A Broadband Model of a Potential Transformer

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Abstract: In this paper, a broadband circuit model of a potential transformer (PT) based on fractional calculus and circuit augmentation method is proposed to improve the high frequency characteristic while providing physical significance of existing traditional equivalent circuit model with simple structure. In order to reflect the characteristics of the hysteresis of the iron core roundly, a fractional model of the magnetic hysteresis loop in iron core measured by a no-load experiment was set up instead of the single-value magnetization curve of previous model. Besides, the original equivalent circuit model was calculated at low frequencies, and the circuit augmentation method (CAM) was employed to extend the compatibility of the original equivalent circuit model in the wide range of frequency. The element values are calculated by the Brune method of synthesis. For validity, comparisons between the simulation and experiment of a 10 kV single-phase oil-immersed PT with lightning overvoltage and switching overvoltage are conducted, respectively. The results show that the broadband model presented in the paper can provide a good accuracy while reserving the physical significance of original equivalent circuit model.

Keywords: Broadband modeling, hysteresis, nonlinear, network synthesis, potential transformer.

1. INTRODUCTION

Power transformer is one of the most important and critical devices in power systems [1, 2]. There are many kinds of transformer-like devices such as generator transformer, main transformer, potential transformers (PT) and current transformers (CT), etc. Potential transformer (PT) plays an important role in the field of power system measurement and protection. Traditional equivalent circuit model with simple structure and explicit physical significance correlate merely well with the measured parameters at lower frequencies. However, it deviates at higher frequencies (e.g. lightning overvoltage and switching overvoltage), so the existing models cannot meet the stringent demand of production and movement. It is of great theoretical significance and practical value to set up an accurately broadband model.

There are several kinds of potential transformer model for decades [3-9]. In general, there are three ways in device modeling. The first one is pure physical method according to the specific structure of the devices, which need to know the detail about the structure and solve the complex electromagnetic field problems. The second one is the black-box method according to the characteristic of external ports. The third one is the gray-box method. However, the existing transformer modeling methods have only the first two, i.e. detailed model and black-box methods. On the one hand, detailed model of the transformer, valid for the dozens of megahertz frequency range, requires that each turn should be represented by all mutual inductance coupling with each other turn, and self-induction and self-capacitance. Solving such a model is very time-consuming. On the other hand, black-box model is a pure mathematics method which cannot reflect the physical significance of equipments. In this paper, we use the third one to establish the broadband circuit model for PT.

The iron core is a nonlinear element under the action of big signal. There are several key factors such as magnetic saturation, hysteresis and eddy current for the nonlinear phenomena of iron core. However, it is difficult to set up a model having the high accuracy for analysis of Ferro-resonant circuit. For modeling of the core excitation system, there are some achievements since 1960. Widger proposed using a rational fraction approximation to represent the magnetization curves in 1969 [10]. Chua and Stromsmoe established the hysteresis model for electronic circuit studies in 1970 [11]. Bailey and Talukdar established the hysteresis model for power system studies in 1976 [12], and Saito et al applied it to the simulation of single phase parallel inverter. Lucas proposed using a non-integer power series to represent the magnetization curves in 1988 [13]. Lucas et al proposed using nonlinear inductance and nonlinear resistors in parallel connection to simulate the iron core magnetization process 1992 [14]. However, the magnetization curves of existing models are single-valued. In order to reflect the characteristics
of the hysteresis of the iron core roundly, there is a need to model the whole magnetic hysteresis loop. In this paper, we establish a fractional order model for the magnetic hysteresis loop of iron core.

2. CIRCUIT AUGMENTATION METHOD

Circuit augmentation method is a gray-box method, and its basic augmentation concept is shown in Fig. (1) [9, 15, 16].

Among the various kinds of augmentation methods, there are three common methods such as branch augmentation, parallel augmentation and cascade augmentation, as shown in Fig. (2).

For m-port networks, the modified nodal analysis (MNA) equations [9, 14, 15] are shown in

\[ \mathbf{A}(j\omega)x = \mathbf{B}v \]  

where \( \mathbf{A}(j\omega) \in \mathbb{C}^{N \times N} \) is the MNA matrix in the frequency \( \omega \), \( \mathbf{B} \in \mathbb{R}^{N \times m} \) is a selection matrix which introduces the port voltages into the node space of MNA equations and \( \mathbf{v} = [0,1]^T \), \( \mathbf{v} \in \mathbb{C}^{m \times 1} \) is a column vector composed by port voltages.

\[ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{C}^{N \times 1} \]  

where \( x_i \in \mathbb{C}^{(N-m) \times 1} \) is a column vector composed by node voltages and additional currents, \( x_j \in \mathbb{C}^{m \times 1} \) is a column vector composed by port currents.

