

A Solution to Non-Linear Equations of Motion of Nambu-Goto String

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Abstract: In this paper we solve the non-linear Lagrange's equations for the Nambu-Goto closed bosonic string.

We show that Ultradistributions of Exponential Type (UET) are appropriate for the description in a consistent way of string and string field theories.

We also prove that the string field is a linear superposition of UET of compact support (CUET), and give the notion of anti-string. We evaluate the propagator for the string field, and calculate the convolution of two of them.

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1. INTRODUCTION

In a series of papers [1-5] we have shown that Ultradistribution theory of Sebastiao e Silva [6-8] permits a significant advance in the treatment of quantum field theory. In particular, with the use of the convolution of Ultradistributions we have shown that it is possible to define a general product of distributions (a product in a ring with divisors of zero) that sheds new light on the question of the divergences in Quantum Field Theory. Furthermore, Ultradistributions of Exponential Type (UET) are adequate to describe Gamow States and exponentially increasing fields in Quantum Field Theory [9-11].

In three recent papers ([12-14]) we have demonstrated that Ultradistributions of Exponential type provide an adequate framework for a consistent treatment of string and string field theories. In particular, a general state of the closed string is represented by UET of compact support, and as a consequence the string field is a linear combination of UET of compact support.

Ultradistributions also have the advantage of being representable by means of analytic functions. So that, in general, they are easier to work with and, as we shall see, have interesting properties. One of those properties is that Schwartz's tempered distributions are canonical and continuously injected into Ultradistributions of Exponential Type and as a consequence the Rigged Hilbert Space with tempered distributions is canonical and continuously included in the Rigged Hilbert Space with Ultradistributions of Exponential Type.

Another interesting property is that the space of UET is reflexive under the operation of Fourier transform (in a similar way of tempered distributions of Schwartz).

In this paper we show that Ultradistributions of Exponential type provides an adequate tool for a consistent treatment of Nambu-Goto closed bosonic string. A general state of the closed Nambu-Goto string is represented by UET of compact support, and the corresponding string field is a linear combination of UET of compact support (CUET).

The motivation that inspired the writing of this paper has been that to quantum level the formulation of Polyakov's bosonic string is not equivalent to the Nambu-Goto string because $(L_0 - a)|\phi\rangle = 0$, $L_m|\phi\rangle = 0$ $m > 0$ and $L_m|\phi\rangle \neq 0$ for $m < 0$ (where L_m is the Virasoro operator and $|\phi\rangle$ is the physical state of the string). This implies that $T_{\alpha\beta}|\phi\rangle \neq 0$ and then the constraints are not satisfied by the theory because in order to satisfy $T_{\alpha\beta}|\phi\rangle = 0$ the Virasoro operators must meet $L_m|\phi\rangle = 0$ for all $m \neq 0$. ($T_{\alpha\beta} = 0$ are the classical constraints of the theory). As a consequence the solutions of the Polyakov string are not true solutions of the nonlinear equations of Nambu-Goto and the resulting theory is not equivalent to the original theory. Another problem presented by the Polyakov string is the presence of a tachyon in its ground state, whose quantification breaks unitarity and causality of the theory.

Moreover, in his book about strings [18], Green, Schwartz and Witten obtain in page 63 (in the proof about the equivalence of Nambu-Goto and Polyakov string)

$$G = \frac{1}{4} h (h^{\alpha\beta} G_{\alpha\beta})^2 \quad (1)$$

where

$$G = |\det G_{\alpha\beta}|$$

$$h = |\det h_{\alpha\beta}|$$

$$G_{\alpha\beta} = \partial_\alpha X_\mu \partial_\beta X^\mu$$

and then concludes

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$$\sqrt{G} = \sqrt{h} h^{\alpha\beta} G_{\alpha\beta} \quad (2)$$

and

$$\int_{\Sigma} \sqrt{G} d^2\sigma = \frac{1}{2} \int_{\Sigma} \sqrt{h} h^{\alpha\beta} G_{\alpha\beta} d^2\sigma \quad (3)$$

In Minkowskian space

$$\int_{\Sigma} \sqrt{|(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2|} d^2\sigma = \frac{1}{2} \int_{\Sigma} \dot{X}^2 - X'^2 d^2\sigma$$

The right hand side of (3) is the Polyakov action. But this is not strictly true because $\sqrt{G} d^2\sigma$ is the surface element of the world sheet. Indeed we have

$$\sqrt{G} = \frac{1}{2} \sqrt{h} |h^{\alpha\beta} G_{\alpha\beta}| \quad (4)$$

and then

$$\int_{\Sigma} \sqrt{G} d^2\sigma = \frac{1}{2} \int_{\Sigma} \sqrt{h} |h^{\alpha\beta} G_{\alpha\beta}| d^2\sigma \quad (5)$$

In Minkowskian space

$$\int_{\Sigma} \sqrt{|(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2|} d^2\sigma = \frac{1}{2} \int_{\Sigma} |\dot{X}^2 - X'^2| d^2\sigma$$

(If x is a real variable $+\sqrt{x^2} = |x|$) Note then that the equations of motion corresponding to (5) are non-linear. This was the reason why we decided to solve the non-linear Nambu-Goto equations directly.

This paper is organized as follows: In section 2 we solve the non-linear Lagrange's equations for closed Nambu-Goto bosonic string. In section 3 we give expressions for the field of the string, the string field propagator and the creation and annihilation operators of a string and a anti-string. In section 4, we give expressions for the non-local action of a free string and a non-local interaction lagrangian for the string field similar to $\lambda\phi^4$ in Quantum Field Theory. Also we show how to evaluate the convolution of two string field propagators. In section 5 we realize a discussion of the principal results. In Appendix A we define the Ultradistributions of Exponential Type and their Fourier transform. In them we give some main results obtained for us and other authors, used in this paper and show that Ultradistributions of Exponential Type are part of a Guelfand's Triplet (or Rigged Hilbert Space [15]) together with their respective dual and a "middle term" Hilbert space. In Appendix B we give a new representation, obtained in [12], for the states of the string using CUET of compact support.

