Wave Propagation in Nanodoped Films

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Abstract: We analyze the propagation of electromagnetic plane waves through a dielectric film endo-wed with a nano doped permittivity made of a sequence of Dirac delta pulses.

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1. INTRODUCTION

The propagation of electromagnetic plane waves through homogeneous and periodically stratified films has been the subject of several important works since the first edition of the Born-Wolf book [1] and after Arzéliès’ publications [2]. We are interested here in the beha-viour of TE, TM harmonic plane waves incident on the z = 0 face of a dielectric film 0 ≤ z ≤ d with a nano doped permittivity, the nanodoping being obtained from a sequence of delta Dirac pulses. We also consider succinctly magnetic composite films doped with magnetic hol-low nanospheres. With the light velocity c = 1, the permeability μ = 1 and exp(-iot) implicit, the components E y, H x, H z of the incident TE wave are [1] in z ≤ 0

\[ E_i^i = A_e \psi_i(x,z), H_i^i = -A_e n_1 \sin \theta_i \psi_i(x,z), \]

\[ \psi_i(x,z) = \exp[i\omega n_1(x \sin \theta_i + z \cos \theta_i)] \] (1a)

while, since \( \theta_i = \pi - \theta_t \), the reflected field is

\[ E_r^r = R_e \psi_r(x,z), H_r^r = R_e n_1 \cos \theta_i \psi_r(x,z), \]

\[ H_r^r = R_e n_1 \sin \theta_i \psi_r(x,z) \] (2a)

with the expressions (1a), (2a) of \( \psi_i, \psi_r \).

2. TE, TM FIELDS INSIDE A FILM WITH NANO DOPED PERMITTIVITY

We consider a dielectric film with permittivity nano doped according to the relation

\[ \varepsilon(z) = \varepsilon_0 + \eta \sum_{m=1}^M \delta(z - z_m), 0 \leq z \leq d, Mz_0 \leq d \]

where \( \varepsilon_0, \eta, z_0 > 0 \) are constant parameters and \( \delta \) the Dirac distribution. This permittivity has the property to have a first derivative null, the relation \( \varepsilon'(z) = 0 \) implying \( \varepsilon(z) = \varepsilon_0 \).

For the TE field, only on x, z, the equations (6, 7) reduce to

\[ \partial_x H_x^r - \partial_z H_z^r + \varepsilon(z) E_y^r = 0, \partial_x E_x^r - \partial_z E_z^r - i \omega n_2(z) \varepsilon(z) E_y^r = 0 \] (8a)

and for the TM field

\[ \partial_x E_x^r - \partial_z E_z^r - i \omega n_2(z) \varepsilon(z) H_y^r = 0 \] (8b)

These fields are consistent with (1-4), just changing \( \varepsilon(z) \) into \( \varepsilon_i \) and using (1a), (2a).

Eliminating \( H^r_x, H^r_z \) from (8a) and taking into account \( \varepsilon'(z) = 0 \) gives for \( E_y^r \) the same wave equation as that obtained for \( H^r_y \) by eliminating \( E_x^r, E_z^r \) from (8b)

\[ \partial_x^2 + \partial_z^2 + \omega^2 n^2(z) \varepsilon(z) [E_y^r, H_y^r] = 0, n^2(z) = \varepsilon(z) \] (9)

We look for the solutions of Eq.(9) in the form

\[ E_y^r(x,z) = T_e \exp[i\omega n_0 x \sin \theta_0] \phi(z) \] a

\[ H_y^r(x,z) = -n_0 T_m \exp[i\omega n_0 x \sin \theta_0] \phi(z) \] b

with \( n_0 = \sqrt{\varepsilon_0}, T_e, T_m \) the field amplitudes. Substituting (10) into (9) gives the differential equation satisfied by \( \phi(z) \)

\[ \partial_x^2 \phi(z) + \omega^2 [n^2(z) - n_0^2 \sin^2 \theta_0] \phi(z) = 0 \] (11)

The solutions of (11) are discussed in Appendix A and assuming \( \eta \ll 1 \) we get to the 0(\eta^2) order.
Substituting (13a) into (8a) gives
\[E_y(0) = A \exp(i \omega z - \beta_1 z)\]
and
\[\phi_m(z) = (i \omega z/2 \pi n_0 \cos \theta^l) \exp(i \omega n_0 \cos \theta^l) \{mz_0 + [z - mz_0]\}\]

