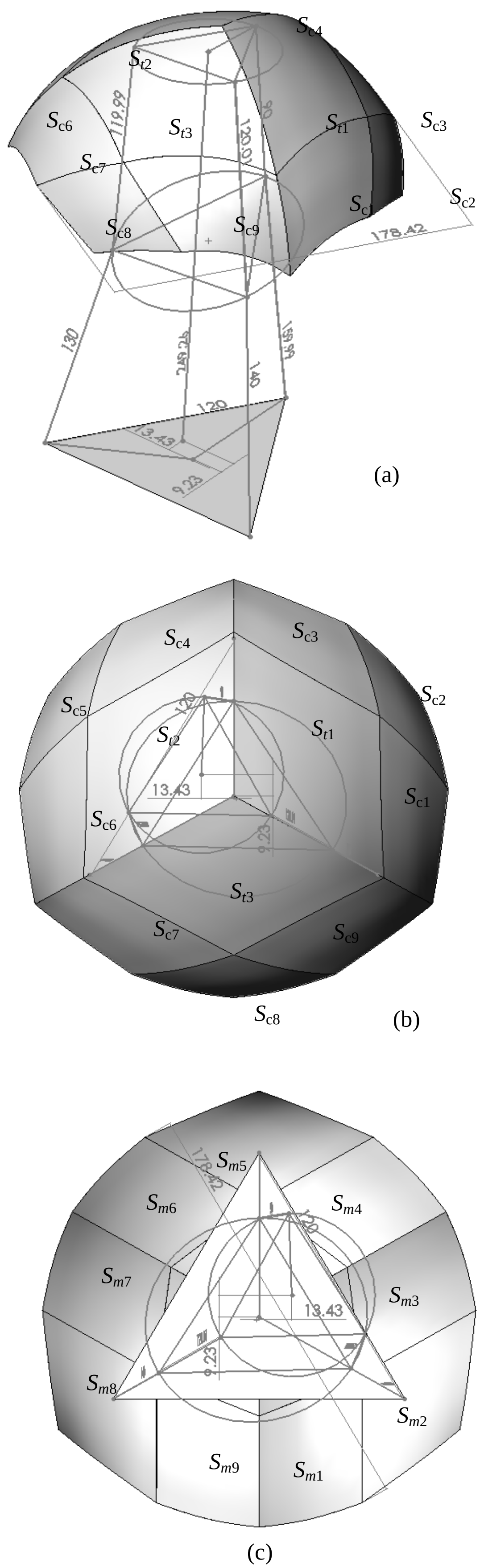

Fig. (2). A reachable workspace *W* of the 3RPS-3SPR S-PM. (**a**) the isometric view, (**b**) the top view, (**c**) the upper view.

extension $r_{b\max}$, the minim extension $r_{b\text{mim}}$ and the increment δr_b of active legs $r_{bi}(i{=}1, 2, 3)$, the maximum extension $r_{c\max}$, the minim extension $r_{c\text{mim}}$ and the increment δr_c of active legs $r_{ci}(i{=}1, 2, 3)$, the reachable workspace *W* of the 3RPS-3SPR S-PM can be constructed by its simulation mechanism.

When four of r_{bi} and r_{ci} (i=1, 2, 3) reach their limited values of (rbmin, $r_{b\max}$, $r_{c\min}$, $r_{c\max}$), varying the remaining two of r_{bi} and r_{ci} (i=1,2,3) from $r_{b\min}$ to $r_{b\max}$ and from $r_{c\min}$ to $r_{c\max}$, respectively, each sub-workspace can be constructed.

The construction processes are described as follows:

Step 1. Set $r_{b3} = r_{b\max}$, $r_{c1}=r_{c2}=r_{c3}=r_{c\max}$.

Step 2. Set $r_{b1} = r_{b\min} +(j-1)\delta r_b$ ($j = 1, \ldots, n_1$),

where $n_1=(r_{b\max} - r_{b\min})/ \delta r_b$.

Step 3. Set j=1 and increase r_{b2} by δr_b at each increment from $r_{b\min}$ to $r_{b\max}$. Solve the position components (X_o Y_o Z_o) by using CAD software. Then, a spatial curve c_1 is formed from the solutions of (X_o Y_o Z_o) of n_1 points.

Step 4. Repeat the steps 2, except that set j = 2, . . . , n1, thus other cj can be constructed. Construct the n_1 spatial curves c_j (j =1,...,n_1) by the loft command. Then the surface S_{t1} can be obtained.

Step 5. Repeat the steps 1-4, except that set r_{bi} and r_{ci} verifying versus Table **6**, the other sub-workspace can be obtained.

The workspace of 3RPS-3SPR S-PM is constructed as shown in Fig. (**2**).

CONCLUSIONS

A novel 3RPS-3SPR serial-parallel manipulator (S-PM) is constructed by connecting a 3RPS parallel manipulator (PM) with a 3SPR PM in series. Some formulae are derived for solving the inverse displacement, velocity, and statics models of this S-PM.

It is known from the analytic solutions that this S-PM has eight inverse solutions. A reasonable solution can be obtained by comparing with the simulation solutions. The inverse velocity and the active forces are computed at this reasonable pose.

A 9×9 Jacobian matrix is derived from the geometrical constraint and the dimension constraint equations. A 6×6 Jacobian matrix is derived for solving inverse velocity by taking the first six rows and the first six arranges out of the 9×9 Jacobian matrix.

The active forces are derived based on the principle of virtual work. Its workspace is constructed by CAD variation geometry approach, this novel S-PM has a large workspace.

This S-PM has some potential applications for the serial–parallel machine tools, the sensors, the surgical manipulators and the satellite surveillance platforms. The method for solving the inverse kinematics and workspace can also be used for other S-PMs.

CONFLICT OF INTEREST

None declared.

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