2. THE CLOSED NAMBU-GOTO STRING

As is known the Nambu-Goto Lagrangian for the closed bosonic string is given by ([16, 17])

$$L_{NG} = T \sqrt{|(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2|} \quad (1)$$

where

$$\left\{ \begin{array}{l} X_{\mu} = X_{\mu}(\tau, \sigma); \dot{X}_{\mu} = \partial_{\tau} X_{\mu}; X'_{\mu} = \partial_{\sigma} X_{\mu} \\ X_{\mu}(\tau, 0) = X_{\mu}(\tau, \pi) \\ -\infty < \tau < \infty; 0 \leq \sigma \leq \pi \end{array} \right. \quad (2)$$

The corresponding action is:

$$S_{NG} = T \int_{-\infty}^{\infty} \int_0^{\pi} \sqrt{|(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2|} d\sigma d\tau \quad (3)$$

If we call

$$L_1 = (\dot{X} \cdot X')^2 - \dot{X}^2 X'^2 \quad (4)$$

The Euler-Lagrange equations are:

$$\begin{aligned} \frac{\partial}{\partial \tau} \left[Sgn(L_1) \frac{(\dot{X} \cdot X') X'_{\mu} - X'^2 \dot{X}_{\mu}}{\sqrt{|L_1|}} \right] + \\ \frac{\partial}{\partial \sigma} \left[Sgn(L_1) \frac{(\dot{X} \cdot X') \dot{X}_{\mu} - \dot{X}^2 X'_{\mu}}{\sqrt{|L_1|}} \right] = 0 \end{aligned} \quad (5)$$

Let X_{μ} be given by:

$$X_{\mu} = Sgn(\dot{Y}^2 - Y'^2) Y_{\mu} \quad (6)$$

where

$$\left\{ \begin{array}{l} Y_{\mu}(\tau, \sigma) = y_{\mu} + l^2 p_{\mu} \tau + \frac{il}{2} \sum_{n=-\infty; n \neq 0}^{\infty} \frac{a_n}{n} e^{-2in(\tau-\sigma)} \\ p^2 = 0 \end{array} \right. \quad (7)$$

or

$$\left\{ \begin{array}{l} Y_{\mu}(\tau, \sigma) = y_{\mu} + l^2 p_{\mu} \tau + \frac{il}{2} \sum_{n=-\infty; n \neq 0}^{\infty} \frac{\tilde{a}_n}{n} e^{-2in(\tau+\sigma)} \\ p^2 = 0 \end{array} \right. \quad (8)$$

Y_{μ} of (7) satisfy

$$\dot{Y}_{\mu} + Y'_{\mu} = p_{\mu} \quad (9)$$

and Y_{μ} of (8):

$$\dot{Y}_{\mu} - Y'_{\mu} = p_{\mu} \quad (10)$$

For both we have:

$$\dot{X}^2 - X'^2 = \dot{Y}^2 - Y'^2 \neq 0 \quad (11)$$

and then

$$L_1 = (\dot{X}^2 - X'^2)^2 = (\dot{Y}^2 - Y'^2)^2 \neq 0 \quad (12)$$

We shall prove that ((6), (7)) or ((6),(8)) are solutions of (5). From(6), (7) we have $\dot{X} = -\dot{X}' = X''$ and (5) transform into:

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X} \cdot X') X'_{\mu} - X'^2 \dot{X}_{\mu}}{\sqrt{|L_1|}} - \frac{(\dot{X} \cdot X') \dot{X}_{\mu} - \dot{X}^2 X'_{\mu}}{\sqrt{|L_1|}} \right] = \quad (13)$$

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X} \cdot X' + \dot{X}^2)X'_\mu - (\dot{X} \cdot X' + X^2)\dot{X}'_\mu}{\sqrt{|L_1|}} \right] = \quad (14)$$

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X}^2 - X^2)X'_\mu - (X^2 - \dot{X}^2)\dot{X}'_\mu}{2\sqrt{|L_1|}} \right] = \quad (15)$$

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X}^2 - X^2)(\dot{X}'_\mu + X'_\mu)}{2\sqrt{|L_1|}} \right] = \quad (16)$$

and finally

$$l^2 \frac{\partial p_\mu}{\partial \tau} = 0 \quad (17)$$

>From (6), (8) we have $\ddot{X} = \dot{X}' = X''$ and (5) transforms into:

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X} \cdot X')X'_\mu - X^2 \dot{X}'_\mu}{\sqrt{|L_1|}} + \frac{(\dot{X} \cdot X')\dot{X}'_\mu - \dot{X}^2 X'_\mu}{\sqrt{|L_1|}} \right] = \quad (18)$$

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X} \cdot X' - \dot{X}^2)X'_\mu + (\dot{X} \cdot X' - X^2)\dot{X}'_\mu}{\sqrt{|L_1|}} \right] = \quad (19)$$

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X}^2 - X^2)\dot{X}'_\mu + (X^2 - \dot{X}^2)X'_\mu}{2\sqrt{|L_1|}} \right] = \quad (20)$$

$$\frac{\partial}{\partial \tau} \left[\frac{(\dot{X}^2 - X^2)(\dot{X}'_\mu - X'_\mu)}{2\sqrt{|L_1|}} \right] = \quad (21)$$

$$l^2 \frac{\partial p_\mu}{\partial \tau} = 0 \quad (22)$$

At quantum level we have for (7):

$$\left\{ \begin{array}{l} Y_\mu(\tau, \sigma) = y_\mu + l^2 p_\mu \tau + \frac{il}{2} \sum_{n=-\infty; n \neq 0}^{\infty} \frac{a_{n\mu}}{n} e^{-2in(\tau-\sigma)} \\ p^2 |\phi\rangle = 0 \end{array} \right. \quad (23)$$

and for (8):

$$\left\{ \begin{array}{l} Y_\mu(\tau, \sigma) = y_\mu + l^2 p_\mu \tau + \frac{il}{2} \sum_{n=-\infty; n \neq 0}^{\infty} \frac{\tilde{a}_{n\mu}}{n} e^{-2in(\tau+\sigma)} \\ p^2 |\phi\rangle = 0 \end{array} \right. \quad (24)$$

where $|\Phi\rangle$ is the physical state of the string.

