Permeability of Fractural Porous Media Determined by Double Percolation Model

X. B. Lu[1,*], W. Zheng[1], Q. J. Liu[2] and X. H. Zhang[1]

[1]*Institute of Mechanics, Chinese Academy of Sciences, Beijing, China, 100190;* [2]*Research Institute of Petroleum Exploration and Development, PetroChina, 100083, China*

Abstract: A model for determining the permeability of fractural porous media based on the double percolation model is presented. The results are close to that by fractural permeability tensor model. The non-uniformity of the permeability and the pore pressure are investigated by parameter study. It is shown that rocks could be divided into "pore-controlling" permeability and "fracture-controlling" permeability according to the fracture length. This model can not only account for the effects of pores and fractures together, but also deal with simulation of fracture network in large scale fast and simply.

Keywords: Fractural porous media, double percolation, permeability.

1. INTRODUCTION

Evaluating permeability of reservoir is very important in engineering practices. For an example, during the geological sequestration of captured CO_2, the permeability of reservoir determines the effects of CO_2 injection during oil and gas production, the permeability of reservoir determines whether oil and gas can be exploited [1-3]. However, reservoirs are fractured porous media, in which the flow passes through pores and fractures.

The conventional approach to study the impact of geological factors on reservoir recovery is to build a detailed reservoir model according to the measured geophysical and geological data, and then perform seepage simulation [4-6]. If fractures in a rock are poorly interconnected and the matrix rock is very low impermeable, the flow may be blocked in the network of fractures. Otherwise, if the matrix is permeable and the fractures are inerratic and highly interconnected, fractures and matrix can be treated as separate continuums occupying the entire domain. In order to estimate the performance parameters, it is necessary to construct reservoir models and run flow simulations to determine the permeability. This method is time costing and computationally expensive [7]. Therefore, there is a great incentive to produce much simpler physically-based models to quickly predict the permeability of a stratum.

Percolation theory is an effective method to investigate the connectivity of reservoir [8-10] which is first developed in the late 1950s [11]. The global static and dynamic properties of such systems are linked to the density of objects (e.g., the fractures and pores in this study) placed randomly in space. In percolation theory a medium is defined as an infinite set of sites. A fluid flows between these sites along paths which connect certain pairs of sites (these paths are called bonds). Then the connectivity and permeability can be estimated directly by using percolation theory. This method can easily estimate the effects of complex geometry in a fraction of a second on a spreadsheet, but it ignores much of the flow physics and subtleties of the heterogeneity distribution, including the effects of fracture's thickness.

This paper reports a method to determine the permeability of fractural porous media based on a double percolation model. To obtain the permeability of a fractural porous media, first the connected cluster in the considered area is analyzed by the double percolation model; then the permeability is solved by only considering the fractures in the connected cluster. The effects of main factors are discussed. This method can simulate the seepage in fractural porous double media in large scale fast and conveniently by combining the pore percolation and fracture percolation. In the second section the basic model of double percolation is introduced; in the third section, the comparison of the results between the seepage tensor theory used in fractural rocks and the double percolation theory by neglecting the pore percolation; in the fourth section, the effects of main factors of the double percolation are investigated.

2. INTRODUCTION OF THE MODEL FOR SOLVING THE PERMEABILITY BY DOUBLE PERCOLATION MODEL

2.1. The Pore Pressure in Fracture Network

As shown in Figs. (**1** and **2**), the sum of the intersection points among fractures Points of A、C、E、F in Fig. (**2**) and connection points among pores Points of B、D in Fig. (**2**) are N in a percolation cluster. These intersection points and connection points form the basic elements for the computation of permeability. There are three types of formation: two neighboring intersections in a fracture, such as the line element (fractural connection element) EF in Fig. (**2**); the neighboring intersection points and connection point in a fracture, such as the elements of AB and AD in Fig. (**2**); two connection points in two neighboring fractures such as the

*Address correspondence to this author at the Institute of Mechanics, Chinese Academy of Sciences, Bei Si Huan Xi Lu 15#, Beijing, 100190, China; Tel: 8610-5441-8290; E-mail: xblu@imech.ac.cn

line element of BD (porous connection element) in Fig. (**2**). If M elements are formed in a percolation cluster, and the two intersection points / connection points are taken as a node in computation, each node is corresponding to a value of coordination and each line element is described by a length δ_i, an angle β_i and a width b_i, which satisfies eq. (1).

