Propagation of Vector-Meson Spectral-Functions in a BUU Type Transport Model: Application to Di-Electron Production

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Abstract: The time evolution of vector-meson spectral-functions is studied within a kinetic theory approach. We implement this formalism in a BUU type transport model. Applications focus on $\rho$ and $\omega$ mesons being important pieces for the interpretation of the di-electron invariant mass spectrum measured by the HADES collaboration for the reaction C + C at 2 AGeV bombarding energy. Since the evolution of the spectral functions is driven by the local density, the in-medium modifications are tiny for small collision systems within this approach, as the life time of the compressed stage is too short.

Keywords: Heavy-ion collisions, compressed nuclear matter, di-electron emission, broad resonance.

1. INTRODUCTION

Di-electrons serve as direct probes of dense and hot nuclear matter stages during the course of heavy-ion collisions [1-3]. The superposition of various sources, however, requires a deconvolution of the spectra by means of models. Of essential interest are the contributions of the light vector mesons $\rho$ and $\omega$. The spectral functions of both mesons are expected to be modified in a strongly interacting environment in accordance with chiral dynamics, QCD sum rules etc. [1, 2, 4-6]. After the first pioneering experiments with the Dilepton Spectrometer DLS [7, 8] now improved measurements with the High-Acceptance Di-Electron Spectrometer HADES [9-11] start to explore systematically the baryon-dense region accessible in fixed-target heavy-ion experiments at beam energies in the 1 - 2 AGeV region. The invariant mass spectra of di-electrons for the reaction C + C at 1 and 2 AGeV are now available [7, 10, 11] allowing us to hunt for interesting many-body effects.

There are several approaches for describing the emission of real and virtual photons off excited nuclear matter:

(i) A piece of matter at rest in thermal equilibrium at temperature $T$ emits $e^+e^-$ pairs with total momentum $q$ at a rate $dN/d^4xd^4q = -\alpha^2/(3M^2x^2)f_\rho Im\Pi_{\rho\rho}(1, 2)$, where $f_\rho(q_0,T)$ is the bosonic thermal distribution function and $\Pi_{\rho\rho}$ denotes the retarded electromagnetic current-current correlator, $\Pi_{\rho\rho} = -i\int d^4xe^{i\Phi(x)}\langle[\{j^\rho(x),j^\rho(0)\}]\rangle$, the imaginary part of which determines directly the thermal emission rate. Here, $\alpha$ stands for the electromagnetic fine structure constant, $M$ means the invariant mass of the di-electron, $j^\rho$ the is electromagnetic current operator, and $\langle[\ldots]\rangle$ denotes the ensemble average encoding the dependence on temperature and density (chemical potential), respectively. This rate may be combined with a global dynamic model which provides us the space averaged quantities as a function of time [12] or even adopting a time average [13-16].

(ii) Some sophistication can be achieved by employing a detailed model for the space-time evolution of baryon density and temperature, e.g., as delivered by hydrodynamics. One may also extract from transport models such parameters, where, however, local off-equilibrium and/or anisotropic momentum distributions hamper a reliable definition of density and temperature. Nevertheless, once the rate is given one has a very concise approach, as realized, e.g., in [17]. A similar approach has been presented in [18].

(iii) Microscopic or kinetic transport models do not require isotropic momentum distributions or local equilibrium. Once general principles are implemented, transport models also provide a detailed treatment of the emission of electromagnetic radiation in heavy-ion collisions.

Our approach belongs to item (iii). The time evolution of single particle distribution functions of various hadrons are evaluated within the framework of a kinetic theory. We focus on the vector mesons $\rho$ and $\omega$. The $\rho$ meson is already a broad resonance in vacuum, while the $\omega$ meson may acquire a noticeable width in nuclear matter [19]. Therefore, we are forced to treat dynamically these resonances and their decays into di-electrons. Resorting to consider only pole-mass dynamics is clearly insufficient in a microscopic approach [20]. Instead, one has to propagate properly the spectral functions of the $\rho$ and $\omega$ mesons. This is the main goal of our paper. We consider our work as being on an explorative level, not yet as a firm and deep theoretically founded prescription of dealing with di-electron emission from excitations with quantum numbers of $\rho$ and $\omega$ mesons off excited nuclear matter.

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Our paper is organized as follows. Essential features of our transport model are outlined in section 2. In Subsection 2.1 we describe how the mean field potentials enter in the relativistic transport equation. Subsection 2.2 introduces the dynamics of broad resonances and its implementation in the test-particle method. The crucial quantities for the spectral functions are the self-energies dealt with in subsection 2.3. Subsections 2.4 and 2.5 are devoted to particle production and di-electron emission, respectively. Numerical results of our simulations and a tentative comparison with published HADES data are presented in sections 3 (2 AGeV) and 4 (1 AGeV). Discussion and summary can be found in section 5.

The present analysis supersedes [21], where a first preliminary comparison of transport model results with the C(2 AGeV) + C data [10] has been presented. Further analyses of HADES data have been performed in [22-27].

2. TREATMENT OF HEAVY-ION COLLISIONS

2.1. The Standard BUU Treatment

The employed BRoBUU computer code for heavy-ion collisions developed by a Budapest-Rossendorf cooperation solves a set of coupled Boltzmann-Uhling-Uhlenbeck (BUU) equations in the quasi-particle limit [28, 29]

\[
\frac{\partial F_i}{\partial t} + \frac{\partial H}{\partial p} - \frac{\partial H}{\partial x} = \sum C_{ij},
\]

\[
H = \sqrt{(m_i + U(p,x))^2 + p^2},
\]

for the one-body distribution functions \( F_i(x,p,t) \) of the various hadron species \( i \), each with rest mass \( m_i \), in a momentum and density dependent mean field \( U(x) \). The scalar field \( U(x) \) is chosen in such a manner that the Hamiltonian \( H \) equals \( H = \sqrt{(m_i + U(p,x))^2 + p^2} \) with a potential \( U^\prime \) calculated in the local rest frame as