In terms of creation and annihilation operators we have:

$$\left\{ \begin{array}{l} Y_\mu(\tau, \sigma) = y_\mu + l^2 p_\mu \tau + \frac{il}{2} \sum_{n>0} \frac{b_{n\mu}}{\sqrt{n}} e^{-2in(\tau-\sigma)} - \frac{b_{n\mu}^+}{\sqrt{n}} e^{2in(\tau-\sigma)} \\ p^2 |\phi\rangle = 0 \end{array} \right. \quad (25)$$

$$\left\{ \begin{array}{l} Y_\mu(\tau, \sigma) = y_\mu + l^2 p_\mu \tau + \frac{il}{2} \sum_{n>0} \frac{\tilde{b}_{n\mu}}{\sqrt{n}} e^{-2in(\tau+\sigma)} - \frac{\tilde{b}_{n\mu}^+}{\sqrt{n}} e^{2in(\tau+\sigma)} \\ p^2 |\phi\rangle = 0 \end{array} \right. \quad (26)$$

where:

$$[b_{\mu m}, b_{\nu n}^+] = \eta_{\mu\nu} \delta_{mn} \quad (27)$$

$$[\tilde{b}_{\mu m}, \tilde{b}_{\nu n}^+] = \eta_{\mu\nu} \delta_{mn} \quad (28)$$

A general state of the string can be written as:

$$|\phi\rangle = [a_0(p) + a_{\mu_1}^{i_1}(p)b_{i_1}^{+\mu_1} + a_{\mu_1\mu_2}^{i_1 i_2}(p)b_{i_1}^{+\mu_1}b_{i_2}^{+\mu_2} + \dots + \dots + a_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(p)b_{i_1}^{+\mu_1}b_{i_2}^{+\mu_2} \dots b_{i_n}^{+\mu_n} + \dots + \dots] |0\rangle \quad (29)$$

or

$$|\phi\rangle = [a_0(p) + a_{\mu_1}^{i_1}(p)\tilde{b}_{i_1}^{+\mu_1} + a_{\mu_1\mu_2}^{i_1 i_2}(p)\tilde{b}_{i_1}^{+\mu_1}\tilde{b}_{i_2}^{+\mu_2} + \dots + \dots + a_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(p)\tilde{b}_{i_1}^{+\mu_1}\tilde{b}_{i_2}^{+\mu_2} \dots \tilde{b}_{i_n}^{+\mu_n} + \dots + \dots] |0\rangle \quad (30)$$

where:

$$p^2 a_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(p) = 0 \quad (31)$$

3. THE STRING FIELD

In this section we generalize the results of [12] and apply these results to the closed Nambu-Goto string. In this case the field of the string is complex.

According to (25), (26) and Appendix B the equation for the string field is given by

$$\square \Phi(x, \{z\}) = (\partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2) \Phi(x, \{z\}) = 0 \quad (1)$$

where $\{z\}$ denotes $(z_{1\mu}, z_{2\mu}, \dots, z_{n\mu}, \dots, \dots)$, and Φ is a CUET in the set of variables $\{z\}$. Any UET of compact support can be written as a development of $\delta(\{z\})$ and its derivatives. Thus we have:

$$\Phi(x, \{z\}) = [A_0(x) + A_{\mu_1}^{i_1}(x)\partial_{i_1}^{\mu_1} + A_{\mu_1\mu_2}^{i_1 i_2}(x)\partial_{i_1}^{\mu_1}\partial_{i_2}^{\mu_2} + \dots + \dots + A_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(x)\partial_{i_1}^{\mu_1}\partial_{i_2}^{\mu_2} \dots \partial_{i_n}^{\mu_n} + \dots + \dots] \delta(\{z\}) \quad (2)$$

where the quantum fields $A_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(x)$ are solutions of

$$\square A_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(x) = 0 \quad (3)$$

The propagator of the string field can be expressed in terms of the propagators of the component fields:

$$\Delta(x-x', \{z\}, \{z'\}) = [\Delta_0(x-x') + \Delta_{\mu_1\nu_1}^{i_1 j_1}(x-x')\partial_{i_1}^{\mu_1}\partial_{j_1}^{\nu_1} + \dots + \dots + \Delta_{\mu_1\dots\mu_n\nu_1\dots\nu_n}^{i_1\dots i_n j_1\dots j_n}(x-x')\partial_{i_1}^{\mu_1} \dots \partial_{i_n}^{\mu_n}\partial_{j_1}^{\nu_1} \dots \partial_{j_n}^{\nu_n} + \dots + \dots] \delta(\{z\}, \{z'\}) \quad (4)$$

For the fields $A_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(x)$ we have:

$$A_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(x) = \int_{-\infty}^{\infty} a_{\mu_1\mu_2\dots\mu_n}^{i_1 i_2 \dots i_n}(k) e^{-ik_\mu x^\mu} + b_{\mu_1\mu_2\dots\mu_n}^{+i_1 i_2 \dots i_n}(k) e^{ik_\mu x^\mu} d^3k \quad (5)$$

We define the operators of annihilation and creation of a string as:

$$a(k, \{z\}) = [a_0(k) + a_{\mu_1}^{i_1}(k)\partial_{i_1}^{\mu_1} + \dots + \dots + a_{\mu_1\dots\mu_n}^{i_1\dots i_n}(k)\partial_{i_1}^{\mu_1} \dots \partial_{i_n}^{\mu_n} + \dots + \dots] \delta(\{z\}) \quad (6)$$

$$a^+(k', \{z'\}) = [a_0^+(k') + a_{v_1}^{+j_1}(k') \partial_{j_1}^{v_1} + \dots + \dots +$$

$$a_{v_1 \dots v_n}^{+j_1 \dots j_n}(k') \partial_{j_1}^{v_1} \dots \partial_{j_n}^{v_n} + \dots + \dots] \delta(\{z'\}) \quad (7)$$

and the annihilation and creation operators for the anti-string

$$b(k, \{z\}) = [b_0(k) + b_{\mu_1}^i(k) \partial_{i_1}^{\mu_1} + \dots + \dots +$$

$$b_{\mu_1 \dots \mu_n}^i(k) \partial_{i_1}^{\mu_1} \dots \partial_{i_n}^{\mu_n} + \dots + \dots] \delta(\{z\}) \quad (8)$$

$$b^+(k', \{z'\}) = [b_0^+(k') + b_{v_1}^{+j_1}(k') \partial_{j_1}^{v_1} + \dots + \dots +$$

$$b_{v_1 \dots v_n}^{+j_1 \dots j_n}(k') \partial_{j_1}^{v_1} \dots \partial_{j_n}^{v_n} + \dots + \dots] \delta(\{z'\}) \quad (9)$$

