Trajectory Research about the Rolling-Pin Belt Transmission

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Abstract: A new belt transmission mechanism, having the function of no-slip driving, is described in the paper. It is composed of pulleys with carved rails and belt with rolled pins spaced out on both sides. Firstly, the trajectory curve is deduced along which pins embedded in the pulley. And then, parametric relationships are established, which are about center offsets of two pulleys, working radius of each pulley, the number of meshing curves on every pulley and the number of pins on the belt. All of those provide a theoretical basis for the design of this type of no-slip belt transmission mechanisms.

Keywords: Rolling pin, separate curve, binding curve, mechanism.

INTRODUCTION

With the priorities of high efficiency and compact layout, mechanical belt transmission plays a very important role in mechanical engineering, and is widely used in automobile engines, gearboxes and other power transmissions [1-3]. As the traditional belt drive belongs to the friction drives, there are many well-known problems such as low transmission power, high slipping rate during transmission, easily failing and so on. Synchronizer gear-belt is a generally used alternative way to solve it [4-7]. However, the strength of the synchronizer gear-belt is limited to a very low level. Therefore, this paper proposes a new flexural transmission mechanism to improve the efficiency and transmitted power, and also to avoid elastic slip by embedding the rolling pins in the belt.

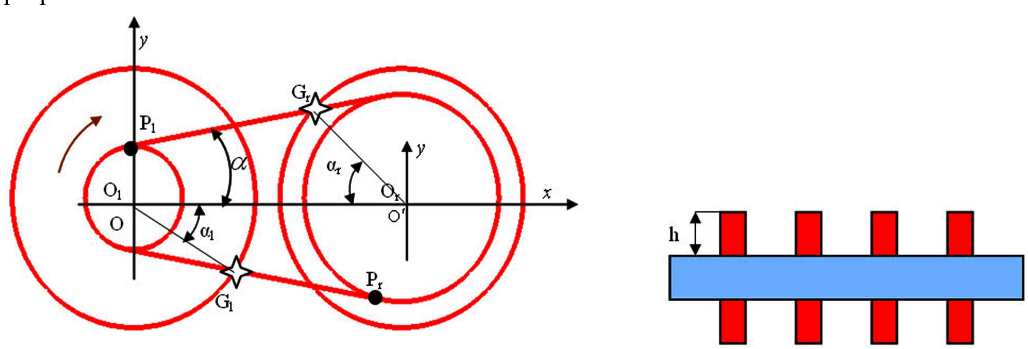

Fig. (1). Architecture diagram of belt transmission.

1. INTRODUCTION TO THE NEW BELT DRIVE STRUCTURE

As shown in Fig. (**1**), the belt is assembled with two pulleys and the belt is embedded with evenly distributed rolling pins. The center coordinate system on each pulley is particularly defined, coordinate system S_l=[O$_l$; x,y] is the left pulley coordinate system, S_r=[O$_r$; x,y] is that of the right one [8]. Coordinate system S=[O; x,y] is a fixed one whose origin superimposes with the left pulley's center, and S'=[O'; x,y] is another fixed coordinate system whose origin locates at the right pulley's center. α_l and α_r are the angles which describe x-axis to the lines connecting the initial integration point with the origin of pulley. So the following relation formulas can be obtained immediately.

$$\alpha = \arcsin\frac{(r_r - r_l)}{a} \tag{1}$$

where, α denotes the angle between belt and x-axis in coordinate system S, r_l and r_r represents working radius of left pulley and the right one respectively, a indicates distance between the center of left pulley and that of the right pulley [9, 10] .

Point P$_l$(x_{0pl},y_{0pl}) is the initial separation point of the left pulley, P$_r$(x_{0pr},y_{0pr}) is the similar point of the right one. From the simple geometric relationship, coordinates of point P$_l$ and P$_r$ are described in the following formulas.

$$x_{0pl} = -r_l \sin(\alpha) \tag{2}$$

$$y_{0pl} = r_l \cos(\alpha) \tag{3}$$

$$x_{0pr} = -r_r \sin(\alpha) \tag{4}$$

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$$y_{0pr} = -r_r \cos(\alpha) \tag{5}$$

where, x_{0pl} and y_{0pl} is x-coordinate and y-coordinate of point P_l in coordinate system S, x_{0p} and y_{0pr} are x-coordinate and y-coordinate of point P_r in coordinate system S, respectively.