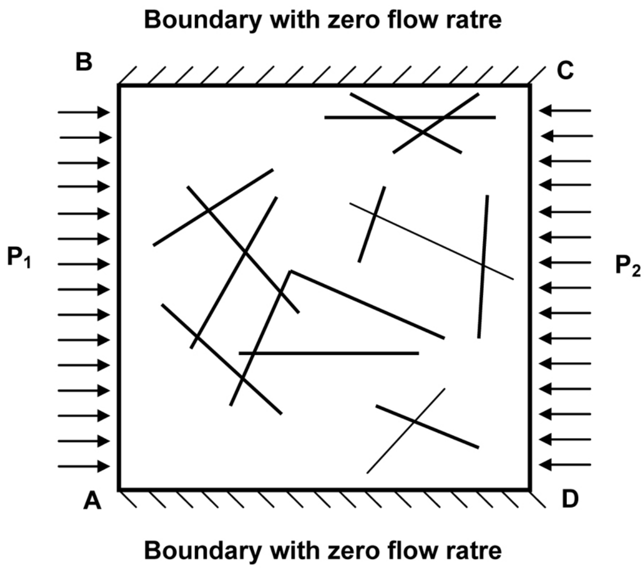

Fig. (1). Seepage model for a double percolation network.

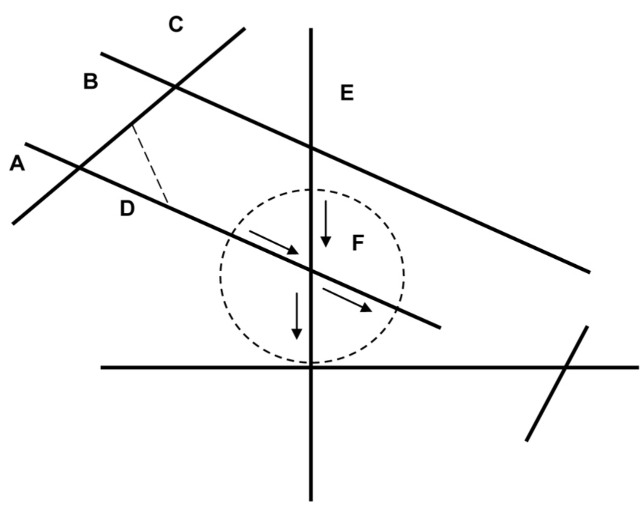

Fig. (2). Intersection and line element in a connected cluster.

Large pores can form connective channel between any two fractures not intersected to cause them connected with some probability, e.g. pore connective probability p. Each fracture's center is mapped as a "site", and the connected pores between any two fractures are mapped as a "bond". In this paper the quadrangle network is used, with each fracture having four "bonds" to connect neighboring fractures. If the probability of connection between any two bonds is larger than a critical value, these two bonds are taken as connected; Furthermore, if any two fractures are intersected, they are considered connected. The main point is to find the conditions required to form a connective cluster.

The fractures in the rock are assumed as a cluster of cylindrical pipes. In each pipe the single fluid obeys the Poiseuille flow, e.g.:

$$q_i = \frac{\Delta P_i b_i^3}{12\mu\delta_i} \tag{1}$$

in which ΔP_i is the pressure difference, μ the viscosity of the fluid, q_i the flow rate.

For any node j, the total flow rate can be expressed as the sum of the flow rates of N' line elements intersected at this node considering that the outflow and the inflow are equal by neglecting the compressibility of the pore fluids:

$$\sum_{i=1}^{N'} q_i = 0 \tag{2}$$

If the double percolation network has N nodes, then we can obtain the following equation by eq. (2):

$$A\hat{q} = 0 \tag{3}$$

in which $\hat{q} = (q_1, q_2, ..., q_N)^T$, $A = \{a_{ij}\}_{N\times M}$ is called as a $N \times M$ matrix of the double percolation network, a_{ij} equals 0,1 or -1 corresponding to three conditions respectively: j element is not connected with the node i, j element is connected with the node i and q_j is in the direction of node i, or j element is connected with the node i and q_j is in the inverse direction of node i.