If we define

$$[a_{\mu_1 \dots \mu_n}^{i_1 \dots i_n}(k), a_{v_1 \dots v_n}^{+j_1 \dots j_n}(k')] = f_{\mu_1 \dots \mu_n v_1 \dots v_n}^{i_1 \dots i_n j_1 \dots j_n}(k) \delta(k - k') \quad (10)$$

the commutations relations are

$$[a(k, \{z\}), a^+(k', \{z'\})] = \delta(k - k') [f_0(k) + f_{\mu_1 v_1}^{i_1 j_1}(k) \partial_{i_1}^{\mu_1} \partial_{j_1}^{v_1} + \dots + \dots$$

$$f_{\mu_1 \dots \mu_n v_1 \dots v_n}^{i_1 \dots i_n j_1 \dots j_n}(k) \partial_{i_1}^{\mu_1} \dots \partial_{i_n}^{\mu_n} \partial_{j_1}^{v_1} \dots \partial_{j_n}^{v_n} + \dots + \dots] \delta(\{z\}, \{z'\}) \quad (11)$$

and for the anti-string:

$$[b_{\mu_1 \dots \mu_n}^i(k), b_{v_1 \dots v_n}^{+j_1 \dots j_n}(k')] = g_{\mu_1 \dots \mu_n v_1 \dots v_n}^{i_1 \dots i_n j_1 \dots j_n}(k) \delta(k - k') \quad (12)$$

the commutations relations are

$$[b(k, \{z\}), b^+(k', \{z'\})] = \delta(k - k') [g_0(k) + g_{\mu_1 v_1}^{i_1 j_1}(k) \partial_{i_1}^{\mu_1} \partial_{j_1}^{v_1} + \dots + \dots$$

$$g_{\mu_1 \dots \mu_n v_1 \dots v_n}^{i_1 \dots i_n j_1 \dots j_n}(k) \partial_{i_1}^{\mu_1} \dots \partial_{i_n}^{\mu_n} \partial_{j_1}^{v_1} \dots \partial_{j_n}^{v_n} + \dots + \dots] \delta(\{z\}, \{z'\}) \quad (13)$$

With this annihilation and creation operators we can write:

$$\Phi(x, \{z\}) = \int_{-\infty}^{\infty} a(k, \{z\}) e^{-ik \cdot x} + b^+(k \{z\}) e^{ik \cdot x} d^3 k \quad (14)$$

4. THE ACTION FOR THE STRING FIELD

4.1. The Case n Finite

In this section we generalize the results of [12] for a complex string field

The action for the free bosonic closed string field is:

$$S_{free} = \oint_{\{\Gamma_1\}} \oint_{\{\Gamma_2\}} \int_{-\infty}^{\infty} \partial_{\mu} \Phi(x, \{z_1\}) e^{\{z_1\} \cdot \{z_2\}} \partial^{\mu} \Phi^+(x, \{z_2\}) d^3 x \{dz_1\} \{dz_2\} \quad (1)$$

A possible interaction is given by:

$$S_{int} = \lambda \oint_{\{\Gamma_1\}} \oint_{\{\Gamma_2\}} \oint_{\{\Gamma_3\}} \oint_{\{\Gamma_4\}} \int_{-\infty}^{\infty} \Phi(x, \{z_1\}) e^{\{z_1\} \cdot \{z_2\}} \Phi^+(x, \{z_2\}) e^{\{z_2\} \cdot \{z_3\}} \Phi(x, \{z_3\}) \times$$

$$e^{\{z_3\} \cdot \{z_4\}} \Phi^+(x, \{z_4\}) d^3 x \{dz_1\} \{dz_2\} \{dz_3\} \{dz_4\} \quad (2)$$

Both, S_{free} and S_{int} are non-local as expected.

4.2. The case $n \rightarrow \infty$

In this case:

$$[S_{free} = \oint_{\{\Gamma_1\}} \oint_{\{\Gamma_2\}} \int_{-\infty}^{\infty} \partial_{\mu} \Phi(x, \{z_1\}) e^{\{z_1\} \cdot \{z_2\}} \partial^{\mu} \Phi^+(x, \{z_2\}) d^3 x \{d\eta_1\} \{d\eta_2\} \quad (3)$$

where

$$d\eta_{i\mu} = \frac{e^{-z_{i\mu}^2}}{\sqrt{\sqrt{2} \pi}} dz_{i\mu} \quad (4)$$

and

$$S_{int} = \lambda \oint_{\{\Gamma_1\}} \oint_{\{\Gamma_2\}} \oint_{\{\Gamma_3\}} \oint_{\{\Gamma_4\}} \int_{-\infty}^{\infty} \Phi(x, \{z_1\}) e^{\{z_1\} \cdot \{z_2\}} \Phi^+(x, \{z_2\}) e^{\{z_2\} \cdot \{z_3\}} \Phi(x, \{z_3\}) \times$$

$$e^{\{z_3\} \cdot \{z_4\}} \Phi^+(x, \{z_4\}) d^3 x \{d\eta_1\} \{d\eta_2\} \{d\eta_3\} \{d\eta_4\} \quad (5)$$

4.3. Gauge Conditions

The gauge conditions for the string field are:

$$\int_{\{\Gamma\}} z_{i_1}^{\mu_1} \dots z_{i_k}^{\mu_k} \partial_{\mu_k} \dots z_{i_n}^{\mu_n} \Phi(x, \{z\}) \{dz\} = 0 \quad (6)$$

$$\partial_{\mu_k} = \partial / \partial x^{\mu_k} ; 1 \leq k \leq n ; n \geq 1$$

With these gauge conditions the number of the components fields of the string field is finite, and the temporal components of all fields are eliminated.