In order to find the relationship between α_l and α, geometric relation between point $G_r(x_{0gr},y_{0gr})$ and coordinate system S is shown in Fig. (**2**) where α_l is represented by formula (6). Similarly, relationship between α_r and α is specified with formula (7).

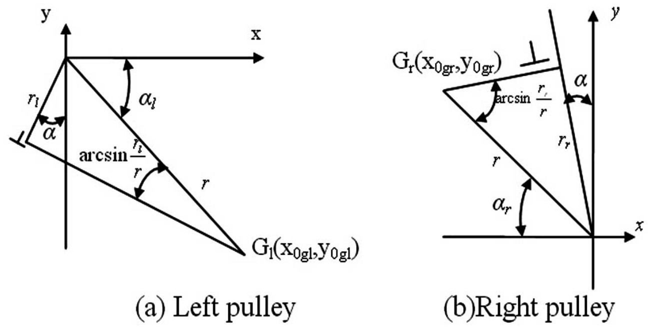

Fig. (2). Geometric relation diagram.

$$\alpha_l = \arcsin\frac{r_l}{r} + \alpha \tag{6}$$

$$\alpha_r = \arcsin\frac{r_r}{r} - \alpha \tag{7}$$

where, r is the radius of each pulley.

$G_l(x_{0gl},y_{0gl})$ is the initial integration point of the left pulley, $G_r(x_{0gr},y_{0gr})$ is the corresponding point of the right one. Immediately, coordinates of point G_l and point G_r can be obtained in the following formulas.

$$x_{0gl} = r\cos(\alpha_l) \tag{8}$$

$$y_{0gl} = -r\sin(\alpha_l) \tag{9}$$

$$x_{0gr} = -r\cos(\alpha_r) \tag{10}$$

$$y_{0gr} = r\sin(\alpha_r) \tag{11}$$

where, x_{0gl} and y_{0gl} are the corresponding x-coordinate and y-coordinate of point G_l in coordinate system S, x_{0gr} and y_{0gr} are the corresponding x-coordinate and y-coordinate of point G_r in coordinate system S.

2. LEFT PULLEY CURVE EQUATION

2.1. Equation of the Left Pulley Separation Curve

In Fig. (**3**), ω_l is the angular velocity of the left pulley, point P_{11} (x_{1pl}, y_{1pl}) and point P_l' are new places of point P_l on belt and pulley in the fix coordinates system S at time t, and θ_l represents the value of $\angle P_lO_lP_l'$. While the rolling pin runs from the separation point P_l to the point P_{11}, point P_{ll}'' on the pulley rotates to the point P_{ll}' which superimposes with point P_{11} in coordinate system S, and point P_l on the belt revolves to the point P_l' in coordinate system S. So the relations about ω_l and θ_l are represented in the following formulas.

$$\omega_l = \frac{\sqrt{(r^2 - r_l^2)}}{r_l t_{0l}} \tag{12}$$

$$\theta_l = \omega_l t \frac{180}{\pi} \tag{13}$$

where, t_{0l} is the total separation time, t denotes running time during separation process.

From kinematics relationship, the speed of rolling pin is $r_l\omega_l$. According to the geometric relationship, coordinate calculating relationships of point P_{11} are described as:

$$x_{1pl} = x_{0pl} + r_l\omega_l t\cos(\alpha) \tag{14}$$

$$y_{1pl} = y_{0pl} + r_l\omega_l t\sin(\alpha) \tag{15}$$

where, x_{1gl} and y_{1gl} are the corresponding x-coordinate and y-coordinate of point P_{ll} in coordinate system S.

At this time, $\angle P_lO_lP_l'$ is the angle that coordinate system S_l rotates relative to coordinate system S, which is equal to the value of $\angle P_{ll}''O_lP_{ll}'$. So it is deduced that the coordinate of P_{ll}'' is the position that P_{11} rotates θ_l counterclockwise around the coordinate origin.

