Theoretical Study on Vibration Control of Symmetric Structure with Shape Memory Alloy (SMA)-Friction Damper

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Abstract: The paper proposed an innovative shape memory alloy (SMA)-friction damper. The damper consisted of the superelastic SMA wire and the friction element in series. According to the working mechanism of the damper, the paper set up the mechanical model of the damper. Seismic elastic-plastic time history response analysis program and energy analysis program of the damped structure were written. The numerical studies on vibration control of a three-story shear-type symmetric structure with the damper were carried out. The results indicate that the damper can decrease the displacement and the inter-story displacement of the structure effectively, but increase the acceleration of the structure comparing with uncontrolled structure. The SMA-friction damper not only can adjust the working status of the energy dissipation elements automatically according to the seismic responses of the structure, but also has such advantages as simple configuration and economical application.

Keywords: Mechanical model, shape memory alloy-friction damper, symmetric structure, vibration control.

1. INTRODUCTION

Shape memory alloy (SMA) is a new intelligent material. It shows a large reversible strain due to superelasticity. In particular, the Ni-Ti-based alloy exhibits some ductility and excellent superelastic strain. SMA has been available applied for electronics, machinery, energy, medical, aerospace, automotive industry at present. Since the 90's of last century, SMA begins to use on the field of civil engineering as intelligent materials. Its application has been continuously developed, especially the use of SMA superelastic dampers on the passive vibration control structure has attracted the attention of the scholars. For example, Zhu and Zhang [1] presented a special type of bracing element termed reusable hysteretic damping brace (RHDB), and the seismic behaviour of a concentrically braced frame system with self-centring capability was researched. Li et al. [2] proposed two types of shape memory alloy (SMA)-based devices, the tension-SMA device (TSD) and the scissor-SMA device (SSD). Parulekar et al. [3] designed and fabricated a damper device. Taking into account the residual martensite accumulation irreversibly due to cyclic forward reverse martensitic transformation, validity study is made using a thermo-mechanical model of SMA. Asgarian and Moradi [4] investigated the seismic performance of steel frames equipped with superelastic SMA braces. Buildings with various stories and different bracing configurations including diagonal, split X, chevron (V and inverted V) bracings were considered. Di Cesare and Ponzo [5] referred to the experimental tests on the model equipped with two different systems based on Hysteretic Dampers (HD) and visco-re-centering devices (SMA+VD). The devices could restrict the inter-story drifts, and the frame yielding is surely prevented. Osman and Stefan [6] proposed a new recentering variable friction device (RVFD), and the seismic response control of a 20-story nonlinear benchmark building was investigated. The energy dissipation capabilities of a variable friction damper (VFD) and the recentering ability of shape memory alloy (SMA) wires all existed in the RVFD.

The experiments and theories have proved that these SMA dampers can effectively limit the seismic response of the structure and ensure the safety of structure. But some problems are not resolved in the past research. Energy dissipation element of damper (SMA or SMA and other energy dissipation materials) acts on the structure at the same time, and these dampers cannot automatically adjust the energy dissipation according to seismic response of the structure. The quantities of SMA wires are determined according to seismic responses of the structure under strong earthquake, and a lot of SMA wires are required. But the abilities of SMA dampers are not fully released under small earthquake or moderate earthquake. It is not economic because the SMA wires are expensive.

The paper proposed an innovative shape memory alloy (SMA)-friction damper. Consisting of the superelastic SMA wire and the friction element in series, the damper not only can adjust the working status of the energy dissipation elements automatically according to the seismic responses of the structure, but also has simple configuration and economical application. According to the working mechanism of the damper, the mechanical model is established. It is written that seismic elastic-plastic time history response analysis program and energy analysis program of the damped structure. The numerical studies on
2. STRUCTURE AND WORKING MECHANISM OF SHAPE MEMORY ALLOY (SMA)-FRICITION DAMPER

2.1. Structure of SMA-Friction Damper

The structure of shape memory alloy (SMA)-friction damper shows in Fig. (1). The damper consists of first level board (1), front side board (2), right moving rod (3), super elastic SMA wire (4), left moving rod (5), friction plate (6), second level board (7), back side board (8), high strength bolt (9), wire wrapped implement (10), fixed screw (11), connecting bolt (12), sleeve (13), fixture (14) and PTFE sheet (15). Numbers in brackets correspond to position of each element of the damper in Fig. (1) [7].

Fig. (1). Schematic sketch of SMA-friction damper.