Another gauge conditions that can be added to (6) are

$$\int_{\{\Gamma\}} z_{i_1}^{\mu_1} \dots z_{i_k}^{\mu_k} \dots z_{i_n}^{\mu_n} \Phi(x, \{z\}) \{dz\} = 0 ; 1 \leq k \leq n ; n \geq 1 \quad (7)$$

$$1 \leq k \leq n ; n \geq 1$$

These additional gauge conditions permit us nullify other component fields according to experimental data. It should be noted that gauge conditions (6) and (7) does not modify the equations of motion of string field.

The convolution of two propagators of the string field is:

$$\hat{\Delta}(k, \{z_1\}, \{z_2\}) * \hat{\Delta}(k, \{z_3\}, \{z_4\}) \quad (8)$$

where * denotes the convolution of Ultradistributions of Exponential Type on the k variable only. With the use of the result

$$\frac{1}{\rho} * \frac{1}{\rho} = -\pi^2 \ln \rho \quad (9)$$

$$(\rho = x_0^2 + x_1^2 + x_2^2 + x_3^2 \text{ in euclidean space})$$

$$\frac{1}{\rho \pm i0} * \frac{1}{\rho \pm i0} = \mp i\pi^2 \ln(\rho \pm i0) \quad (10)$$

$$(\rho = x_0^2 - x_1^2 - x_2^2 - x_3^2 \text{ in minkowskian space})$$

the convolution of two string field propagators is finite.

5. DISCUSSION

In this paper we have shown that UET are appropriate for the description in a consistent way of string and string field theories. We have solved the non-linear Lagrange's equations corresponding to Nambu-Goto Lagrangian. Also we have obtained the equations of motion for the field of the string and solve it with the use of CUET. We have proved that this string field is a linear superposition of CUET. We have evaluated the propagator for the string field, and calculate the convolution of two of them, taking into account that string field theory is a non-local theory of UET of an infinite number of complex variables, For practical calculations and experimental results we have given expressions that involve only a finite number of variables.

We have decided to include, for the benefit of the reader, a first appendix with a summary of the main characteristics of Ultradistributions of Exponential Type and their Fourier transform used in this paper and a second appendix with the representation of the states of the closed string obtained in [12].

As a final remark we would like to point out that our formulas for convolutions follow from general definitions. They are not regularized expressions.

APPENDIX A

1.1. Ultradistributions of Exponential Type

Let S be the Schwartz space of rapidly decreasing test functions. Let Λ_j be the region of the complex plane defined as:

$$\Lambda_j = \{z \in \mathbb{C} : |\Im(z)| < j : j \in \mathbb{N}\} \tag{1}$$

According to ref. [6, 8] be the space of test functions $\hat{\phi} \in V_j$ is constituted by all entire analytic functions of S for which

$$\|\hat{\phi}\|_j = \max_{k \leq j} \left\{ \sup_{z \in \Lambda_j} \left[e^{(j\Re(z))} |\hat{\phi}^{(k)}(z)| \right] \right\} \tag{2}$$

is finite.

The space Z is then defined as:

$$Z = \bigcap_{j=0}^{\infty} V_j \tag{3}$$

It is a complete countably normed space with the topology generated by the system of semi-norms $\{\|\cdot\|_j\}_{j \in \mathbb{N}}$. The dual of Z , denoted by B , is by definition the space of ultradistributions of exponential type (ref. [6, 8]). Let S the space of rapidly decreasing sequences. According to ref. [15] S is a nuclear space. We consider now the space of sequences P generated by the Taylor development of $\hat{\phi} \in Z$

$$P = \left\{ Q : Q \left(\hat{\phi}(0), \hat{\phi}'(0), \frac{\hat{\phi}''(0)}{2}, \dots, \frac{\hat{\phi}^{(n)}(0)}{n!}, \dots \right) : \hat{\phi} \in Z \right\} \tag{4}$$

The norms that define the topology of P are given by:

$$\|\hat{\phi}\|_p' = \sup_n \frac{n^p}{n} |\hat{\phi}^n(0)| \tag{5}$$

P is a subspace of S and therefore is a nuclear space. As the norms $\|\cdot\|_j$ and $\|\cdot\|_p'$ are equivalent, the correspondence

$$Z \Leftrightarrow P \tag{6}$$

is an isomorphism and therefore Z is a countably normed nuclear space. We can define now the set of scalar products

$$\langle \hat{\phi}(z), \hat{\psi}(z) \rangle_n = \sum_{q=0}^n \int_{-\infty}^{\infty} e^{2m|x|} \overline{\hat{\phi}^{(q)}(z)} \hat{\psi}^{(q)}(z) dz = \sum_{q=0}^n \int_{-\infty}^{\infty} e^{2m|x|} \overline{\hat{\phi}^{(q)}(x)} \hat{\psi}^{(q)}(x) dx \tag{7}$$

This scalar product induces the norm

$$\|\hat{\phi}\|_n'' = [\langle \hat{\phi}(x), \hat{\phi}(x) \rangle_n]^{1/2} \tag{8}$$

The norms $\|\cdot\|_j$ and $\|\cdot\|_n''$ are equivalent, and therefore Z is a countably hilbertian nuclear space. Thus, if we call now Z_p the completion of Z by the norm p given in (8), we have:

$$Z = \bigcap_{p=0}^{\infty} Z_p \tag{9}$$

where

$$Z_0 = \mathbf{H} \tag{10}$$

is the Hilbert space of square integrable functions.

As a consequence the "nested space"

$$U = (Z, \mathbf{H}, B) \tag{11}$$

is a Guelfand's triplet (or a Rigged Hilbert space=RHS. See ref. [15]).

Any Guelfand's triplet $G = (\Phi, \mathbf{H}, \Phi')$ has the fundamental property that a linear and symmetric operator on Φ , admitting an extension to a self-adjoint operator in \mathbf{H} , has a complete set of generalized eigen-functions in Φ' with real eigenvalues.

B can also be characterized in the following way (refs. [6, 8]): let E_ω be the space of all functions $\hat{F}(z)$ such that:

- I** - $\hat{F}(z)$ is analytic for $\{z \in \mathbb{C} : |\Im(z)| > p\}$.
- II** - $\hat{F}(z)e^{-p|\Re(z)|} / z^p$ is bounded continuous in $\{z \in \mathbb{C} : |\Im(z)| \geq p\}$, where $p = 0, 1, 2, \dots$ depends on $\hat{F}(z)$.