\[
U^\prime(x,p) = A \frac{n}{n_0} + B \left( \frac{n}{n_0} \right)^2 + C \frac{2}{n_0} \int \frac{d^3 p}{(2\pi)^3} \frac{F_i(x,p')}{1 + \left( \frac{p-p'}{\Lambda} \right)^2},
\]

where the parameters \( A, B, C, \Lambda \) define special types of potentials, while \( n, n_0 \) and \( F_i \) stand for the baryon number density, saturation density and nucleon distribution function. We use the momentum dependent soft potential defined by \( A = 0.120 \) GeV, \( B = 0.151 \) GeV, \( \tau = 1.23 \), \( C = -0.0645 \) GeV, \( \Lambda = 2.167 \) GeV. The BRoBUU code propagates in the baryon sector the nucleons and 24 \( \Delta \) and \( N^\prime \) resonances and additionally \( \pi, \eta, \sigma, \omega \) and \( R \) mesons. Different particle species are coupled by the collision integral \( C_{ij} \), which also contains the Uhling-Uhlenbeck terms responsible for Pauli blocking of spin-1/2 baryons in the collision as well as particle creation and annihilation processes.

\[ \frac{d\tilde{c}}{dt} = \frac{1}{1 - C \frac{1}{2} m_i^2 + m_0^2} \left( 2 \frac{c}{\tilde{c}} + \frac{\partial}{\partial x} Re \Sigma \tilde{c} + \frac{m_i^2}{\tilde{c}} Re \Sigma \tilde{c} - \frac{\partial}{\partial x} Re \tilde{c} \right), \]

\[ \frac{d\tilde{p}}{dt} = -\frac{1}{1 - C \frac{1}{2} m_i^2 + m_0^2} \left( \frac{\partial}{\partial x} Re \Sigma \tilde{p} + \frac{m_i^2}{\tilde{p}} Re \Sigma \tilde{p} - \frac{\partial}{\partial x} Re \tilde{p} \right), \]

\[ \frac{dE}{dt} = \frac{1}{1 - C \frac{1}{2} m_i^2 + m_0^2} \left( \frac{\partial}{\partial x} Re \Sigma \tilde{E} + \frac{m_i^2}{\tilde{E}} Re \Sigma \tilde{E} - \frac{\partial}{\partial x} Re \tilde{E} \right), \]

Equations of motion for test particles follow from the transport equation. We use the relativistic version of the equations which have been derived in Ref. [37]:

For bosons the spectral function is related to the self-energy \( \Sigma \) via

\[
A(p) = \frac{\tilde{\Gamma}(x,p)}{(E^2 - \tilde{p}^2 - m_0^2 - Re \Sigma^\prime(x,p))^2 + \frac{1}{4} \tilde{\Gamma}^2(x,p)^2},
\]

where the resonance widths \( \Gamma \) and \( \tilde{\Gamma} \) obey \( \tilde{\Gamma}(x,p) = -2 Im \Sigma^\prime \approx 2m_0 \Gamma \), and \( m_0 \) is the vacuum pole mass of the respective particle. The spectral function (3) is normalized as \( \int dp^2 A = 2\pi \). (We omit here the label denoting the particle type for simplicity).

To solve numerically the Kadanoff-Baym equations one may exploit the above test-particle ansatz for a modified retarded Green function (see Refs. [37, 38]). This function can be interpreted as a product of particle number density multiplied with the spectral function \( A \). The spectral function can significantly change in the course of the heavy-ion collision process. Therefore, the standard test-particle method, where the test-particle mass is a constant of motion, must be extended by treating the energy \( E = p^0 \) of the four-momentum \( p \) as an independent variable.

Equations of motion for test particles follow from the transport equation. We use the relativistic version of the equations which have been derived in Ref. [37]:

\[
\frac{d\tilde{c}}{dt} = \frac{1}{1 - C \frac{1}{2} m_i^2 + m_0^2} \left( 2 \frac{c}{\tilde{c}} + \frac{\partial}{\partial x} Re \Sigma \tilde{c} + \frac{m_i^2}{\tilde{c}} Re \Sigma \tilde{c} - \frac{\partial}{\partial x} Re \tilde{c} \right),
\]

\[
\frac{d\tilde{p}}{dt} = -\frac{1}{1 - C \frac{1}{2} m_i^2 + m_0^2} \left( \frac{\partial}{\partial x} Re \Sigma \tilde{p} + \frac{m_i^2}{\tilde{p}} Re \Sigma \tilde{p} - \frac{\partial}{\partial x} Re \tilde{p} \right), \]

\[
\frac{dE}{dt} = \frac{1}{1 - C \frac{1}{2} m_i^2 + m_0^2} \left( \frac{\partial}{\partial x} Re \Sigma \tilde{E} + \frac{m_i^2}{\tilde{E}} Re \Sigma \tilde{E} - \frac{\partial}{\partial x} Re \tilde{E} \right),
\]
with the renormalization factor

\[ C = \frac{1}{2E} \left( \partial_t \sqrt{\frac{m^2 - p^2 - \Sigma^{\text{ret}}}{\Gamma}} \partial_t \sqrt{\frac{m^2 - p^2 - \Sigma^{\text{ret}}}{\Gamma}} \right). \]  

(7)

In the above, \( m = \sqrt{E^2 - p^2} \) is the mass of an individual test-particle of a given hadron species. The self-energy \( \Sigma^{\text{ret}} \) is considered to be a function of density \( n \), energy \( E \), and momentum \( \vec{p} \); thus the dependence on time and position comes only from its density dependence. Partial derivatives with respect to any of the four variables, \( t, \vec{x}, E, \vec{p} \), are understood by fixing the three other ones. The quantity \( 1 - C \) has been introduced to ensure that the test particles describe a conserved quantity [38].