2.2. Working Mechanism of SMA-Friction Damper

The damper can adjust the working status of the energy dissipation elements automatically when sliding friction force of the damper is more than the maximum restoring force of the SMA wires. The working mechanism of the damper under pullout load is described.

1) On the small load, the displacement of first level board is less than which of second level board. The first level board drives the right moving rod moving, and the left moving rod is stopped by the front side board and the back side board. The SMA wires are stretched. As unloading load, the right moving rod and first level board reset by the restoring force of the SMA wires. The less energy is consumed in the process of stretch or retracting of the SMA wires.

2) On the big load, the displacement of first level board is more than which of second level board. The left moving rod and right moving rod approach to contact with the first level board, the front side board and back side board. If the contact force is more than the maximum static friction force of the damper, the front side board and back side board which are stuck on friction sheet have relative sliding displacement to the second level board. The friction force is produced and a lot of energy is consumed. Meanwhile, the SMA wires keep the maximum stretch length and its abilities are fully released. As unloading load, the stretch length reset and the friction sliding length is residual displacement.

3. MECHANICAL MODEL OF SMA-FRICTION DAMPER

3.1. Graesser's Constitutive Equations of the Superelastic SMA

Graesser's constitutive equations [8] are used to describe the stress $\sigma$ -strain $\varepsilon$ relationship of the SMA wires. The formulas are:

$$\dot{\sigma} = E \times \left[ \dot{\varepsilon} - \left| \varepsilon \right| \left( \frac{\sigma - \beta}{Y} \right)^n \right]$$

(1)

$$\beta = E \times \alpha \times \left[ \varepsilon - \frac{\sigma}{E} + f_t \varepsilon^a \right] \text{erf} \left( a \varepsilon \right) \left[ u \left( -\varepsilon \varepsilon \right) \right]$$

(2)

where $E$ is elastic modulus, $Y$ is yielding stress, $\alpha = E \left( 1 - E' \right)$, $E'$ is the slope of $\sigma$-$\varepsilon$ curve as SMA wire yielding, $n$, $f_t$, $a$, $c$ are constant that related to the materials, erf$(\ )$ and $u(\ )$ are respectively the error function and the unit step function, the expressions are:

$$\text{erf} \left( y \right) = \frac{2}{\sqrt{\pi}} \int_0^y \text{e}^{-t^2} \, dt$$

(3)

$$u \left( y \right) = \begin{cases} 1 & \left( y \geq 0 \right) \\ 0 & \left( y < 0 \right) \end{cases}$$

(4)

3.2. Mechanical Model of SMA-Friction Damper

According to the working mechanism of the damper, the restoring force can be written as:

$$f_d = \begin{cases} f_{\text{SMA}} & \left( x'_e > x'_e < x'_e \right) \\ f_{\text{PRI}} & \left( x'_e \geq x'_e \right) \\ -f_{\text{PRI}} & \left( x'_e < x'_e \right) \end{cases}$$

(5)

where $x'_e$ and $f_d$ are respectively displacement and restoring force of the damper, $x'_e$ and $x'_e$ are critical displacement. As the residual displacement of the damper equals zero, $x'_e = x'_e = x_e$, $x_e$ is maximum stretch length of the single SMA wire. As the residual displacement of the damper doesn't equal zero, $x'_e$ and $x'_e$ are respectively corresponding displacement which the tensile load and pressure load begin to be unloaded, satisfy $x'_e - x'_e = 2x_e$. $f_{\text{PRI}}$ is sliding friction force, $f_{\text{SMA}}$ is the restoring force of the SMA wires, $f_{\text{SMA}} = \sigma \times A$, $A$ is the total cross-sectional area of SMA wires.

4. VIBRATION ANALYSIS OF SYMMETRIC DAMPED STRUCTURE

Seismic elastic-plastic time history response analysis program and energy analysis program of the damped structure are written by Matlab.
4.1. Motion Equations of Symmetric Damped Structure

A p-story shear-type symmetric structure with the damper is studied, shown in Fig. (2). The hypotheses are given:

1) The slab is infinitely rigid in its own plane;
2) The quality of all the members of the structure piles up on the slab according to the proximity principle;
3) Bottom structure and foundation perfectly fixed, and joint action of upper structure and foundation is not considered.
4) The damper and brace in series are installed on the structure, and stiffness of brace is infinity.