From formula (1), we get the short-circuit admittance matrix of the m-port equivalent circuit

\[ \mathbf{Y} = \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B} \]  

We use \( \mathbf{Y}_{\text{mean}} \) to denote the measured short-circuit admittance matrix. The purpose of circuit augmentation method is to obtain modified model \( \hat{\mathbf{Y}} \) such that it matched the entire broadband spectrum by adding the corresponding branches \( \Delta \mathbf{Y} \), i.e. \( \hat{\mathbf{Y}} = \mathbf{Y} + \Delta \mathbf{Y} \).

Then, we get the following error function

\[ \varepsilon(j\omega) = \left\| \mathbf{Y}_{\text{mean}}(j\omega) - \hat{\mathbf{Y}}(j\omega) \right\| 
= \left\| \mathbf{Y}_{\text{mean}}(j\omega) - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B} - \Delta \mathbf{Y}(j\omega) \right\| \]  

In this paper, the \( \mathbf{Y}_{\text{mean}} \) parameters can also be indirectly deduced with scatting parameters \( \mathbf{S}_{\text{meas}} \) measured by Agilent 4395A working as network analyzer, as shown in Fig. (3), and the measurement range of frequency with 87512A is from DC to 2 GHz.

According to two-port theory, the \( \mathbf{Y}_{\text{meas}} \) parameter matrix of PT/CT can be obtained from the \( \mathbf{S}_{\text{meas}} \).

\[ \mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \left[ \mathbf{R} + \mathbf{S}(\mathbf{E} + \mathbf{S}) \right]^{-1} \left( \mathbf{E} - \mathbf{S} \right) \]

\[ = 1 \frac{1}{\mathbf{E}} \frac{1}{\mathbf{S}} \times \begin{bmatrix} 1 - S_{11} & -2S_{12} \\ -2S_{21} & 1 - S_{22} \end{bmatrix} \]  

\[ (1 - S_{11})(1 + S_{22}) + S_{12}S_{21} - 2S_{12} \]
\[ -2S_{21} \]

3. ORIGINAL EQUIVALENT MODEL OF PT

We calculate and measure a JDJ-10 of 10 kV single-phase oil-immersed PT.

The iron core of electromagnetic voltage transformer is a typical nonlinear element under operating overvoltage or lightning overvoltage. Firstly, we establish a fractional order model of iron core in the large signals. Experimental principles are shown in Fig. (4), where the primary side of PT is disconnected.

According to the law of electromagnetic induction and transformation ratio of PT, we get

\[ \psi = \int_0^\infty u dt + C, \quad \psi = \frac{\dot{u}}{j\omega} \quad \text{and} \quad i = n_2 / n_1 \]  

And the relationship between current and flux linkage can be calculated by experimental results.
The basic principle describing the relationship between input and output in dynamic systems is

$$y(t) + b_1 \frac{d^{\beta_1}}{dt^{\beta_1}} y(t) + \cdots + b_n \frac{d^{\beta_n}}{dt^{\beta_n}} y(t) = a_n x(t) + a_{n-1} \frac{d^{\alpha_n}}{dt^{\alpha_n}} x(t) + \cdots + a_1 \frac{d^{\alpha_1}}{dt^{\alpha_1}} x(t)$$  \hspace{1cm} (7)

Then we use the fractional polynomial of formula (8) to fit the hysteresis loop

$$B = a_0 H + \sum_{i=1}^{N} a_i D^{\alpha_i} H$$  \hspace{1cm} (8)

where $D^{\alpha} H = \frac{d^a H}{dt^a}$, $0 < \alpha < 1$.

Combined with $B = \frac{k}{NS} \int u \, dt$, $H = \frac{Ni}{Lk} \cdot \frac{d\lambda}{dt}$, we get the fitting result of flux linkage and current

$$u = a_0 \frac{di}{dt} + a_1 \frac{d^{1+\alpha_1} i}{dt^{1+\alpha_1}} + a_2 \frac{d^{1+\alpha_2} i}{dt^{1+\alpha_2}} + \cdots + a_n \frac{d^{1+\alpha_n} i}{dt^{1+\alpha_n}}$$  \hspace{1cm} (9)

The circuit is shown in Fig. (5), and for the sake of brevity, we use Fig. (6) to instead. Comparison result between measurement and simulation are shown in Fig. (7). And the values of coefficient $a_i$ and their correspondent $\alpha_i$ are shown in Table 1.

![Fig. (5). Equivalent circuit of iron core.](image)

![Fig. (6). Contracted notation.](image)

![Fig. (7). Comparison result between measurement and simulation.](image)
core is linear and the low frequency model of PT/two-winding transformer can be shown in Fig. (10).