Assuming that the flow-rates on the boundaries of AD and BC are zero, and the pressures on the boundaries of AD and BC Fig. (**1**) are constants, then the mathematical formulation of the double percolation network is:

$$\begin{cases} \sum_{i=1}^{N'} q_i = \frac{\Delta P_i b_i^3}{12\mu_{CO2}\delta_i} = 0 \\ B.C.: \quad P|_{AB} = P_1 \\ B.C.: \quad P|_{CD} = P_2 \\ I.C.: \quad \frac{\partial P}{\partial n}|_{AD,BC} = 0 \end{cases} \tag{4}$$

A set of linear algebra equations can be formed by building an equation for each node i according to eq. (4). The pore pressure at each node can be obtained by solving this set of equations.

3. THE METHOD FOR SOLVING PERMEABILITY

Snow [11] presented a seepage tensor model for describing the fractural rocks based on the statistics of fractures. This model took the fractural rocks as pure fracture systems with singular geometrical configuration, and the permeability due to porosities were neglected. In this model, the scale, shapes and positions were all considered.

The seepage tensor of a rock with n groups of fractures is as follows by summing up the permeability in each direction Fig. (**3**):

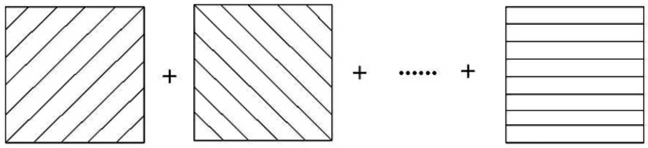

Fig. (3). Model of seepage tensor: accumulated by n groups of fractures.

$$[k]=\begin{bmatrix} \sum_{i=1}^{n} k_{ei}\left(1-\cos^2\alpha_{1i}\right) & -\sum_{i=1}^{n} k_{ei}\cos\alpha_{1i}\cos\alpha_{2i} \\ -\sum_{i=1}^{n} k_{ei}\cos\alpha_{2i}\cos\alpha_{1i} & \sum_{i=1}^{n} k_{ei}\left(1-\cos^2\alpha_{2i}\right) \end{bmatrix} \tag{5}$$

$$k_{ei}=k_{fi}\frac{b_i}{d_i}, k_{fi}=\frac{\gamma b_i^2}{12\mu_g}$$

in which k_{fi}, b_i, d_i are respectively the equivalent permeability, average fractural width and average distance of i group fractures; and $\cos\alpha_{1i}$, $\cos\alpha_{2i}$ are the corresponding direction cosines.

For the fractural porous meida, assuming that there are N_0 fractures with equal lengths and widths, and N_1 fractures belongs to the percolation cluster. The randomly distributed N_1 fractures are divided into x groups. Factures in each group are in the same directions (α_1, α_2, …, α_x), $\alpha_2-\alpha_1=\alpha_3-\alpha_2=\cdots=\alpha_x-\alpha_{x-1}=\alpha_0$. Any fracture is included in the 1th group with the direction angle of α_i if its direction angle is in the range $\left(\alpha_i-\frac{1}{2}\alpha_0,\alpha_i+\frac{1}{2}\alpha_0\right)$ ($\alpha_i\subset\{\alpha_1,\alpha_2,\cdots\alpha_x\}$). So, each group has the fractures of N_1/x. Each group is equivalently simplified as y parallel and equal-distance fractures according to the rule that the total fracture length is equal. Each fracture can penetrate the whole area. The distance of the ith group fractures is denoted by d_i. So if the relation of $\alpha_i\sim d_i$ is obtained, the equivalent seepage tensor can be computed by eq. (5).

d_i can be obtained by the way of equivalent area Fig. (**4**). The computing area is in the square formed by bold line. Dashed lines denote the ith group parallel fractures. Thin-line square is a square, whose two boundaries are coincided with the parallel fractures for the supplement of computation. The rule of the method is that the ratio of the total length of the fractures in the bold-line square to that in the thin-line square is equal to the area ratio of the two squares.