The structure generates elastic-plastic deformation under strong earthquake. The inter-story restoring forces are computed according to the Bou-Wen model [9]:

\[ f_i = \delta k_i x_c + (1-\delta) k_j x_{iy} \]  \hspace{1cm} (6)

\[ \ddot{z} = \frac{1}{x_{iy}} [\lambda \dddot{x}_i - \mu \dddot{x}_i |\dddot{x}_i|^\gamma - \gamma \dddot{x}_i |\dddot{x}_i|^\eta ] \]  \hspace{1cm} (7)

where \( k_i \) is inter-story initial elastic stiffness of structure, \( \delta \) is the ratio of inter-story yielding stiffness to the initial elastic stiffness, \( x_c \) is the inter-story displacement, \( x_{iy} \) is inter-story yielding displacement, \( \lambda \), \( \mu \), \( \gamma \) and \( \eta \) are parameters that control the shape and smooth of \( f_i - x_{iy} \) curve.

\[ \text{Fig. (2). Model of structure.} \]

The equations of motion of damped structure are written:

\[ M_i \dddot{X}(t) + C_i \dot{X}(t) + H F_i(t) + H F_d(t) = -M_i \dddot{x}_c(t) \]  \hspace{1cm} (8)

where \( X(t) \), \( \dot{X}(t) \) and \( \dddot{X}(t) \) are respectively column vector of the displacement, column vector of the speed and column vector of the acceleration, \( \dddot{x}_c(t) \) is ground acceleration, \( I \) is p dimensional 1 column vector, \( M_i \) is mass matrix of structure, \( C \) is Rayleigh damping matrix, \( C = a_r M_i + a_z K_w \), \( K_w \) is initial elastic stiffness matrix of structure, \( a_r \) and \( a_z \) are coefficients that are calculated according to the ratio of the preceding two mode shapes damping \( \xi \) to \( \xi_2 \), \( F_i(t) \) is column vector of the inter-story restoring force, \( F_d(t) \) is column vector of the damper restoring force, if the i story has no damper, the i line of \( F_d(t) \) equals 0, \( H \) is \( p \times p \) matrix, its expression is:

\[ H = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -1 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \]  \hspace{1cm} (9)

4.2. Energy Balance Equation of Damped Structure

Each component in the formula (8) integrates with the relative displacement, and the energy balance equation of structure can be given [10]:

\[ E_s(t) + E_c(t) + E_i(t) + E_d(t) = E_m(t) \]  \hspace{1cm} (10)

where \( E_s(t) \) is structural kinetic energy, \( E_c = \frac{1}{2} \dot{X}(t)^T M_c \dot{X}(t) \), \( E_i(t) \) is structural internal damping energy, \( E_s = \int \dot{X}(t)^T C_c \dot{X}(t) dt \), \( E_d(t) \) is structural strain energy, \( E_d = \int [\dddot{X}(t)]^T H F_d(t) \dddot{X}(t) dt \), it includes hysteretic dissipation energy and elastic strain energy. When the structure tends to be static after earthquake, elastic strain energy will approach to 0. \( E_s(t) \) is dissipation energy of the damper, \( E_s = \int [\dddot{X}(t)]^T H F_d(t) \dddot{X}(t) dt \), \( E_m(t) \) is the total input energy to structure when earthquake happened, \( E_m = \int [-M_i \dddot{x}_c(t)]^T \dot{X}(t) dt \).

5. VIBRATION CONTROL CALCULATION OF SYMMETRIC STRUCTURE WITH SMA-FRICTION DAMPER

5.1. Model Introduction

The three-story shear-type symmetric structure is studied. The quality of each story is \( m = 1 \times 10^3 \text{kg} \), the inter-story restoring force model parameters of each story are as follows:

\( k_c = 5 \times 10^7 \text{N/m} \), \( \delta = 0.3 \), \( \lambda = 1 \), \( \mu = 0.9 \), \( \gamma = 0.1 \), \( \eta = 95 \), the first story and the second story \( x_{iy} = 0.014 \text{m} \), the third story \( x_{iy} = 0.010 \text{m} \), \( x_c = 0.05 \). The self vibration period of the structure which are calculated are respectively 0.631 s, 0.225 s and 0.156 s.

The mechanical model parameters of the SMA-friction damper are as follows: \( A = 55.292 \text{mm}^2 \), \( E = 3402.7 \text{GPa} \), \( Y = 47.85 \text{GPa} \), \( \alpha = 0.13 \), \( n = 5 \), \( f_1 = 0.23 \), \( a = 197 \), \( c = 0.09 \), \( f_{fri} = 75.9 \text{kN} \), \( x_c = 12 \text{mm} \). The length of the single SMA wire is 200 mm.