**Table 1. Coefficients and orders.**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0=0.00254393050655755$</td>
<td>null</td>
</tr>
<tr>
<td>$a_1=3.60492387516746$</td>
<td>$\alpha_1=0.076804$</td>
</tr>
<tr>
<td>$a_2=-0.791579409744237$</td>
<td>$\alpha_2=0.23351$</td>
</tr>
<tr>
<td>$a_3=1.87912230687602$</td>
<td>$\alpha_3=0.47205$</td>
</tr>
<tr>
<td>$a_4=0.70116586198962$</td>
<td>$\alpha_4=0.53542$</td>
</tr>
<tr>
<td>$a_5=-0.253480210360523$</td>
<td>$\alpha_5=0.59612$</td>
</tr>
<tr>
<td>$a_6=0.0440853220991656$</td>
<td>$\alpha_6=0.64206$</td>
</tr>
</tbody>
</table>

**Fig. (8).** Scattering parameters of PT.

**Fig. (9).** Transfer characteristic of PT.

So the equivalent circuit in primary side can be calculated by

\[ Z_{11} - Z_{12} = R_1 + j\omega L_1 \]  \[ Z_{22} - Z_{12} = n^2 (R_2 + j\omega L_2) \]  \[ (10) \]

At medium frequency (\(\leq 100 \text{ kHz}\)), the equivalent circuit model of PT/two-winding transformer can be shown in Fig. (11).

**Fig. (10).** Low frequency model of PT/two-winding transformer.

**Fig. (11).** Medium frequency model of PT/two-winding transformer.

$Y$ is the admittance matrix, as shown in formula (11)

\[
Y = \begin{bmatrix}
  A_{11}B_{11} - A_{12}B_{21} & A_{22}B_{12} - A_{22}B_{22} \\
  -A_{11}B_{11} + A_{12}B_{21} & -A_{22}B_{12} + A_{22}B_{22}
\end{bmatrix}
\]  \[ (11) \]

where $A_{11} = j\omega (L_1 + Z_m + R_1)$, $A_{12} = j\omega Z_m$, $A_{21} = j\omega Z_m$, $A_{22} = j\omega (n^2 L_2 + Z_m + n^2 R_2)$, $B_{11} = 1 - \omega^2 \left[C_3 (L_1 + R_1) + C_1 (L_1 + R_1 + Z_m)\right]$, $B_{12} = \omega^2 \left[C_2 (L_1 + R_1) - Z_m \cdot C_2\right]$, $B_{21} = \omega^2 \left[C_3 (n^2 L_2 + n^2 R_2) - Z_m \cdot C_3\right]$, $B_{22} = 1 - \omega^2 \left[C_3 (n^2 L_2 + n^2 R_2) + C_2 (n^2 L_2 + n^2 R_2 + Z_m)\right]$.

Spurious capacitors can be calculated by formula (11).

Finally, the optimized element values were obtained with particle swarm optimization. The components of original equivalent circuit model are $C_1 = 0.15 \times 10^{-9} \text{F}$, $C_2 = 3.6607 \times 10^{-9} \text{F}$, $C_3 = 0.89 \times 10^{-9} \text{F}$, $R_1 = 1.6271 \Omega$, $R_2 = 0.00126 \Omega$, $L_1 = 3.5272 \text{H}$, $L_2 = 0.0089 \text{H}$, for respectively.

**4. AUGMENTATION BRANCHES**

The spurious capacitors in Fig. (11) can be regard as a delta connection and meanwhile the rest in Fig. (11) can be regard as a star connection scheme. Using the star-delta switching, we simplify the spurious capacitors into the rest. And the original equivalent circuit model and its augmentation in this paper is shown in Fig. (12).

$Y_{\text{Aug 1}}, Y_{\text{Aug 2}}$ and $Y_{\text{Aug 3}}$ can be calculated by the short-circuit admittance $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ of original equivalent circuit model, as shown in formula (12).
\[
Y_{\text{aug1}}(\omega) = Y_{11,\text{aug}}(\omega_s) + Y_{21,\text{aug}}(\omega_s) - Y_{11,\text{cmm}}(\omega) - Y_{21,\text{cmm}}(\omega_s) \\
Y_{\text{aug2}}(\omega) = Y_{12,\text{aug}}(\omega_s) - Y_{12,\text{cmm}}(\omega_s) \\
Y_{\text{aug3}}(\omega) = Y_{22,\text{aug}}(\omega_s) + Y_{12,\text{aug}}(\omega_s) - Y_{22,\text{cmm}}(\omega_s) - Y_{12,\text{cmm}}(\omega_s)
\]

(12)

Fig. (12). The original equivalent circuit model and its augmentation in this paper.