Now, in the dielectric film, the field reflected on the z = d face has to be taken into account so that according to (10) the components \(E_x(x,z), H_y(x,z)\) are
\[E_x(x,z) = n_0 \beta_1^l(x) [T_m^l \phi(z) + T_{m}^{l^*} \phi^2((d-z))]\]
and
\[H_y(x,z) = -n_0 \beta_2^l(x) [T_m^l \phi(z) + T_{m}^{l^*} \phi^2((d-z))]\]
in which
\[\beta_1^l(x) = \exp(i \omega n_0 x \sin \theta^l)\]

Substituting (13a) into (8a) gives
\[H_y(x,z) = 1 \omega \beta_1^l(x) [T_m^l \phi(z) + T_{m}^{l^*} \phi^2((d-z))]\]
and
\[H_y(x,z) = n_0 \beta_2^l(x) [T_m^l \phi(z) + T_{m}^{l^*} \phi^2((d-z))]\]

Similarly with (13b) substituted into (8b)
\[E_x(x,z) = \{i \omega \epsilon(z) \} \beta_1^l(x) [T_m^l \phi(z) + T_{m}^{l^*} \phi^2((d-z))]\]
\[E_x(x,z) = [i \omega / \epsilon(z)] \sin \theta^l \beta_1^l(x) [T_m^l \phi(z) + T_{m}^{l^*} \phi^2((d-z))]\]

We now have all the ingredients to analyze the electromagnetic plane wave propagation through the nano doped dielectric film.

3. ELECTROMAGNETIC WAVE PROPAGATION

3.1. TE Field

The amplitudes of the TE field must satisfy boundary conditions at \(z = 0\) and \(z = d\). Then, noting first that the Descartes-Snell relation \(n_1 \sin \theta^l = n_0 \sin \theta^l\) transforms (14) into
\[\beta_1^l(x) = \exp(i \omega n_1 x \sin \theta^l)\] (17)

We have at \(z = 0\)
\[E_y(0) + E_y^r(0,0) = E_y^r(0,0)\]
\[H_y(0) + H_y^r(0,0) = H_y^r(0,0)\]
and, taking into account (1), (1a), (2), (2a) and (13a), (15a) together with (17), we get from (18)
\[R_e + A_e = T_e^l \phi(0) + T_{e}^{l^*} \phi^2(d)\]
\[n_1 \cos \theta^l (R_e - A_e) = i[T_m^l \phi'(0) + T_{m}^{l^*} \phi^2'(d)]\]

Now, to get the TE field \(E_y, H_y, H_y^r\) outside the film for \(z > d\), one has just to change in (1) the amplitude \(A_e\) into \(A_{e,c}\) so that the boundary conditions at \(z = d\) are
\[E_y(0, x) = E_y^r(0, x)\]
\[H_y(0, x) = H_y^r(0, x)\]
and, still using (1), (1a) and (13a), (15a), (17), we get
\[T_e^l \phi(0) + T_{e}^{l^*} \phi^2(0) = \gamma A_{e,c}\]
\[i[T_m^l \phi'(0) + T_{m}^{l^*} \phi^2'(0)] = -n_1 \cos \theta^l \gamma A_{e,c}\]
in which
\[\gamma = \exp(i \omega n_1 d \sin \theta^l)\] (21a)

So, we get from (19) and (21) four relations to determine the four unknown amplitudes \(R_e, T_e^l, T_{e}^{l^*}, A_{e,c}\); this set of equations is solved in Appendix B.

3.2. TM Field

The boundary conditions for the TM field are at \(z = 0\)
\[H_y^t(0, x) + H_y^r(0, x) = H_y^t(0, x)\]
\[E_y^t(0, x) + E_y^r(0, x) = E_y^t(0, x)\]

Then, using (3), (4) with (1a), (2a) together with (13b), (16a), taking into account (17), we get since \(\epsilon(0) = \epsilon_0 = n_0^2\)
\[n_1 (R_m + A_m) = n_0 [T_m^l \phi(0) + T_{m}^{l^*} \phi^2(d)]\]
\[n_0 \cos \theta^l (R_m - A_m) = i[T_m^l \phi'(0) + T_{m}^{l^*} \phi^2'(d)]\] (23)

Now the TM field in \(z > d\) has the expression (3) with \(A_m\) changed into \(A_{m,c}\) so that the boundary conditions at \(z = d\) are
\[H_y^t(x, d) = H_y^t(0, x)\]
\[E_y^t(x, d) = E_y^t(0, x)\]
(24)
implying with \(\gamma^l\) given by (21a) since \(\epsilon(d) = \epsilon_0 = n_0^2\)
\[i[T_m^l \phi'(d) + T_{m}^{l^*} \phi^2'(d)] = -n_0 \cos \theta^l \gamma A_{m,c}\]
(25)