The dampers which are installed on the structure are shown as Fig. (2). In order to limit the vibration of each
story, the quantities of damper are determined: 11 on the first story, 7 on the second story, 11 on the third story.

El-Centro wave (1940.5.18) is selected as seismic dynamic recording. The strong earthquake during \( t = 0 \text{s} \to 10 \text{s} \) is selected. The peak acceleration is adjusted to 4 m/s\(^2\). In order to evaluate the restoring function of damper, the structure is allowed to freely vibrate 10 s after earthquake wave stop.

5.2. Calculation Results and Analysis

The peak displacement \( x_{\text{max}} \), the peak inter-story displacement \( x_{\text{cmax}} \), the peak acceleration \( \ddot{x}_{\text{max}} \) and reduction ratio are shown in Table 1. The inter-story displacement time history curves of structure are shown in Fig. (3). The displacement and inter-story displacement of structure can be restricted by damper, and reduction ratios are respectively 44.92\%~36.66\% and 44.92\%~18.20\%. The acceleration of structure is enlarged to 52.94\%~7.27\% because the dampers give additional structural stiffness. Comparing with uncontrolled structure, residual displacement of the first story and second story increase and their value are respectively 6.31 mm and 2.53 mm in damped structure. Residual displacement of third story decreases and approximates to 0.

To understand the elastic-plastic hysteretic behavior and the mechanism of damper, inter-story force-displacement curves of damped structure are shown as Fig. (4), and the force-displacement curves of SMA-friction damper on damped structure are shown as Fig. (5), and energy time history curves of damped structure and uncontrolled structure are shown as Fig. (6).

1) The first story and the second story are on the elastic-plastic stage, and the third story still on the elastic stage in Fig. (4).

2) In Fig. (5), the SMA wires and friction element of the first story and the second story damper act on the structure in sequence. The hysteretic loop is full and energy dissipation is high. The result indicates that the vibration of structure is controlled but there exists the residual displacement after earthquake. Only the SMA wires of the third story damper act on the structure, and the hysteretic loop is narrow, and energy dissipation is low. So the reduction ratio is smaller but an excellent self-centering capacity.

![Fig. (3). Inter-story displacement time history of structure.](image_url)
3) In Fig. (6), the input energy of damped structure (\(E_i = 167.46\text{kJ}\)) decreases 25.03% than which of uncontrolled structure (\(E_i = 223.36\text{kJ}\)) because the structure vibration is restricted by damper. In uncontrolled structure, the structure consumes total input energy (\(E_s = 130.21\text{kJ}, E_c = 93.15\text{kJ}\)) because of its hysteresis behavior and internal damping. In damped structure, 50.46% input energy (\(E_d = 84.50\text{kJ}\)) is consumed by damper, and hysteretic energy and internal damping energy must be decreased (\(E_s = 26.51\text{kJ}, E_c = 56.45\text{kJ}\)), and the structure safety is ensured.

![Fig. (4). Inter-story force-displacement curve of damped structure.](image)

![Fig. (5). Force-displacement curve of sma-friction damper on damped structure.](image)

**CONCLUSION**

An innovative shape memory alloy (SMA)-friction damper is proposed. Consisting of the superelastic SMA wires and the friction element in series, the damper can adjust the working status of the energy dissipation elements automatically according to the seismic responses of the structure. Only SMA wires act on under small load, and the SMA wires and friction element act on in sequence under large load. The configuration of damper is simple and economical. According to the working mechanism of damper, its mechanical model is given. Seismic elastic-plastic time history response analysis program and energy analysis program of the damped structure are written by Matlab. Studies on vibration control of a three-story shear-type symmetric structure with the damper are carried out. The results indicate that the damper can effectively decrease the displacement and the inter-story displacement of the structure, but increase the acceleration of the structure.
Furthermore, the SMA wires can present low energy dissipation but an excellent self-centering capacity, and the friction element can present a considerable energy but make the structure have the residual displacement after earthquake.

![Energy time history of structure](image)

**CONFLICT OF INTEREST**

The authors confirm that this article content has no conflict of interest.

**ACKNOWLEDGMENTS**

This work is supported by the General Program Foundation of NSFC (No. 50978081), the Central Universities Independent Research Fund, China (No. DC201502040302).

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