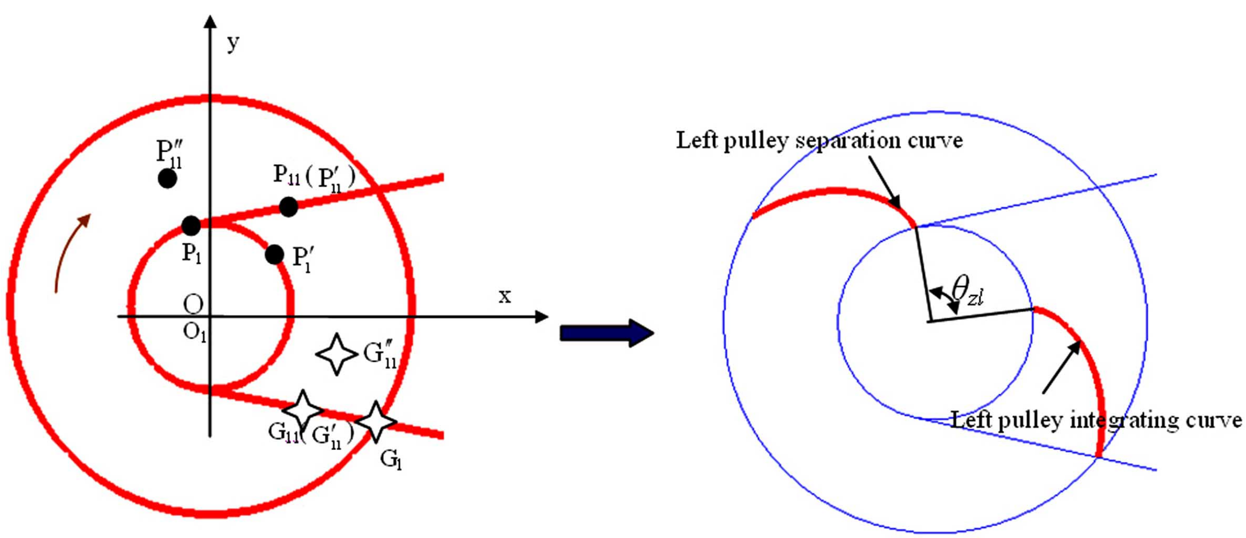

Fig. (3). Points relation diagram of left pullet.

Defining the coordinate of point P''_{11} as (x_{pl}, y_{pl}) in coordinate system S_l and coordinate transformation matrix as A_l, the coordinate (x_{pl}, y_{pl}) is described as:

$$\begin{pmatrix} x_{pl} & y_{pl} \end{pmatrix}^{\mathbf{T}} = \mathbf{A}_l \begin{pmatrix} x_{1pl} & y_{pl1} \end{pmatrix}^{\mathbf{T}} \tag{16}$$

where, A_l is the coordinate transformation matrix from coordinate system S to S_l [11, 12], the matrix is described as,

$$A_l = \begin{bmatrix} \cos\theta_l & -\sin\theta_l \\ \sin\theta_l & \cos\theta_l \end{bmatrix} \tag{17}$$

2.2. Equation the Left Pulley Integrating Curve

As described in the Fig. (**3**), point $G_l(x_{0gl}, y_{0gl})$ is the initial integration point between the left pulley and belt, $G_{11}(x_{1gl}, y_{1gl})$ is some point before the end of integration. Because point G_l is starting integration while point P_l starts separation, rotating speed and the time are equal that two points spend on respective integrating and separating, which are named as t_{0l} and ω_l above. According to the geometric relationship, the coordinate of Point G_{11} is described as:

$$x_{1gl} = x_{0gl} - r_l\omega_l t\cos(\alpha) \tag{18}$$

$$y_{1gl} = y_{0gl} + r_l\omega_l t\sin(\alpha) \tag{19}$$

where, x_{1gl} and y_{1gl} are corresponding x-coordinate and y-coordinate of point G_{11} in coordinate system S.