We get from (23), (25) four relations to determine \(R_m, T_m^l, T_{m}^{l^*}, A_{m,c}\) which is made in Appendix B

4. DISCUSSION

High-k dielectrics are used for instance in semiconductor manufacturing process to replace silicon gate dielectrics, allowing a miniaturization of microelectronics component with better performances in thin materials such as dielectric films. Nano doped dielectrics offer the possibility of high-k dielectrics. Here for instance, the mean value of the dielectric constant is
\[\epsilon = 1/d \int_0^d \epsilon(z) \, dz\]
(26)
that is substituting (5) into (26)
\[\epsilon = \epsilon_0 + \eta/\omega d \int_0^d [U(z) - U(z-d)] \sum_m^M \delta(z-mz_0)\]
\[= \epsilon_0 + \eta/\omega d \sum_m^M [U(mz_0) - U(mz_0-d)]\]
\[= \epsilon_0 + M\eta/\omega d\]
(26a)
taking great values when \(M/\omega d\) is high.

Incidentally, the sum in (5) is the truncated series of the Dirac distribution [4, 5]
\[\pi \delta(\pi z/z_0) = \sum_n \delta(z-nz_0)\], \(n \) integer in \((-\infty, \infty)\)
(27)

The matrix technique [1,2] used to analyze the propagation of electromagnetic plane waves through homogeneous and periodically stratified dielectric films is not suitable for TE, TM fields inside a film with the permittivity (5) which is neither homogeneous nor stratified be-cause the dielectric constant is only perturbed by the Dirac pulses at local points. The impartance of \(\epsilon'(z) = 0\), to get the wave equation (9) must be stressed.
We have obtained in Appendix A an $0(\eta^2)$ approximation of TE, TM fields in which the Green’s function of the 1D-
Helmholtz equation intervenes rather naturally. The object of this approximation was only to get a perception of the TE,
TM behaviour, but it is clear that an important numerical analysis has to be performed when $\eta$ is not very small.

Finally, it has been assumed that 1D-nano doping may be described by a sequence of delta Dirac pulses, the nano dots
being assimilated to points. This postulate could be generalized to 2D and 3D nano doping from the relations [5]

$$\delta(r) / \pi r = \delta(x) \delta(y) = (x^2 + y^2)^{1/2}$$

$$\delta(r) / 2\pi r^2 = \delta(x) \delta(y) \delta(z) = (x^2 + y^2 + z^2)^{1/2} \quad (28)$$

The following generalization of (5) could be used to describe nanodoped photonic crystals made of multilayer films [6]

$$e_j(z) = e_0 + z_0 \sum_{m=1}^{M} \eta_j \delta(z - mz_0), \quad j = 1, 2, \ldots \tilde{J} \quad (29)$$

in which $\tilde{J}$ is the number of layers.

5. MAGNETIC NANO COMPOSITE FILMS

Magnetic nano composite films are used specially to enhance the field coercivity [7-11]. That is their resistance to
becoming demagnetized. The non existence of magnetic monopoles prevents to imagine the doping of these films as
made of nano dots and, we have instead to consider magnetic hollow nano spheres [12-17]. Then, the permeability $\mu(z)$ in
$0 \leq z \leq d$ may be represented by the expansion

$$\mu(z) = \mu_0 + \sum_{m=1}^{M} \nu(mz_0) \delta(z - mz_0) \quad (30)$$

$z_0$ is the center of a hollow nano sphere and $\nu(mz_0)$ depends on its nature and on its radius [14, 15]. This permittivity satisfies also the condition $\mu(0) = 0$.