Let N be: $N = \{\hat{F}(z) \in E_\omega : \hat{F}(z) \text{ is entire analytic}\}$. Then B is the quotient space:

$$\mathbf{III} - B = E_\omega / N$$

Due to these properties it is possible to represent any ultradistribution as (ref. [6, 8]):

$$\hat{F}(\hat{\phi}) = \langle \hat{F}(z), \hat{\phi}(z) \rangle = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz \quad (12)$$

where the path Γ_j runs parallel to the real axis from $-\infty$ to ∞ for $Im(z) > \zeta$, $\zeta > p$ and back from ∞ to $-\infty$ for $Im(z) < -\zeta$, $-\zeta < -p$. (Γ surrounds all the singularities of $\hat{F}(z)$).

Formula (12) will be our fundamental representation for a tempered ultradistribution. Sometimes use will be made of "Dirac formula" for exponential ultradistributions (ref. [6]):

$$\hat{F}(z) \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{t-z} dt \equiv \frac{\cosh(\lambda z)}{2\pi i} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{(t-z)\cosh(\lambda t)} dt \quad (13)$$

where the "density" $\hat{f}(t)$ is such that

$$\oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz = \int_{-\infty}^{\infty} \hat{f}(t) \hat{\phi}(t) dt \quad (14)$$

(13) should be used carefully. While $\hat{F}(z)$ is analytic on Γ , the density $\hat{f}(t)$ is in general singular, so that the r.h.s. of (14) should be interpreted in the sense of distribution theory.

Another important property of the analytic representation is the fact that on Γ , $\hat{F}(z)$ is bounded by an exponential and a power of z (ref. [6, 8]):

$$|\hat{F}(z)| \leq C |z|^p e^{p\Re(z)} \quad (15)$$

where C and p depend on \hat{F} .

The representation (12) implies that the addition of any entire function $\hat{G}(z) \in N$ to $\hat{F}(z)$ does not alter the ultradistribution:

$$\oint_{\Gamma} \{\hat{F}(z) + \hat{G}(z)\} \hat{\phi}(z) dz = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz + \oint_{\Gamma} \hat{G}(z) \hat{\phi}(z) dz$$

But:

$$\oint_{\Gamma} \hat{G}(z) \hat{\phi}(z) dz = 0$$

as $\hat{G}(z) \hat{\phi}(z)$ is entire analytic (and rapidly decreasing),

$$\therefore \oint_{\Gamma} \{\hat{F}(z) + \hat{G}(z)\} \hat{\phi}(z) dz = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz \quad (16)$$

Another very important property of B is that B is reflexive under the Fourier transform:

$$B = F_c \{B\} = F \{B\} \quad (17)$$

where the complex Fourier transform $F(k)$ of $\hat{F}(z) \in B$ is given by:

$$F(k) = \Theta[\Im(k)] \int_{\Gamma_+} \hat{F}(z) e^{ikz} dz - \Theta[-\Im(k)] \int_{\Gamma_-} \hat{F}(z) e^{ikz} dz = \Theta[\Im(k)] \int_0^{\infty} \hat{f}(x) e^{ikx} dx - \Theta[-\Im(k)] \int_{-\infty}^0 \hat{f}(x) e^{ikx} dx \quad (18)$$

Here Γ_+ is the part of Γ with $\Re(z) \geq 0$ and Γ_- is the part of Γ with $\Re(z) \leq 0$. Using (18) we can interpret Dirac's formula as:

$$F(k) \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{f(s)}{s-k} ds \equiv F_c \{F^{-1} \{f(s)\}\} \quad (19)$$

The treatment for ultradistributions of exponential type defined on \mathbb{C}^n is similar to the case of one variable. Thus

$$\Lambda_j = \{z = (z_1, z_2, \dots, z_n) \in \mathbb{C}^n : |\Im(z_k)| \leq j \quad 1 \leq k \leq n\} \quad (20)$$

$$\|\hat{\phi}\|_j = \max_{k \leq j} \left\{ \sup_{z \in \Lambda_j} \left[e^{\left[\sum_{p=1}^n \Re(z_p) \right]} \left| D^{(k)} \hat{\phi}(z) \right| \right] \right\} \quad (21)$$

where $D^{(k)} = \partial^{(k_1)} \partial^{(k_2)} \dots \partial^{(k_n)}$ $k = k_1 + k_2 + \dots + k_n$

B^n is characterized as follows. Let E_{ω}^n be the space of all functions $\hat{F}(z)$ such that:

I - $\hat{F}(z)$ is analytic for

$$\{z \in \mathbb{C}^n : |\Im(z_1)| > p, |\Im(z_2)| > p, \dots, |\Im(z_n)| > p\}.$$

II - $\hat{F}(z) e^{-\left[\sum_{j=1}^n \Re(z_j) \right]} / z^p$ is bounded continuous in $\{z \in \mathbb{C}^n : |\Im(z_1)| \geq p, |\Im(z_2)| \geq p, \dots, |\Im(z_n)| \geq p\}$, where $p = 0, 1, 2, \dots$ depends on $\hat{F}(z)$.

Let N^n be: $N^n = \{\hat{F}(z) \in E_{\omega}^n : \hat{F}(z) \text{ is entire analytic at minus in one of the variables } z_j \quad 1 \leq j \leq n\}$ Then B^n is the quotient space:

III - $B^n = E_{\omega}^n / N^n$ We have now

$$\hat{F}(\hat{\phi}) = \langle \hat{F}(z), \hat{\phi}(z) \rangle = \oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz_1 dz_2 \dots dz_n \quad (22)$$

$\Gamma = \Gamma_1 \cup \Gamma_2 \cup \dots \cup \Gamma_n$ where the path Γ_j runs parallel to the real axis from $-\infty$ to ∞ for $Im(z_j) > \zeta$, $\zeta > p$ and back from ∞ to $-\infty$ for $Im(z_j) < -\zeta$, $-\zeta < -p$. (Again Γ surrounds all the singularities of $\hat{F}(z)$). The n-dimensional Dirac's formula is

$$\hat{F}(z) = \frac{1}{(2\pi i)^n} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{(t_1 - z_1)(t_2 - z_2) \dots (t_n - z_n)} dt_1 dt_2 \dots dt_n \quad (23)$$

where the "density" $\hat{f}(t)$ is such that

$$\oint_{\Gamma} \hat{F}(z) \hat{\phi}(z) dz_1 dz_2 \dots dz_n = \int_{-\infty}^{\infty} f(t) \hat{\phi}(t) dt_1 dt_2 \dots dt_n \quad (24)$$

and the modulus of $\hat{F}(z)$ is bounded by

$$|\hat{F}(z)| \leq C |z|^p e^{\left[p \sum_{j=1}^n \Re(z_j) \right]} \quad (25)$$

where C and p depend on \hat{F} .