While rolling pin runs from the initial integration point G_l to the point G_{11}, point G''_{11} on the pulley rotates to the point G'_{11} which superimposes with point G_{11} in the fixed coordinate system S, and the pulley coordinate system S_l also rotates angle θ_l relative to the original fixed coordinate system S. So it is deduced that the coordinate of G''_{11} is the position that G_{11} rotates θ_l counterclockwise around the point O_l. According to the geometric relationship, the coordinate of point G''_{11} is described as:

$$\begin{pmatrix} x_{gl} & y_{gl} \end{pmatrix}^{\mathbf{T}} = \mathbf{A}_l \begin{pmatrix} x_{1gl} & y_{1gl} \end{pmatrix}^{\mathbf{T}} \tag{20}$$

where, x_{gl} and y_{gl} are corresponding x-coordinate and y-coordinate of point G''_{11} in coordinate system S_l.

3. EQUATION OF THE RIGHT PULLEY CURVE

3.1. Separation Curve Equation

The derivation of the right pulley separation curve equation is similar to that of the left one. As shown in Fig. (**4**), ω_r is the angular velocity of the right pulley and t_{0r} is the time the separation process spends, θ_r is the angle offset of coordinate system S_r during the separation process. So calculating relations about ω_r and θ_r are obtained in the following formulas.

$$\omega_r = \frac{\sqrt{(r^2 - r_r^2)}}{r_r t_{0r}} \tag{21}$$

$$\theta_r = \omega_r t\frac{180}{\pi} \tag{22}$$

Point P_r (x_{0pr}, y_{0pr}) is the starting separating point of belt and pulley, P_{1r} (x_{1pr}, y_{1pr}) is any point before the end of the separation. Based on kinematics relationship, the speed of rolling pin can be described as $r_r\omega_r$. According to the geometric relationship, the coordinate of Point P_{1r} is described as:

$$x_{1pr} = x_{0pr} - r_r\omega_r t\cos(\alpha) \tag{23}$$

$$y_{1pr} = y_{0pr} + r_r\omega_r t\sin(\alpha) \tag{24}$$

where, x_{1pr} and y_{1pr} are the corresponding x-coordinate and y-coordinate of point P_{1r} in coordinate system S'.

As left pulley's derivation, defining the coordinate of point P''_{1r} as (x_{pr}, y_{pr}) in coordinate S_r and the coordinate transformation matrix as A_r, the coordinate (x_{pr}, y_{pr}) is described as:

$$\begin{pmatrix} x_{pr} & y_{pr} \end{pmatrix}^{\mathbf{T}} = \mathbf{A}_{\mathbf{r}} \begin{pmatrix} x_{1pr} & y_{p1r} \end{pmatrix}^{\mathbf{T}} \tag{25}$$

where, A_r is the coordinate transformation matrix from coordinate system S' to S_l, the matrix is described as,