The area of the bold-line square is: $A_1=L^2=\lambda^2$

The area of the thin-line square is: $A_2=\left[(\sin\alpha+\cos\alpha)L\right]^2=\left[(\sin\alpha+\cos\alpha)\lambda\right]^2$

The total length of the fractures in the thin-line square is: $(\sin\alpha+\cos\alpha)^2\cdot\mu\lambda N_1/x$

The equivalent quantity of fractures is: $|\sin\alpha+\cos\alpha|\cdot\mu N_1/x$

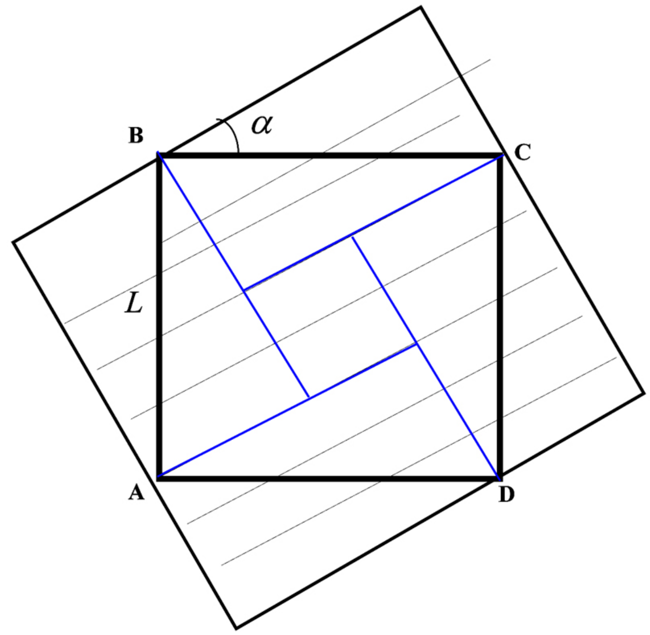

Fig. (4). Sketch of the equivalent area method.

So the distance of d_i can be obtained:

$$d_i=\frac{|\sin\alpha+\cos\alpha|\lambda}{|\sin\alpha+\cos\alpha|\cdot\mu N_1/x-1} \tag{6}$$

or

$$\frac{1}{d_i}=\frac{1}{\lambda}\cdot\left[\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha+\cos\alpha|}\right] \tag{7}$$

Then the parameter k_{ei} can be obtained:

$$k_{ei}=k_{fi}\frac{b_i}{d_i}=\frac{k_{fi}b_i}{\lambda}\cdot\left[\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha+\cos\alpha|}\right]=\frac{\gamma b_i^3}{12\mu\lambda}\cdot\left[\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha+\cos\alpha|}\right] \tag{8}$$

Instituting eq.(8) into eq. (5), the tensor of the permeability coefficients can be obtained as follows:

$$[k]=\begin{bmatrix} \sum_{i=1}^{n} k_{ei}\left(1-\cos^2\alpha_{1i}\right) & -\sum_{i=1}^{n} k_{ei}\cos\alpha_{1i}\cos\alpha_{2i} \\ -\sum_{i=1}^{n} k_{ei}\cos\alpha_{2i}\cos\alpha_{1i} & \sum_{i=1}^{n} k_{ei}\left(1-\cos^2\alpha_{2i}\right) \end{bmatrix}$$
$$=\frac{\gamma b_i^3}{12\mu_g\lambda}\cdot\begin{bmatrix} \sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\left(1-\cos^2\alpha_{1i}\right) & -\sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\cos\alpha_{1i}\cos\alpha_{2i} \\ -\sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\cos\alpha_{2i}\cos\alpha_{1i} & \sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\left(1-\cos^2\alpha_{2i}\right) \end{bmatrix} \tag{9}$$

The physical permeability is:

$$[K]=\frac{b_i^3}{12\lambda}\cdot\begin{bmatrix} \sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\left(1-\cos^2\alpha_{1i}\right) & -\sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\cos\alpha_{1i}\cos\alpha_{2i} \\ -\sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\cos\alpha_{2i}\cos\alpha_{1i} & \sum_{i=1}^{n}\left(\frac{\mu N_1}{x}-\frac{1}{|\sin\alpha_{2i}+\cos\alpha_{2i}|}\right)\cdot\left(1-\cos^2\alpha_{2i}\right) \end{bmatrix} \tag{10}$$

To decrease the non-uniformity due to the simple grouping of fractures, the physical permeability is assumed as average of that in x and y directions:

$$K=\sqrt{K_{11}K_{22}} \tag{11}$$

4. COMPUTING RESULTS

In computation, the parameters are adopted as follows: the area is 100m×100m, e.g. $\lambda = 100$, the number of fractures is set as 900 ($N_0 = 900$), the angles and positions of the fractures are randomly distributed and $\alpha = 1.57\left(90^{\circ}\right)$, the lengths of the fractures range 5~16m (It is assumed disconnected when the length is less than 8m, so the permeability is zero.), the fracture width is 0.1mm, the connection probability of the pores is zero, e.g. $p = 0$, the viscosity coefficient of the pore water is 0.001Pa.s, the density of the pore water is 1000kg/m^3. The pores' diameters are all assumed as 0.001mm .