1.2. The Case $N \rightarrow \infty$

When the number of variables of the argument of the Ultradistribution of Exponential type tends to infinity we define:

$$d\mu(x) = \frac{e^{-x^2}}{\sqrt{\pi}} dx \quad (26)$$

Let $\hat{\phi}(x_1, x_2, \dots, x_n)$ be such that:

$$\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} |\hat{\phi}(x_1, x_2, \dots, x_n)|^2 d\mu_1 d\mu_2 \dots d\mu_n < \infty \quad (27)$$

where

$$d\mu_i = \frac{e^{-x_i^2}}{\sqrt{\pi}} dx_i \quad (28)$$

Then by definition $\hat{\phi}(x_1, x_2, \dots, x_n) \in L_2(\mathbb{R}^n, \mu)$ and

$$L_2(\mathbb{R}^\infty, \mu) = \bigcup_{n=1}^{\infty} L_2(\mathbb{R}^n, \mu) \quad (29)$$

Let $\hat{\psi}$ be given by

$$\hat{\psi}(z_1, z_2, \dots, z_n) = \pi^{n/4} \hat{\phi}(z_1, z_2, \dots, z_n) e^{\frac{z_1^2 + z_2^2 + \dots + z_n^2}{2}} \quad (30)$$

where $\hat{\phi} \in Z^n$ (the corresponding n-dimensional of Z).

Then by definition $\hat{\psi}(z_1, z_2, \dots, z_n) \in G(\mathbb{C}^n)$,

$$G(\mathbb{C}^\infty) = \bigcup_{n=1}^{\infty} G(\mathbb{C}^n) \quad (31)$$

and the dual $G'(\mathbb{C}^\infty)$ given by

$$G'(\mathbb{C}^\infty) = \bigcup_{n=1}^{\infty} G'(\mathbb{C}^n) \quad (32)$$

is the space of Ultradistributions of Exponential type.

The analog to (11) in the infinite dimensional case is:

$$W = (G(\mathbb{C}^\infty), L_2(\mathbb{R}^\infty, \mu), G'(\mathbb{C}^\infty)) \quad (33)$$

If we define:

$$F : G(\mathbb{C}^\infty) \rightarrow G(\mathbb{C}^\infty) \quad (34)$$

via the Fourier transform:

$$F : G(\mathbb{C}^n) \rightarrow G(\mathbb{C}^n) \quad (35)$$

given by:

$$F\{\hat{\psi}\}(k) = \int_{-\infty}^{\infty} \hat{\psi}(z_1, z_2, \dots, z_n) e^{ikz + \frac{k^2}{2}} d\rho_1 d\rho_2 \dots d\rho_n \quad (36)$$

where

$$d\rho(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz \quad (37)$$

we conclude that

$$G'(\mathbb{C}^\infty) = F_c\{G'(\mathbb{C}^\infty)\} = F\{G'(\mathbb{C}^\infty)\} \quad (38)$$

where in the one-dimensional case

$$F_c\{\hat{\psi}\}(k) = \Theta[\Im(k)] \int_{\Gamma_+} \hat{\psi}(z) e^{ikz + \frac{k^2}{2}} d\rho - \Theta[-\Im(k)] \int_{\Gamma_-} \hat{\psi}(z) e^{ikz + \frac{k^2}{2}} d\rho \quad (39)$$

APPENDIX B

1.1. A Representation of the States of the Closed String

1.1.1. The Case n Finite

For an ultradistribution of exponential type, we can write:

$$G(k) = \oint_{\Gamma_z} \{ \Theta[\Im(k)] \Theta[\Re(z)] - \Theta[-\Im(k)] \Theta[-\Re(z)] \} \hat{G}(z) e^{ikz} dz$$

$$\hat{G}(z) = \frac{1}{2\pi} \oint_{\Gamma_k} \{ \Theta[\Im(z)] \Theta[-\Re(k)] - \Theta[-\Im(z)] \Theta[\Re(k)] \} G(k) e^{-ikz} dk \quad (1)$$

and

$$G(\phi) = \oint_{\Gamma_k} G(k) \phi(k) dk =$$

$$\oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\Im(k)] \Theta[\Re(z)] - \Theta[-\Im(k)] \Theta[-\Re(z)] \} \hat{G}(z) \phi(k) e^{ikz} dk dz = (2)$$

$$-i \oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\Im(k)] \Theta[\Im(z)] - \Theta[-\Im(k)] \Theta[-\Im(z)] \} \hat{G}(-iz) \phi(k) e^{kz} dk dz \quad (3)$$

where the path Γ'_z is the path Γ_z rotated ninety degrees counterclockwise around the origin of the complex plane.