The change of the test-particle mass \( m \) can be more clearly seen combining Eqs. (5) and (6) to

\[ \frac{dm^2}{dt} = \frac{1}{1 - C} \left( \frac{d}{dt} \sqrt{\frac{m^2 - \Sigma^{\text{ret}}}{\Gamma}} \right), \]  

(8)

with the comoving derivative \( d / dt = \partial_t + \vec{p} / E \partial_{\vec{x}} \). This equation means that the square of the particle mass tends to reach a value shifted by the real part of the self-energy within a range of the value of \( \Gamma \). Thus, the vacuum spectral function is recovered when the particle leaves the medium. This ensures the smooth transition from the in-medium behavior to the vacuum properties, as discussed in detail in [22, 38].

The equations of motions of the test particles have to be supplemented by a collision term which couples the equations for the different particle species. It can be shown [38] that this collision term has the same form as in the standard BUU treatment.

The off-shell transport has been implemented in simulations for the propagation of \( \rho \) and \( \omega \) mesons to study the di-electron production in \( \gamma A \) and \( p \Lambda \) reactions. Early approaches [39, 40] did not automatically provide the correct asymptotic behavior of the spectral function and an auxiliary potential was introduced to cure this problem. The above equations of motion, which do not have this deficit, were applied to di-electron production in \( p \Lambda \) collisions in [41] and for \( AA \) collisions in [22]. For further developments of transport models we refer the interested reader to [42, 43].

### 2.3. Self-Energies

To solve the Eqs. (4)-(6) one needs the knowledge of the self-energies. Here one faces the need to decide which effects to take into account in the expression for the retarded self-energy \( \Sigma^{\text{ret}} \) in the medium. That is because the BUU transport equations themselves already contain some part of in-medium effects that usually are considered in theoretical models in local density and local equilibrium approximations [44-46]. For instance models for in-medium effects of \( \rho \) mesons usually take into account the \( N(1520) \)-nucleon-hole loop for the self-energy, the corresponding vertices are accounted for in BUU via \( \rho \)-nucleon scattering and absorption through the \( N(1520) \) resonance.

In our calculations we employ a simple schematic form of the self-energy of a vector meson \( V \) which accounts for density effects:

\[ Re \Sigma^{\text{ret}}_V = 2m_v \Delta m_v \frac{n}{n_0}, \]  

(9)

\[ Im \Sigma^{\text{ret}}_V = m_v \left( \frac{\Gamma^{\text{vac}}_V + \frac{m_v \sigma}{\sqrt{1 - v^2}}}{\Gamma^{\text{ret}}_V} \right). \]  

(10)

(The Lorentz factor in the last equation emerges from the time transformation from the laboratory frame to the comoving frame, since the width corresponds to the lifetime in the eigenframe. At given energies and for the massive vector mesons the velocity \( v \) stays sufficiently below unity.)

Equation (9) causes a "mass shift" \( \Delta m_v = \sqrt{m_v^2 + Re \Sigma^{\text{ret}}_V} - m_v \), roughly proportional to the density \( n \) of the surrounding matter. The imaginary part contains the vacuum width \( \Gamma^{\text{vac}}_V \), the energy dependence of which is described by a form factor [47]. For instance, for \( R \to NM \) decays we use the parameterization

\[ \Gamma(q) = \Gamma \frac{M q}{\mu} \left( \frac{q^2}{q_t^2 + \delta^2} \right)^{2(\ell + 1)} \left( \frac{q^2 + \delta^2}{q^2 + \delta^2} \right)^{\ell+1}, \]  

(11)

where \( \ell \) is the angular momentum of the meson, \( q \) is the momentum of the meson in the resonance cm system, \( r \) index refers to the quantities at the resonance peak and \( \delta = 0.3 \) GeV. \( \Gamma^{\text{vac}}_V \) is accordingly given by Eq. (11) with \( \ell = 1 \) for vector mesons.

The second term in Eq. (10) results from the collision broadening which depends on density, relative velocity \( v \) and the cross section \( \sigma_v \) of the vector meson in matter. This cross section \( \sigma_v \) is calculated via the Breit-Wigner formula

\[ \sigma_v = \frac{4\pi}{q_{\text{in}}} \sum \frac{2J + 1}{2J + 1} \frac{s \Gamma^{\text{vac}}_V \Gamma^{\text{ret}}_V}{(s - m^2)^2 + s \Gamma^{\text{ret}}_V} 2J + 1 \]  

(12)

for forming resonances with masses \( m_R \), angular momenta \( J_R \), partial widths \( \Gamma_{v,R} \), total widths \( \Gamma^{\text{ret}}_V \) with energy \( \sqrt{s} \) and relative momentum \( q_{\text{in}} \) in the entrance channel. In vacuum the baryon density \( n \) vanishes and the resulting spectral function \( A_{\text{vac}} = A(n = 0) \) is solely determined by the energy dependent width \( \Gamma^{\text{vac}}_V \). We remark that the decay of a test particle is, furthermore, reduced by the absorption during the two-particle collisions which characterizes the total width \( Im \Sigma^{\text{ret}}_V \).

In our actual numerical implementation we assume that the spectral function of the \( \rho \) (\( \omega \)) meson vanishes below two (three) times the pion mass, respectively. (For a discussion of this issue see, for instance, [48]). If a \( \rho \) meson is generated at normal nuclear matter density \( n_0 \), its mass is distributed in accordance with the spectral function (see Eq. (17) below). If the meson propagates into a region of higher density then the mass will be lowered according to the action
of $\text{Re} \Sigma^{\text{tot}}$ in Eq. (8). However if the meson comes near the threshold the width $\Gamma$ becomes small and the second term of the right hand side of Eq. (8) dominates and reverses this trend leading to an increase of the mass.