Proceeding as in (26a), we get from (30) for the mean value permittivity $\mu$ of this magnetic nano composite film giving
the possibility to check its coercivity performance.

$$\mu = \mu_0 + 1/\text{iod} \sum_m \nu(mz_0) \quad (30a)$$

Then, using (30) and assuming $\varepsilon = 1$, it is easily checked that the equations (8a,b) for TE, TM fields transform into

$$\partial_t E_x^\perp = - \varepsilon_0 \mu(z) H_y^\perp, \quad \partial_t E_y^\perp = \varepsilon_0 \mu(z) H_x^\perp, \quad \partial_t H_x^\perp - \partial_t H_y^\perp + i \omega E_y^\perp = 0 \quad (31a)$$

$$\partial_t H_y^\perp = i \omega E_x^\perp, \quad \partial_t H_x^\perp = - i \omega E_x^\perp, \quad \partial_t E_x^\perp - \partial_t E_y^\perp - i \omega \mu(z) H_y^\perp = 0 \quad (31b)$$

so that since $\mu(z) = 0$, the components $E_x^\perp, H_y^\perp$ are still solutions of the wave equation (9) in which now $n''(z) = \mu(z)$ and they take the form (13) with $\phi(z)$ satisfying the differential equation (12). Substituting (13) into (31a,b) gives the other two components of the TE, TM fields with according to (31a)

$$E_x^\perp(x,z) = [i/\mu(z)] \beta^\perp(x) [T_{m}^{-1} \phi((d-z))] \quad (a)$$

$$E_y^\perp(x,z) = [n_0/\mu(z)] \sin\theta \beta^\perp(x) [T_{m}^{-1} \phi((d-z))] \quad (b) \quad (32)$$

and from (31b)

$$E_x^\perp(x,z) = \mu_0 \beta^\perp(x) [T_{m}^{-1} \phi((d-z))] \quad (a)$$

$$E_y^\perp(x,z) = \mu_0 \sin\theta \beta^\perp(x) [T_{m}^{-1} \phi((d-z))] \quad (b) \quad (33)$$

From there, we may proceed as in Sec.3, using the boundary conditions at $z = 0$ and $z = d$ to get four equations to determine the four unknown amplitudes.

As previously stated, $\phi(z)$ is solution of Eq.(11) with $n''(z) = \mu(z)$. Let $\nu = \text{Max}_0 \nu(mz_0)$ then assuming $\nu << 1$ and consequently $\nu(mz_0) << 1$ we have to the $0(\nu^2)$ order

$$\phi(z) = \phi_0(z) + \nu \sum_{m=1}^{M} \phi_m(z) + 0(\nu^2) \quad (34)$$

with $\phi_0(z), \phi_m(z)$ given by (12a,b).

This analysis of magnetic nano composite films reposes on magnetic hollow nano sphere whose existence requires further
works.

APPENDIX A

We discuss here the solutions of Eq.(11) rewritten for convenience

$$\partial_z^2 \phi(z) + \omega^2 [n''(z) - n_0^2 \sin^2\theta] \phi(z) = 0 \quad (A.1)$$

in which, according to (5):

$$n''(z) = n_0^2 + \eta z_0 \sum_{m=1}^{M} \delta(z - mz_0), \quad Mz_0 \leq d \quad (A.2)$$

We start this analysis with the simple refractive index

$$n''(z) = n_0^2 + \eta z_0 \delta(oz - mz_0) \quad (A.3)$$

so that the equation (A.1) becomes

$$\phi''(z) + \omega^2 n_0^2 \cos^2\theta \phi(z) = - \omega^2 \eta z_0 \delta(z - mz_0) \phi(z) \quad (A.4)$$

We assume $\eta << 1$ very small and we look for the solutions of (A.4) to the $0(\eta^2)$ order in the

the form

$$\phi(z) = \phi_0(z) + \eta \phi_m(z) + 0(\eta^2) \quad (A.5)$$

Substituting (A.5) into (A.4) gives

$$\phi_{m}''(z) + \omega^2 n_0^2 \cos^2\theta \phi_m(z) = - \omega^2 \eta z_0 \delta(z - mz_0) \phi_m(z) \quad (A.6)$$

supplying the two equations

$$\phi_{m}''(z) + \omega^2 n_0^2 \cos^2\theta \phi_m(z) = 0 \quad (a)$$

$$\phi_{m+1}''(z) + \omega^2 n_0^2 \cos^2\theta \phi_m(z) = - \omega^2 \eta z_0 \delta(z - mz_0) \phi_m(z) \quad (b) \quad (A.7)$$