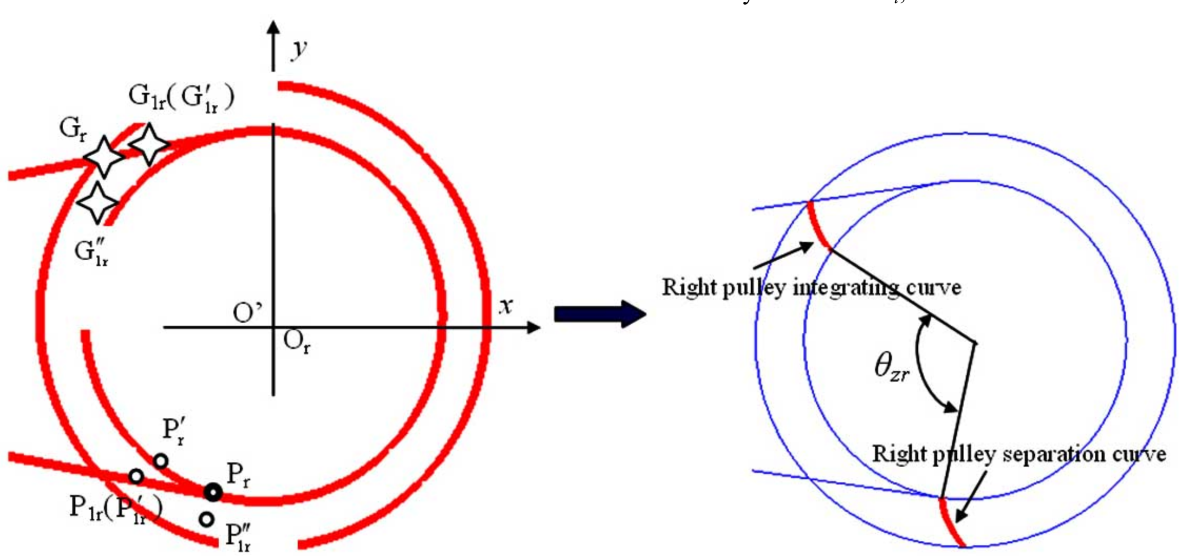

Fig. (4). Points relation diagram of right pullet.

$$A_r = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \tag{26}$$

When describing relations in the fixed coordinate system S, the offset from the right pulley coordinate origin to the left is a. So the coordinate equation is described as:

$$\begin{pmatrix} x_{pr} & y_{pr} \end{pmatrix}^T = A_r \begin{pmatrix} x_{1pr} & y_{1pr} \end{pmatrix}^T + a\begin{pmatrix} 1 & 0 \end{pmatrix}^T \tag{27}$$

3.2. Integration Curve Equation

Similar to the deduction of the left pulley, right pulley's rotation speed, time cost in the integration process and the angle right pulley rotating during integrating period, are the same as those of right pulley during separation process, whose parameters are t_{0r}, ω_r and θ_r. As shown in Fig. (**4**), point $G_r(x_{0gr}, y_{0gr})$ is the initial integration point between the left pulley and belt, point $G_{1r}(x_{1gr}, y_{1gr})$ is some point before the end of integration. According to the geometric relationship, the coordinate of point G_{1r} is described as:

$$x_{1gr} = x_{0gr} + r_r\omega_r t\cos(\alpha) \tag{28}$$

$$y_{1gr} = y_{0gr} + r_r\omega_r t\sin(\alpha) \tag{29}$$

where, x_{1pr} and y_{1pr} are the corresponding x-coordinate and y-coordinate of point G_{1r} in coordinate system S'.

In a similar way, defining the coordinate of point G''_{1r} as (x_{gr}, y_{gr}) in coordinate S_r, the coordinate of point G''_{1r} is described as:

$$\begin{pmatrix} x_{gr} & y_{gr} \end{pmatrix}^{\mathbf{T}} = \mathbf{A_r}\begin{pmatrix} x_{1gr} & y_{1gr} \end{pmatrix}^{\mathbf{T}} \tag{30}$$

When describing relations in the fixed coordinate system S, the coordinate equation is described as:

$$\begin{pmatrix} x_{gr} & y_{gr} \end{pmatrix}^T = A_r \begin{pmatrix} x_{1gr} & y_{1gr} \end{pmatrix}^T + a\begin{pmatrix} 1 & 0 \end{pmatrix}^T \tag{31}$$

4. MESHING CURVE AND ROLLING PIN NUMBER

During driving, sliding shouldn't occur between pulleys and belt. In other words, rolling pin shouldn't slide on the pulley. So, it is deduced that initial separation point of the separation curve must superimposes with the starting point of the integration curve and the whole meshing curve is the combination of above two curves. Fig. (**5**) shows several meshing curves placed evenly on the left pulley and the thickened curve is one single meshing curve.

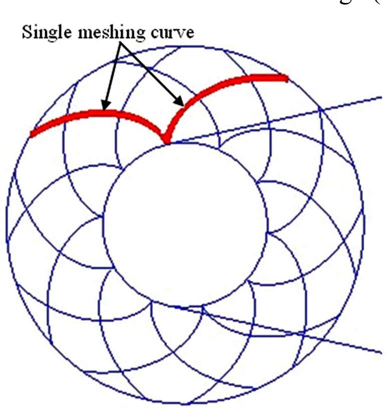

Fig. (5). Evenly placed meshing curves on the left pulley.