The life time of unstable particles is also accessible in the framework of the transport equations for resonances. As it was shown in Ref. [49] this description leads to a life time $\tau = d\delta / dE$, where $\delta$ is the energy dependent scattering phase in the formation or the respective decay of the resonance. Although this relation is known for a long time [50] it was introduced only recently in the context of a BUU transport treatment [51]. This prescription is very different from the commonly used formula $\tau = h / \Gamma$. Especially if the resonance is a $p$ wave resonance the life time tends to small values near the threshold in the former case, while it approaches large values in the latter one. If the resonance decays into several channels, the total width is the relevant quantity which describes the phase of the amplitude common to all decay channels:

$$\tan \delta = -\frac{1}{2} \frac{\Gamma}{p^2 - M^2_0 - \text{Re} \Sigma^{\text{tot}}}.$$  \hspace{1cm} (13)

Therefore, the decay rate into a special channel $c$ is given by the partial width $\Gamma_c = b_c \Gamma^{\text{tot}}$ according to

$$\tau^{-1} = b_c \Gamma^{\text{tot}} = b_c (d\delta / dE)^{-1}$$ \hspace{1cm} (14)

with $b_c$ being the branching ratio of the decay into channel $c$. If we do not mention otherwise, we use the standard prescription for the life time, $\tau = h / \Gamma$, but in some cases we study the effect of using Eq. (14), as well.

### 2.4 Particle Production

In most instances, a vector meson $V$ is created by the decay of a baryon resonance $R$ in the BrOBUU code. Thus, mesons are created in two-step processes like $NN \leftrightarrow NR$ with subsequent decay $R \leftrightarrow VN$. As mentioned above the BrOBUU model includes 24 non-strange baryon resonances. Their parameters (mass, width and branching ratios) are determined by a global fit to pion-nucleon scattering data, while resonance production cross sections are fitted to inelastic nucleon-nucleon scattering cross sections [47]. Since there are very few $n\rho \rightarrow RN$ data we assume that

$$\sigma_{n\rho \rightarrow RN} = a \sigma_{n\rho \rightarrow RN}^\text{tot},$$ \hspace{1cm} (15)

where $a$ is a channel independent (except one, see below) constant, and its value $a = 1.34$ is obtained from a fit to the few existing data. We use that prescription for all resonances (R) except for N(1535), the main source of $\eta$ meson, where experimental data indicate a much higher value of $a_{N(1535)} = 5$.

Our approach is in contrast to other ones (e.g. [22, 23]) where individual elementary hadron reaction channels are parameterized independently from one another. Using such a coupled channel approach could allow us to obtain cross sections for not measured or poorly measured channels.

The in-medium spectral functions of $\omega$ and $\rho$ mesons also have to be taken into account when their test particles are created. In the resonance decay the mass distribution of the generated test particles for mesons results from an interplay of phase-space effects and the in-medium spectral functions $A$ of the created meson. For the decay of a resonance of mass $m_R$ in a meson of mass $m$ and a baryon of mass $m_N$ we use the phase space distribution in the final state with a constant matrix element squared $|M|^2$

$$\Gamma = N \int dp_\pi \delta(p^2_\pi - m^2_R) \int dp_\nu \frac{1}{2\pi} A(p_\nu) |M|^2$$ \hspace{1cm} (16)

from which the distribution

$$\frac{dN^{R \rightarrow VN}}{dm_\nu} = N m_\nu \lambda^{1/2}(m^2_R, m^2_R, m^2_N) A(m_\nu)$$ \hspace{1cm} (17)

results, where $\lambda$ is the triangle function $\lambda(a^2, b^2, c^2) = (c^2 - a^2 - b^2)^2 - (2ab)^2$. $N$ is an appropriate normalization factor.

We also include meson emission during a transition $R \rightarrow R'V$ from a resonance state $R$ to another resonance $R'$ with $R' = \Lambda(1232)$, $N(1440)$, $N(1520)$, $N(1535)$.

In Fig. (1) we compare the cross section calculated with our parameter set with data measured by the collaborations

![Fig. (1)](image-url) Production cross sections of $\omega$ (left), $\rho^0$ (middle) and $\eta$ (right) mesons in $pp$ collisions as a function of the excess energy in comparison with data [52-60].
SATURN, COSY and DISTO [52-60]. The relevant range for collisions in the 1 - 2 AGeV region is at excess energies below 0.5 GeV; very nearly the respective threshold, however, the cross sections are tiny and do not contribute noticeably [61]. We recognize that the $\omega$ production in $pp$ collisions is well reproduced by our model parameters. The $pn \to p\pi^0$ cross sections are about 1.5 times larger than the $pp$ cross sections. The one-boson exchange model in [62] predicts even a ratio of two.

For the $\rho^0$ production near threshold there are not many $pp$ measurements. Such measurements are hampered by the large $\rho$ width which make it difficult to discriminate $\rho$ mesons from sequential two pion emission. At an excess energy of 0.33 GeV a $\rho^0$ cross section of $23 \pm 9 \mu b$ [56] has been measured, where $\rho^0$ mesons were identified by pion pairs with masses above 0.6 GeV. The employed global fit including many elementary channels overestimates this cross section by a factor of 2.3. As a consequence we will reduce the $\rho$ production by this factor in the following.

With respect to $\eta$ production our model describes well the production in $pn$ collision, however it seems to underestimate the production in $pp$ collisions. Since the cross sections in $pp$ collisions are anyhow smaller than those of $pn$ collisions, this fact will not seriously affect our final results.

Furthermore, the $\rho$ mesons can also be created in pion annihilation processes $\pi^+\pi^-\to\rho$ (see below). The dominating channel, however, is $\pi N \leftrightarrow \rho N$ via an intermediate resonance $R$, as mentioned above.

### 2.5. Di-Electron Production

The di-electron production from direct vector meson decays $V \to e^+e^-$ is calculated by integrating the local decay probabilities along their trajectories in accordance with Eq. (14). The branching ratios $b_i$ of the vector mesons are taken from experimental data at their pole masses. The mass dependence of this branching ratio is assumed to behave proportional to $m_v^3$ in accordance with the vector meson dominance model, where the photon propagator leads to a $m_v^3$ dependence and the phase space of decaying virtual photon modifies this to $m_v^3$.