The \( Y_{\text{aug}} \) can express by rational fractional function

\[
Y_{\text{aug}}(s) = \sum_{i=1}^{n} \frac{k_i}{s - p_i} + d + se
\]

(13)

Then we get the CAM model \( Y_{\text{cam}}(s) \) with \( Y_{\text{aug}} \), as shown in

\[
Y_{\text{cam}}(s) = Y_{\text{cmm}}(s) + Y_{\text{aug}}(s)
\]

(14)

Using the vector fitting technology [22], the formula (13) is obtained. Next, the Brune method of synthesis [17, 18] is executed for the formula (13), and makes passivity correction before network synthesis if necessary. In this paper, the element values of Brune realizations are shown in Tables 2-4.

### Table 2. Element values of brune realizations in \( Y_{\text{aug1}} \).

<table>
<thead>
<tr>
<th>Element Order Number</th>
<th>( R_i ) (Ohm)</th>
<th>( L_{\rho} ) (H)</th>
<th>( L_s ) (H)</th>
<th>( C ) (F)</th>
</tr>
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<tbody>
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<td>null</td>
<td>null</td>
<td>null</td>
<td>3.63674300e-11</td>
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<tr>
<td>2</td>
<td>1.266848967</td>
<td>3.69518403e-5</td>
<td>5.66089730e-5</td>
<td>1.28305786e-11</td>
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<tr>
<td>3</td>
<td>1.878957500e+2</td>
<td>null</td>
<td>null</td>
<td>4.04340177e-11</td>
</tr>
<tr>
<td>4</td>
<td>4.39667737e+2</td>
<td>4.575835e-6</td>
<td>5.17399620e-6</td>
<td>1.03299110e-11</td>
</tr>
<tr>
<td>5</td>
<td>4.635046485e-12</td>
<td>1.90412930e-5</td>
<td>4.03339100e-4</td>
<td>2.63177329e-11</td>
</tr>
<tr>
<td>6</td>
<td>2.169917945e+2</td>
<td>2.67160370e-4</td>
<td>2.66012790e-4</td>
<td>3.48479594e-13</td>
</tr>
<tr>
<td>7</td>
<td>4.584115244e+2</td>
<td>6.43917540e-4</td>
<td>6.55973110e-4</td>
<td>3.21056683e-13</td>
</tr>
<tr>
<td>8</td>
<td>4.181628600e+2</td>
<td>2.86914197e-1</td>
<td>1.67583400e-2</td>
<td>4.02803017e-9</td>
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<tr>
<td>9</td>
<td>3.271638000e+2</td>
<td>1.21779058e-5</td>
<td>5.93108426e-6</td>
<td>8.67269309e-12</td>
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<td>10</td>
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<td>1.30438753e-6</td>
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### Table 3. Element values of brune realizations in \( Y_{\text{aug2}} \).

<table>
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<tr>
<th>Element Order Number</th>
<th>( R_i ) (Ohm)</th>
<th>( L_{\rho} ) (H)</th>
<th>( L_s ) (H)</th>
<th>( C ) (F)</th>
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<tr>
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<td>4.2215000100e-12</td>
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<td>3</td>
<td>4.453645181e+1</td>
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<td>5.9765143964e+1</td>
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<tr>
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<tr>
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<td>2.9523395100e+5</td>
<td>null</td>
<td>null</td>
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</tr>
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</table>
Further, we can make the global perturbed optimization [19-21] in PSpice, and the principle is consist in formula (15)

\[
Y_{con}(s) = \sum_{i=1}^{Q_{aug}} \frac{k_i + \nabla \hat{k}_i}{s - (\hat{p}_i + \Delta \hat{p}_i)} + \sum_{i=1}^{Q_{aux}} \frac{\hat{k}_i}{s - \hat{p}_i} + \hat{d} \tag{15}
\]

5. EXPERIMENTAL RESULTS

We measure the PT with lightning overvoltage and switching overvoltage, and compare with simulation result. The lightning overvoltage waveform is shown in Fig. (13), and the switching overvoltage is shown in Fig. (14).

As shown in Figs. (15, 16), through the comparison of the measurement and simulation, the accuracy is sacrificed.
CONCLUSION

In this paper, a broadband circuit model of PT was proposed for the power system measurement and protection in the wide range of frequencies. According to the no-load experiment data, we obtained the hysteresis loop curve and established a fractional order model for the iron core of PT. Moreover, according to the S parameters measured by the Agilent 4395A analyzer with 87512A, we calculated the element values with the circuit augmentation method and Brune realization of synthesis for the augmented parts of PT.

To verify the performance of the proposed broadband model of the PT, experiments of lightning overvoltage and switching overvoltage had been conducted. A good agreement has been obtained between the simulation and measurement under the lightning overvoltage and switching overvoltage respectively, and the results show that the proposed method enables the engineers to retain the original equivalent circuit models while providing physical significance to obtain the high frequency characteristic accurately.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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