The initial separation point and the ending integration point of each pulley are in the same respective working cycle, and offset angles in their respective coordinates are expressed as θ_{zl} and θ_{zr}, as shown in Fig. (**2**) and Fig. (**3**). With affine transformation, ending integration point could superimpose with initial separation point on each pulley by means of counterclockwise rotation. So there is the following relationship between (x_{0pl}, y_{0pl}) and (x_{1gl}, y_{1gl})

$$\begin{pmatrix} x_{0pl} & y_{0pl} \end{pmatrix}^{\mathbf{T}} = A_{zl}\mathbf{A}_l \begin{pmatrix} x_{1gl} & y_{1gl} \end{pmatrix}^{\mathbf{T}}\Big|^{t=t_{0l}} \tag{32}$$

where, A_{zr} is affine transformation matrix of left pulley from the ending integration point to the initial separation point. It is described as:

$$A_{zl} = \begin{bmatrix} \cos\theta_{zl} & -\sin\theta_{zl} \\ \sin\theta_{zl} & \cos\theta_{zl} \end{bmatrix} \tag{33}$$

The relationship between (x_{0pr}, y_{0pr}) and (x_{1gr}, y_{1gr}) is similar.

$$\begin{pmatrix} x_{0pr} & y_{0pr} \end{pmatrix}^{\mathbf{T}} = A_{zr}\mathbf{A}_l \begin{pmatrix} x_{1gr} & y_{1gr} \end{pmatrix}^{\mathbf{T}}\Big|^{t=t_{0r}} \tag{34}$$

where, A_{zr} is affine transformation matrix of right pulley from the ending integration point to the initial separation point. It is described as:

$$A_{zr} = \begin{bmatrix} \cos\theta_{zr} & -\sin\theta_{zr} \\ \sin\theta_{zr} & \cos\theta_{zr} \end{bmatrix} \tag{35}$$

The complete meshing equation is shown as follows:

$$(x \quad y)^{\mathrm{T}} = \begin{cases} \begin{cases} A_l\left(x_{1pl} \quad y_{1pl}\right)^{\mathrm{T}} & \text{Left pulley separation curve} \\ A_{zl}A_l\left(x_{1pl} \quad y_{1pl}\right)^{\mathrm{T}} + a(1 \quad 0)^T & \text{Left pulley integrating curve} \end{cases} \\ \begin{cases} A_r\left(x_{1pr} \quad y_{1pr}\right)^{\mathrm{T}} & \text{Right pulley separation curve} \\ A_{zr}A_l\left(x_{1pr} \quad y_{1pr}\right)^{\mathrm{T}} + a(1 \quad 0)^T & \text{Right pulley integrating curve} \end{cases} \end{cases} \tag{36}$$

According to the geometric relationship, the belt length is calculated as:

$$L = r_l\pi(1 - \frac{\alpha}{90}) + 2a\cos(\alpha) + r_r\pi(1 + \frac{\alpha}{90}) \tag{37}$$

Suppose the number of rolling pins is n, and meshing curves are evenly distributed in the pulley, and the number of meshing curves is n_l on the left pulley and n_r on the right, the relations are as follows:

$$l_n = \frac{L}{n} \tag{38}$$

$$l_n n_l = 2\pi r_l \tag{39}$$

$$l_n n_r = 2\pi r_r \tag{40}$$

Then, the parameter n can be solved and expressed as follows.

$$n = \frac{r_l \pi (1 - \frac{\alpha}{90}) + 2a\cos(\alpha) + r_r \pi (1 + \frac{\alpha}{90})}{2\pi r_l} n_l = \frac{\pi (1 - \frac{\alpha}{90}) + 2\frac{a}{r_l}\cos(\alpha) + \frac{r_r}{r_l}\pi (1 + \frac{\alpha}{90})}{2\pi} n_l \quad (41)$$

By setting priorities and adjusting values of three parameter r_l , r_r and a, a series of discrete solutions are calculated to meet integer parameters: n , n_l and n_r.

CONCLUSIONS

Improving driving torque and reducing slip rate have important and practical significance in power transmission industries. This paper presents a new way that is evenly placed rolling pins on the belt and engraved curve trajectories on pulley. Firstly, separation and integration curves are deduced in the paper, and then, meshing curve equations are established. Finally, the method to determine the number of trajectories and rolling pins is obtained. All those can be used for mechanism design and further research.

CONFLICT OF INTEREST

None declared.

ACKNOWLEDGEMENT

None declared.

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