The subleading so-called direct channel $\pi\pi \to \rho \to e^+e^-$ is treated with the $\rho$ meson formation cross section

$$\sigma(M) = \frac{\pi}{3p^2} 2m_\rho \Gamma_\rho(p)\Lambda_\rho$$

with $m_\rho$ being the actual pole mass of the $\rho$ meson and $\Gamma_\rho(p)$ the vacuum width of the $\rho$ resonance. The in-medium effects are encoded in the spectral function $\Lambda_\rho$.

The $\rho$ meson produces di-electrons with a rate of

$$\frac{dN}{dt} = \left(\frac{m_\rho^3}{m_\rho}\right) b_i \Gamma_\rho(p).$$

We also include into our simulations a bremsstrahlung contribution which is guided by a one-boson exchange model adjusted to $pp$ virtual bremsstrahlung and transferred to $pn$ virtual bremsstrahlung [63]. Actually, we use

$$\frac{d\sigma}{dM} = \frac{\sigma_\omega \alpha^2}{M} \int \frac{d^4q}{q_0} R_\omega(s).$$

(20)

Here $M$ is again the $e^+e^-$ invariant mass, $R_\omega$ denotes the two-particle phase space volume, $\sqrt{s}$ stands for the c.m.s. energy; $s = M^2 + 2q_0\sqrt{s}$ (with $q_0$ as the di-electron energy in the c.m.s.) is the reduced energy squared after the di-electron emission, and $\sigma_\omega(s)$ is the transverse cross section. Equation (20) can be approximated:

$$\frac{d\sigma}{dM} = \frac{\alpha^2 \sigma_{tot} s - (m_1 + m_2)^2}{3\pi^2} |\ln(\frac{q_{max}^2 - q_{min}^2}{M^2})|.$$  

(21)

This approximation, employed in [23] too, is applied to $pn$ and $\pi N$ collisions using the respective corresponding total cross sections $\sigma_{tot}$; $e_m$ stands for the energy of the charged particle in the rest system of the colliding particles with masses $m_1$ and $m_2$, $q_{max}^2 = (s + M^2 - (m_1 + m_2)^2)/2\sqrt{s}$ is the maximum di-electron energy, and $q_{max}^2 = \sqrt{q_{min}^2 - M^2}$ denotes the maximum di-electron momentum. It should be noted, however, that this cross section is still rather uncertain and needs experimental control.

An essential di-electron contribution comes from the Dalitz decays of $\pi^0$, $\eta$, $\omega$ mesons and the excited baryon resonances emitting a di-electron together with a photon or nucleon. The decay rate $\Gamma_{Dal}$ for a di-electron of mass $M$ for mesons can quite generally be brought into the form [64].

$$\frac{d\Gamma_{Dal}}{dM} = \frac{4\alpha}{3\pi M} \Gamma^\rho \left(1 - \frac{M^2}{m_\rho^2}\right)^3 F(M)^2.$$  

(22)

The Dalitz decay rates are assumed to be given by the photon partial width $\Gamma^\rho$, which have been taken from experiment [65]. The relevant form factors $F(M)$ for the mesons considered are summarized in [66].

The Dalitz decay of the baryon resonances is treated as in [67]. The most important contribution to the di-electron spectra of these come from the $\Delta(1232)$ resonance. There are also other models [32, 66, 68] for the Dalitz decay. As can be seen on the left panel of Fig. (2), these models agree very well for resonance decays from the peak mass, however they differ substantially for $\Delta$ resonances with energies (masses) relevant for the vector meson region. There is another uncertainty concerning these high-energy (mass) $\Delta(1232)$ resonances. The width and consequently the spectrum of these resonances are sensitive on the cut-off for high masses. Here we show two possible parameterizations (see right panel of Fig. 2): one from Moniz [69] (solid curve) and the other one from Manley (dotted curve) used in the Particle Data Book [65].
The number of Λ(1232) resonances at energy (mass) around 1.85 GeV may depend on the cut-off prescription by a factor of 3. Their Dalitz decay (see left of Fig. 2) panel may differ by a factor of 4. So the Dalitz decay contribution of the Λ(1232) resonance is uncertain by more than an order of magnitude in the vector meson region. This uncertainty may only be clarified by a detailed comparison of the calculation for $pp \rightarrow ppe^+e^−$ with forthcoming experimental data. Using their angular dependence one can localize the different channels and then fix their magnitude. Here we would like to mention that different groups use different prescriptions for that channel which is one reason why the predictions, especially for the Λ(1232) Dalitz decay contribution, are different. We follow [67] and, accordingly, may somewhat underestimate the di-electron contribution from the Λ channel.

3. RESULTS FOR 2 AGeV

3.1. Spectral Function Dynamics

We employ the above described code for the reaction C(2 AGeV) + C, where data from HADES are at our disposal [10]. In the present exploratory study we are going to contrast simulations with and without medium modifications of ρ and ω mesons to elucidate to which degree medium effects may become visible in the light collision system under consideration. In doing so we use fairly schematic medium effects (to be considered as an extreme upper limit) condensed in a “mass shift” described by the above parameter $Δm_ω = −50$ MeV in Eq. (9) for the ω meson. Previous CB-TAPS data [70] suggested indeed such a $m_ω$ mass shift. (See, however, [6] and [71] for a critical discussion of this data). This problem is also investigated experimentally [72] and theoretically [44, 73-75] in calculating the ω spectral function. The use of QCD sum rules [76] then can be utilized to translate this shift into a significantly larger shift for the ρ meson (dictated essentially by the Landau damping term); we use here $Δm_ρ = −100$ MeV. We are aware of experiments as reported in Ref. [77-79] which do not observe a noticeable shift of the ω meson excitation strength. Nevertheless, several theoretical attempts are made to predict a possible ρ ”mass shift” during the last decade. Many of them predict a fairly large shift of strength of ρ excitation to lower energy [4, 5], see also [44, 46, 48, 80-83]. Thus, we keep this (presumably too large a) value to illustrate whether it would have a significant imprint on the observed spectra.