If $F(z)$ is an UET of compact support we can define:

$$\langle \hat{F}(z), \phi(z) \rangle =$$

$$\oint_{\Gamma_k} \oint_{\Gamma'_z} \{ \Theta[\Im(k)] \Theta[\Im(z)] - \Theta[-\Im(k)] \Theta[-\Im(z)] \} \hat{F}(z) \phi(k) e^{kz} dk dz \quad (4)$$

then:

$$\langle \hat{F}'(z), \phi(z) \rangle =$$

$$\oint_{\Gamma_k} \oint_{\Gamma'_z} \{ \Theta[\Im(k)] \Theta[\Im(z)] - \Theta[-\Im(k)] \Theta[-\Im(z)] \} \hat{F}'(z) \phi(k) e^{kz} dk dz =$$

$$- \oint_{\Gamma_k} \oint_{\Gamma'_z} \{ \Theta[\Im(k)] \Theta[\Im(z)] - \Theta[-\Im(k)] \Theta[-\Im(z)] \} \hat{F}(z) k \phi(k) e^{kz} dk dz =$$

$$\langle \hat{F}(z), -z\phi(z) \rangle \quad (5)$$

If we define:

$$a = -z \quad ; \quad a^+ = \frac{d}{dz} \tag{6}$$

we have

$$[a, a^+] = I \tag{7}$$

Thus we have a representation for creation and annihilation operators of the states of the string. The vacuum state annihilated by z_μ is the UET $\delta(z_\mu)$, and the orthonormalized states obtained by successive application of $\frac{d}{dz_\mu}$ to $\delta(z_\mu)$ are:

$$F_n(z_\mu) = \frac{\delta^{(n)}(z_\mu)}{\sqrt{n!}} \tag{8}$$

On the real axis:

$$\langle \hat{F}(z), \phi(z) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{f}(x) \phi(k) e^{kx} dx dk \tag{9}$$

where $\bar{f}(x)$ is given by Dirac's formula:

$$\hat{F}(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\bar{f}(x)}{x-z} dx \tag{10}$$

A general state of the string can be written as:

$$\begin{aligned} \phi(x, \{z\}) = & [a_0(x) + a_{\mu_1}^{i_1}(x) \partial_{i_1}^{\mu_1} + a_{\mu_1 \mu_2}^{i_1 i_2}(x) \partial_{i_1}^{\mu_1} \partial_{i_2}^{\mu_2} + \dots + \dots \\ & + a_{\mu_1 \mu_2 \dots \mu_n}^{i_1 i_2 \dots i_n}(x) \partial_{i_1}^{\mu_1} \partial_{i_2}^{\mu_2} \dots \partial_{i_n}^{\mu_n} + \dots + \dots] \delta(\{z\}) \end{aligned} \tag{11}$$

where $\{z\}$ denotes $(z_{1\mu}, z_{2\mu}, \dots, z_{n\mu}, \dots)$, and ϕ is a UET of compact support in the set of variables $\{z\}$. The functions $a_{\mu_1 \mu_2 \dots \mu_n}^{i_1 i_2 \dots i_n}(x)$ are solutions of

$$\square a_{\mu_1 \mu_2 \dots \mu_n}^{i_1 i_2 \dots i_n}(x) = 0 \tag{12}$$

1.1.2. The Case $n \rightarrow \infty$

In this case

$$G(k) = \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{Z}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{Z}(z)] \} \hat{G}(z) e^{ikz + \frac{k^2 z^2}{2}} \frac{dz}{\sqrt{2\pi}}$$

$$\hat{G}(z) = \oint_{\Gamma_k} \{ \Theta[\mathfrak{Z}(z)] \Theta[-\mathfrak{X}(k)] - \Theta[-\mathfrak{Z}(z)] \Theta[\mathfrak{X}(k)] \} \times$$

$$G(k) e^{-ikz + \frac{z^2 k^2}{2}} \frac{dk}{\sqrt{2\pi}} \tag{13}$$

$$G(\phi) = \oint_{\Gamma_k} G(k) \phi(k) e^{-k^2} \frac{dk}{\sqrt{\pi}} =$$

$$\oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{X}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{X}(z)] \} \times$$

$$\hat{G}(z) \phi(k) e^{ikz - \frac{z^2 k^2}{2}} \frac{dk dz}{\sqrt{2\pi}} = \tag{14}$$

$$-i \oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{Z}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{Z}(z)] \} \times$$

$$\hat{G}(-iz) \phi(k) e^{kz + \frac{z^2 k^2}{2}} \frac{dk dz}{\sqrt{2\pi}} \tag{15}$$

If $F(z)$ is an CUET we can define:

$$\langle \hat{F}(z), \phi(z) \rangle =$$

$$\oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{Z}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{Z}(z)] \} \times$$

$$[\hat{F}(z) e^{-\frac{3z^2}{2}}] \phi(k) e^{kz + \frac{z^2 k^2}{2}} \frac{dk dz}{\sqrt{2\pi}} = \tag{16}$$

$$\oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{Z}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{Z}(z)] \} \times$$

$$\hat{F}(z) \phi(k) e^{kz - z^2 - k^2} \frac{dk dz}{\sqrt{2\pi}} = \tag{17}$$

and then

$$\langle -2z\hat{F}(z) + \hat{F}'(z), \phi(z) \rangle =$$

$$\oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{Z}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{Z}(z)] \} \times$$

$$[-2z\hat{F}(z) + \hat{F}'(z)] \phi(k) e^{kz - z^2 - k^2} \frac{dk dz}{\sqrt{2\pi}} =$$

$$-\oint_{\Gamma_k} \oint_{\Gamma_z} \{ \Theta[\mathfrak{Z}(k)] \Theta[\mathfrak{Z}(z)] - \Theta[-\mathfrak{Z}(k)] \Theta[-\mathfrak{Z}(z)] \} \times$$

$$\hat{F}(z) k \phi(k) e^{kz - z^2 - k^2} \frac{dk dz}{\sqrt{2\pi}} =$$

$$\langle \hat{F}(z), -z\phi(z) \rangle \tag{18}$$

If we define:

$$a = -z \quad ; \quad a^+ = -2z + \frac{d}{dz} \tag{19}$$

we have

$$[a, a^+] = I \tag{20}$$

The vacuum state annihilated by a is $\delta(z)e^{z^2}$. The orthonormalized states obtained by successive application of a^+ are:

$$\hat{F}_n(z) = 2^{\frac{1}{4}} \pi^{\frac{1}{2}} \frac{\delta^{(n)}(z) e^{z^2}}{\sqrt{n!}} \tag{21}$$

On the real axis we have

$$\langle \hat{F}(z), \phi(z) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{\hat{f}}(x) \phi(k) e^{kx - x^2 - k^2} \frac{dx dk}{\sqrt{2} \pi} \quad (22)$$

where $\overline{\hat{f}}(x)$ is given by Dirac's formula:

$$\hat{F}(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\overline{\hat{f}}(x)}{x - z} dx \quad (23)$$

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