Taking as solution of (A.7a)

$$\phi_0(z) = e^{i\gamma z} \quad (A.8)$$

the equation (A.7b) becomes

$$\phi_m''(z) + \omega^2 n_0^2 \cos^2\theta \phi_m(z) = - \omega^2 \eta z_0 \exp[i\gamma z] \delta(z - mz_0) \quad (A.9)$$

which is in fact the equation satisfied by the Green’s function of the 1D-Helmholtz equation and this equation has the solution [3] for the infinite domain

$$\phi_m(z) = \left(\exp[i\gamma z] \delta(z - mz_0) \right) \quad (A.10)$$

Now, with the refractive index (A.2), the equation (A.1) becomes

$$\phi''(z) + \omega^2 n_0^2 \cos^2\theta \phi(z)$$

$$= - \omega^2 \eta z_0 \sum_{m=1}^{M} \delta(z - mz_0) \phi(z) \quad (A.11)$$
and we look for its solutions in the form

\[ \phi(z) = \phi_0(z) + \eta \sum_{m=1}^{M} \phi_m(z) + O(\eta^2) \quad (A.12) \]

Substituting (A.12) into (A.11) supplies the equations (A.7a) and (A.7b) for \( m = 1, 2 \ldots M \) with the solutions (A.10) which achieves to determine (A.12) to the 0(\eta^2) order.

**APPENDIX B**

To obtain the amplitudes \( R_e, T_e^1, T_e^2, A_{t,e} \) for the TE field, we introduce the functions

\[ \rho_m(z) = n_1 \cos \theta_1 \phi(z) - i \phi'(z), \quad \sigma_m(z) = n_1 \cos \theta_1 \phi(z) + i \phi'(z) \quad (B.1) \]

Then, eliminating \( R_e \) from (19) and \( A_{t,e} \) from (21) gives:

\[ \rho_1(0) T_e^1 + \rho_1(d) T_e^2 = 2 n_1 \cos \theta_1 A_e, \quad \sigma_1(d) T_e^1 + \sigma_1(0) T_e^2 = 0 \quad (B.2) \]

from which \( T_e^1 \) and \( T_e^2 \) are obtained

\[ T_e^1 = 2 n_1 \cos \theta_1 \chi_e \sigma_1(0) A_e, \quad T_e^2 = -2 n_1 \cos \theta_1 \chi_e \sigma_1(d) A_e \quad (B.3) \]

\[ \chi_e = \frac{\rho_1(0) \sigma_1(0) - \rho_1(d) \sigma_1(d)}{\rho_1(0) \sigma_1(0) + \rho_1(d) \sigma_1(d)} \quad (B.3a) \]

so that we get at once from the first relation (21)

\[ A_e = 2 n_1 \cos \theta_1 \chi_e [\sigma_1(0) \phi(d) - \sigma_1(d) \phi(0)] \gamma^{-1} A_e \quad (B.4) \]

while eliminating \( A_e \) from (19) and taking into account (B3) give

\[ R_e = \chi_e [\sigma_e^2(0) - \sigma_e^2(d)] A_e \quad (B.5) \]

We proceed similarly for the TM field with the functions

\[ \rho_m(z) = n_0^2 \cos \theta_1 \phi(z) - i \phi' (z), \quad \sigma_m(z) = n_0^2 \cos \theta_1 \phi(z) + i \phi' (z) \quad (B.6) \]

Eliminating \( R_m \) from (23) and \( A_{t,m} \) from (25) gives

\[ \rho_m(0) T_m^1 + \rho_m(d) T_m^2 = 2 n_0 n_1 \cos \theta_1 A_m, \quad \sigma_m(d) T_m^1 + \sigma_m(0) T_m^2 = 0 \quad (B.7) \]

from which we get

\[ T_m^1 = 2 n_0 n_1 \cos \theta_1 \chi_m \sigma_m(0) A_m, \quad T_m^2 = -2 n_0 n_1 \cos \theta_1 \chi_m \sigma_m(d) A_m \quad (B.8) \]

\[ \chi_m = \frac{\rho_m(0) \sigma_m(0) - \rho_m(d) \sigma_m(d)}{\rho_m(0) \sigma_m(0) + \rho_m(d) \sigma_m(d)} \quad (B.8a) \]

and, substituting (B.8) into the first relation (25), it comes

\[ A_{t,m} = 2 n_0^2 \cos \theta_1 \chi_m [\sigma_m(0) \phi(d) - \sigma_m(d) \phi(0)] \gamma^{-1} A_m \quad (B.9) \]

while eliminating \( A_e \) from (22) gives nто account (B7)

\[ R_m = \chi_m [\sigma_m^2(0) - \sigma_m^2(d)] A_m \quad (B.10) \]

To achieve to determine the TE and TM fields we have just to express \( \phi(0) \) and \( \phi(d) \) in terms of the solutions of Appendix A.

**CONFLICT OF INTEREST**

None declared.

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None declared.

**REFERENCES**