Given the present experimental situation [1, 2, 6, 77-79] of not yet having found a solid hint to significant mass shifts we also discuss the option $Δm_ω = 0$, i.e. collision broadening only.

The spectral function for ρ and ω mesons are shown in Fig. (3) at two different densities of nuclear matter and two meson velocities in comparison with the vacuum spectral function. We would like to emphasize the strong velocity dependence of the widths, in particular for the ω mesons.

Fig. (2). Left panel: Dalitz- decay of Λ(1232) for two energies (masses) $m_λ$ (lower set: 1.232 GeV, upper set: 1.85 GeV) according to the prescriptions [32] (“Wolf”; used, e.g., in [23]), [67] (“ZW”), [68] (“Krivo.”; used, e.g., in [24-26]), and [66] (“Ernst”, used, e.g., in [22]). Right panel: spectral function with two different cut-off prescriptions according to [69] (“Moniz”, solid curve) and [65] (“Manley”, dotted curve).

Fig. (3). Influence of medium effects on the spectral functions for $ρ$ (left panel) and $ω$ (right panel) mesons at various nuclear matter densities and velocities in matter.
Despite the excitations of various baryon resonances the spectral functions appear as relatively smooth distributions.

Let us now consider the effect of the mass evolution of the \( \omega \) and \( \rho \) vector mesons given by the equations of motion Eqs. (4)-(6). The ensemble of test particles of mesons is generated in dense matter where their masses are distributed in accordance with their broadened and mass shifted spectral function. In Fig. (4) we show the time evolution of a small ensemble of \( \omega \) test particles. In the calculations the nuclei touch each other at a time of 2.5 fm/c while the density peaks at about 6 fm/c and drops at 8 fm/c below saturation density (see lower part of the figure). At maximum density most of the vector mesons are created, afterwards the mass distribution gets narrower. Only a few of the \( \omega \) mesons decay in the dense phase where their masses deviate strongly from the pole mass value. If a low density is reached the vacuum spectral function dominates the di-electron decays which leads to a sharp peak at the pole mass. The right hand part of Fig. (4) shows the test particles with a spectral function with both the mentioned mass shift and collision broadening, while the distribution on the left hand part is calculated for vacuum spectral functions. Note the apparent squeezing of \( m_\omega \) for the in-medium case which is caused by the reduced masses since the main decay channel is \( \omega \to \rho \pi \), and the \( \rho \)'s are populating the tail of their distribution.

Fig. (5) displays the analog behavior of the test particles of \( \rho \) mesons. In the high density stage one recognizes the large spread of the \( \rho \) masses of about 300 MeV due to the imaginary part of the self-energy. Most of the \( \rho \) mesons decay rapidly (life time \( \tau < 2 \) fm/c). If we use the relation \( \tau = h/\Gamma \) for the life time then essentially the high-mass particles with their large width decay rapidly and, hence, have little chances to radiate di-electrons during their short life time. They look like flashes occurring only in a narrow time interval, thus not causing longer paths of adjacent points in the mass vs. time plot. New mesons are readily created at later times and lower densities. A few low-mass \( \rho \) mesons survive these periods and can still be found at 25 fm/c. In case of \( \Delta m_\rho = 0 \) there is no change of the real part of the self-energy. The imaginary part due to collision broadening is small because of the smallness of the cross section for low masses, and its gradient is even smaller. This all together causes only tiny changes of the mass. The same holds for test particles with masses nearby the assumed threshold, where the width goes to zero, see middle panel of Fig. (4). Therefore, one expects a shift of the di-electron spectra to lower masses. Quite a different picture shows the right hand panel where the life time is calculated accordingly to Eq. (14). Here the low mass particles have a shorter life time than the more massive ones.

Fig. (5). Time evolution of the masses of about 400 test particles of \( \rho \) mesons in a \( C + C \) collision at 2 AGeV kinetic beam energy at an impact parameter of 1 fm. The particles in the left panel are calculated with a spectral function without a mass shift while the middle and the right ones the indicated mass shift were used. Note the different prescription for the life time indicated in the legends.
In Fig. (6) we exhibit the resulting di-electron spectra from \( \rho \) and \( \omega \) decays for the different prescriptions of the life time. The green dash-dotted curve shows the \( \rho \) and \( \omega \) spectra if the masses of the test particles are kept fixed to the values at the instant of creation. In this case the di-electron spectrum reflects the initial mass distribution which contains the high density spectral function. The effect is best visible for \( \omega \) mesons near the pole mass: here, the peak at the pole mass would nearly disappear when disregarding the mass evolution. In fact, the sharp \( \omega \) vacuum peak (red dashed curve) is down shifted and broadened due to averaging over some density interval (cf. Eq. (9)). This is in agreement with the schematic model in [86] for a narrow resonance. However, the time evolution of the off-shell propagation pushes the resulting di-electron spectra towards their vacuum spectral function. If the life time of the vector mesons follows the standard expression \( \tau = \hbar / \Gamma \) then the low-mass vector mesons have sufficient time to reach their pole mass. This behavior is also clearly seen (solid curve) for the \( \omega \) meson. However, if Eq. (14) controls the decay this shift is hindered by an earlier decay of the low mass mesons, see blue dotted curve. The peak position for di-electrons stemming from direct \( \omega \) decays is nevertheless the same as for a vacuum spectral function. Only for significantly heavier collision systems, which achieve much longer lasting higher maximum compressions, the in-medium modifications may become sizeable (see subsection 3.3).

The direct decays \( \rho \rightarrow e^+ e^- \) reflect to some extent the distribution of \( \rho \) excitations: even for a vacuum spectral function a broad distribution emerges. With the assumed strong mass shift, strength is shifted to lower invariant masses, most notably for the life time prescription Eq. (14). Also the "static" scenario (green dash-dotted curve) points to a strong down shift of strength. All together, the various scenarios differ by changes within a factor of two. For heavier collision systems the differences become more pronounced, see subsection 3.3.

Up to now we have contrasted the two extreme scenarios "vacuum" with vacuum spectral functions and "matter" with spectral functions including both the assumed strong mass shifts and the collision broadening. The more realistic option of "collision broadening only" is employed in Fig. (6) for two life time prescriptions which cause small differences in the di-electron yields. In what follows we contrast the two extreme scenarios "vacuum" and "matter" which are expected to bracket the realistic case.

Finally we investigate the distribution of the emitted di-electrons as a function of the density of the emitting region.

![Fig. (6).](image)

**Fig. (6).** Di-electron spectra from direct decays of \( \omega \rightarrow e^+ e^- \) (left panel) and \( \rho \rightarrow e^+ e^- \) (right panel) mesons calculated with different assumptions for the dynamics and the spectral functions. "vacuum": vacuum spectral function, "matter": collision broadening and mass shifts with two different assumption of the life time, "static": in-medium spectral function while the mass evolution of the mesons is switched off.

![Fig. (7).](image)

**Fig. (7).** As Fig. 6 but for collision broadening only. Two life time prescriptions are employed (dashed curves: \( \tau = 1 / \Gamma \), dot-dashed curves: \( \tau = d\bar{\sigma} / dE \)).
We consider the effect of the $\omega$ mesons in three different density regions: (i) the density $n < n_0 / 3$, (ii) $n_0 / 3 < n < n_0$, and (iii) $n > n_0$. For the light C + C system di-electrons from all regions have similar masses and therefore can be hardly disentangled in experiment, see Fig. (8). Let us first consider the scenario with the extreme mass shift of -50 MeV and life time prescription $\tau = h / \Gamma$. The $\omega$ peak (blue curve) is determined by $e^+e^-$ stemming from $\omega$ decays at density smaller than one third of saturation density (read dashed curve). The contribution from densities between one third and saturation density (green dotted curve) shows clearly the expected mass shift, but its contribution at maximum is less than 10% of the total yield, i.e. when including the other di-electron sources considered in subsection 3.2. The yield from the high compression stage (red dash-dotted curve) is again a factor of ten smaller.

The situation changes qualitatively for the life time prescription according to Eq. (14), see right panel in Fig. (8).

However, the down shifted strengths are hidden under the much stronger yields from other sources considered in the next subsection.

### 3.2. Comparison with HADES Data

While Figs. (4 – 8) refer to the central point of our work, we now look at how the evolving $\rho$ and $\omega$ spectral functions compare with data. In doing so the other di-electron sources have to be included. The obtained di-electron spectra are represented in Figs. (9) and (10). In Fig. (9) the results exhibited are obtained by including the above described pole-mass shifts as an additional medium modification of $\rho$ and $\omega$ mesons. In Fig. (10) only the vacuum spectral function is employed. For comparison with the data, the HADES filter has been applied [84] accounting for the geometrical acceptance, momentum cuts and pair kinematics. The filter causes a reduction of the strength and a smearing of the invariant masses of the di-electrons. The result of this filtering is always shown on the right hand panel of the figures.

![Fig. (8).](image1.png) **Fig. (8).** Contribution to the di-electron yield from $\omega$ mesons in various density regions compared with the total yield (thick solid line labelled by "total"). The left picture is calculated with the standard life time for $\omega$ mesons, while the right panel shows the effect when using the life time $\tau = d\delta / dE$.

![Fig. (9).](image2.png) **Fig. (9).** Di-electron invariant mass spectra for C(2 AGeV) + C calculated with in-medium ("matter") spectral functions. Individual contributions are depicted. Left panel: Full phase space. Right panel: With experimental filter [84] and compared to HADES data, cf. [10] (two data points at very low invariant masses have been discarded).
In these figures we show various contributions to the di-electron rate. Important low-mass di-electron sources are $\pi^0$ and $\eta$ Dalitz decays which are proportional to the multiplicities of their parents. The TAPS collaboration has measured [85] the $\pi^0$ and $\eta$ production cross sections of $707 \pm 72$ mb and $25 \pm 4$ mb which have to be compared to our calculations of $870$ mb and $23$ mb in the same reaction at the same energy. While the values for pion production are overestimated the $\eta$ production is quite in agreement with the data. (Note that the presently employed cross sections rely on a global fit of many elementary reactions which is not optimized for special channel). The Dalitz decays of $\rho$ and $\omega$ mesons and nucleon resonances do not contribute noticeably.

Comparing Figs. (9) and (10), the mass shifts of the vector mesons do not have a noticeable effect on the overall shape of the di-electron spectra. In particular, the contribution of the cocktail of the other sources mask the effect of the $\rho$ mesons. Furthermore, most of the $\omega$ mesons decay outside the dense zone and are therefore not very sensitive for medium effects. Since the fine structure ($\omega$ peak) is not yet resolved in the data a conclusive decision cannot be made.

Progress could be made if the di-electron mass resolution is improved to identify the $\omega$ peak. However, our calculations do not point to the possibility of a two-peak structure (resulting from a superposition of vacuum decays and in-medium decays) or a substantial smearing of the $\omega$ peak due to a density dependent shift analog to the consideration of the $\phi$ meson (see [86]).

The present set-up provides a reasonable description of the HADES data [9] for di-electron masses below 0.6 GeV. In the higher mass region some overestimation of the data is recognized, as also found in [22, 23, 27]. With respect to the uncertainties of the cross section $pp \rightarrow ppp$ at threshold and generally the $pn$ channel as well as the role of the resonance channels one could try to improve the agreement by rescaling the $\rho$ and $\omega$ contributions. In doing so one could assume unaltered spectral shapes. In fact, agreement with data is achieved by decreasing artificially the cross sections for vector meson production by factors 0.2 for $\omega$ and 0.8 for $\rho$.

The transverse momentum spectra for three invariant mass bins are exhibited in Fig. (11). One recognizes a good agreement with these multi-differential data with the exception of the low-$p_t$ region for invariant masses $M > 0.55$ GeV.

3.3. Effects in Larger Collision Systems

The C $+$ C system is rather light and a consequence of it is that the maximum density is about 2.5 $n_0$ (see Fig. 4). A heavy system has, in particular, a longer living high-density stage reaching densities of about 3.5 $n_0$. In Fig. (12) we display the result of our calculations for a collision of Au $+$
Au at 2 AGeV. Comparing the left and the right hand part one recognizes the larger effect of the assumed pole mass shifts of the mesons in contrast to the C + C case shown in Figs. (9) and (10). Furthermore, it is clearly seen that the amount of di-electrons coming from \( \rho \) mesons is relatively larger than for light systems. This can be understood by the fact that during the longer collision time \( \rho \) mesons can rapidly decay and regenerate.

The effects discussed with respect to Fig. (6) (life time prescription, vacuum vs. in-medium ("matter") spectral functions, time evolution of spectral functions) are more clearly seen for a larger collision system. Fig. (13) shows the dramatic change of the di-electron spectra emitted from \( \rho \) and \( \omega \) mesons in central collisions Au + Au for different assumptions on the spectral functions. The vacuum spectral function of the \( \rho \) meson (right panel) still shows the peak near the pole mass despite the \( m_\rho^3 \) dependence, which causes, together with the \( \rho \) excitation, the long flat tail towards the two-pion threshold implemented here. In case of medium modification the shape differs strongly from the vacuum one, with the exception of the standard life time prescription for \( \omega \), see left panel in Fig. (13).

Repeating the same analysis as in Fig. (8) (tracing back the contributions from various density stages) for central Au + Au collisions system we find that \( \omega \) decay di-electrons from the dense region (red dot-dashed curves) have low masses around 600 MeV and contribute roughly 10% to the total \( \omega \) yield (see Fig. 14). There is a remarkable difference between the outcome of the standard life time expression and...
Eq. (14): While for the standard life time prescription the assumed density dependent mass shift is clearly visible (see left panel in Fig. 14), and the prescription of Eq. (14) causes a strong down shift of strength, reducing eventually the peak height of the \( \omega \) signal.

4. RESULTS AT 1 AGeV

Recently new HADES data has become available also for C + C collisions at a bombarding energy of 1 GeV per nucleon [11]. The excess energy is about 450 MeV, thus only low energy tails of the \( \rho \) and \( \omega \) mesons play a role. Nevertheless the \( \rho \) mesons contribute essentially to the di-electron spectrum above an invariant mass of 500 MeV since other sources are even much smaller, see Figs. (15) and (16).

We obtain a reasonable agreement with the measured HADES [11] (Fig. 15) and DLS [7] (Fig. 16) data, but the data at an invariant mass around 400 MeV are underestimated. The shoulder in the data at 400 MeV could only be explained by a higher contribution of di-electrons coming from the Dalitz decay of the \( \eta \) mesons. We calculate a production cross section \( \sigma_\eta = 1.8 \text{ mb} \) which has to be compared to the value \( \sigma_\rho = 450 \text{ mb} \). The experimental values measured by [85], \( \sigma_\eta = 1.5 \pm 0.4 \text{ mb} \) and \( \sigma_\rho = 287 \pm 21 \text{ mb} \), give a ratio which is a factor of 1.3 larger than our calculated value. Such an increase of the \( \eta \) yield could hardly improve the total di-electron spectra and explain the shoulder at 400 MeV. (It should be noted that a rescaling of \( \pi^0 \) and \( \eta \) contributions in accordance with the TAPS data [85] would result in a better agreement with HADES data [11]). It should be emphasized that, for invariant masses around 350 MeV, the three contributions from \( \eta \) and \( \Delta \) decays as well as bremsstrahlung are comparable. A strong bremsstrahlung contribution in the \( pn \) channel has been advocated in [22] as a convincing solution to the "DLS puzzle". [23] used the same parametrization of bremsstrahlung as we employ here, cf. (21) which in turn is supported by the one-boson exchange model in [63].

Fig. (17) exhibits the transverse momentum spectra for three invariant mass bins. A good agreement with available data can be stated. The figure (cf. middle panel) exhibits again the competition of the mentioned three sources with a window at small \( p_T \), where the bremsstrahlung dominates.

5. SUMMARY

In summary we have considered the propagation of broad resonances within a kinetic theory (transport) approach to heavy-ion collisions. Vector mesons are described by spectral functions and these are evolved in space and time by a test-particle method. The motivation for this work is a new generation of data on di-electrons. The corresponding experiments are aimed at seeking imprints of chiral symmetry restoration as particular aspect of in-medium modifications of hadrons. This lets us focus on the treatment of \( \rho \) and \( \omega \) mesons. The wide-spread predictions call for an experimental clarification, but still heavy-ion data need often to be compared with models to extract the wanted information from data.

We have utilized here the transport equations from Ref. [37] which are approximations of the much more involved Kadanoff-Baym equations [93]. Compared to an approach...
where the spectral function is frozen in after creation the present framework let the spectral functions evolve rapidly towards the vacuum spectral functions. Therefore, the in-medium modifications are washed out, in particular, for the $\omega$ meson. In contrast to earlier expectations the $\omega$ peak does not suffer a significant modification, even when assuming a strong hypothetical shift of the peak position.

Within the employed framework, medium modifications of $\rho$ and $\omega$ mesons are hardly seen in the di-electron spectra of small collision systems, even when using fairly strong and schematic assumptions for them. Only heavy collision systems (which achieve somewhat higher compression at a significantly longer time interval) seem to allow us to still identify the wanted medium modifications. The elementary channels which contribute to the overall yields need better control to arrive at firm conclusions on interesting many-body effects.

**ACKNOWLEDGEMENTS**

We gratefully acknowledge the continuous information by the HADES collaboration, in particular R. Holzmann for delivering and assisting us in using the acceptance filter routines. The work is supported by the German BMBF 06DR136, GSI-FE and the Hungarian OTKA T48833 and T71989.

**REFERENCES**


Propagation of Vector-Meson Spectral-Functions


Received: October 08, 2009
Revised: July 23, 2010
Accepted: August 04, 2010

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