The following conclusions can be obtained from Figs. (**5**~**8**): (1) the average length corresponding to the connection decreases when p increases; (2) the equivalent permeability is very small when the average length of the fractures is just equal to the percolation threshold because in this case the pores are the main contributor to the permeability and some zones are not connected; and it increases fast when the average length equals the percolation threshold at $p = 0$ because in this case the fractures has form a connected network and the flow rate in the fracture is larger than that in pores. In the double percolation media, the permeability of the pores is smaller than that of the fractures. When the fractures are too short to form a network of percolation, the percolation is controlled by pore connection. Otherwise, it is controlled by fracture connection.

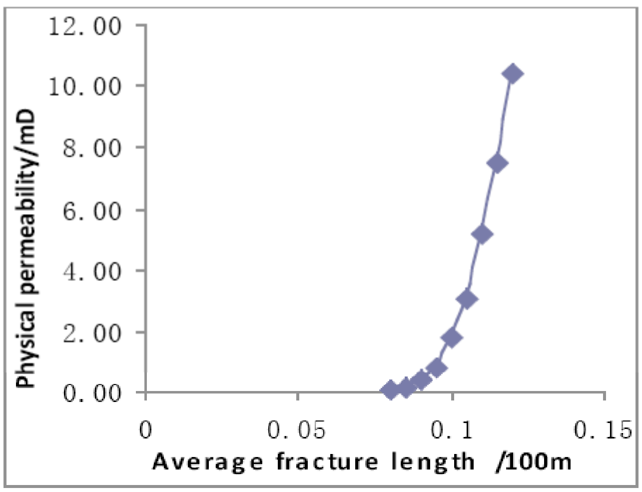

Fig. (5). Permeability versus average length($p = 0.0$).

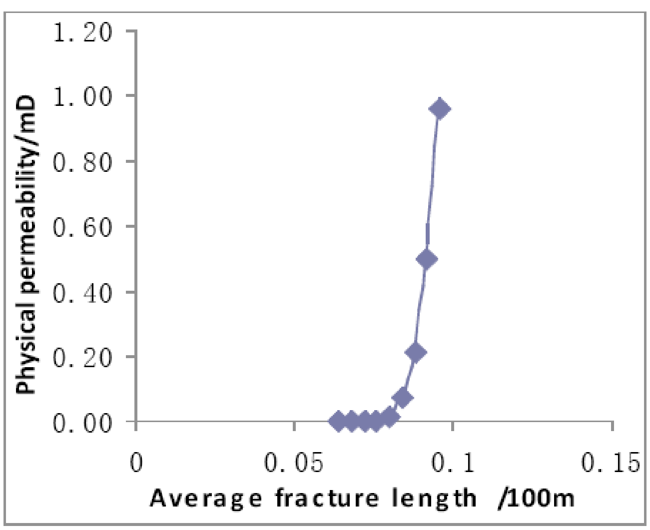

Fig. (6). Permeability versus average length($p = 0.1$).

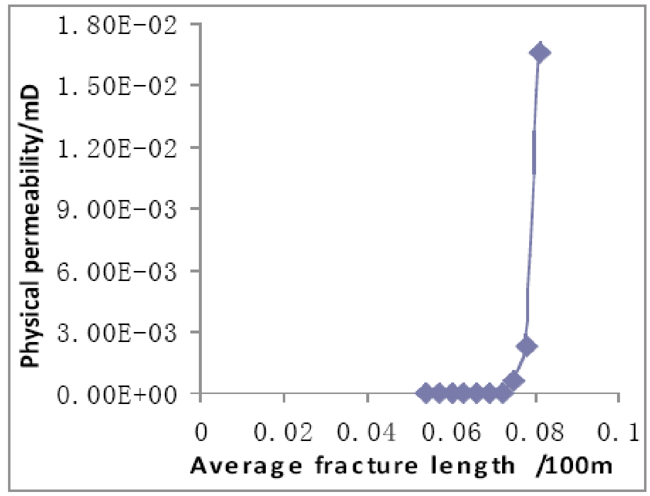

Fig. (7). Permeability versus average length ($p = 0.2$).

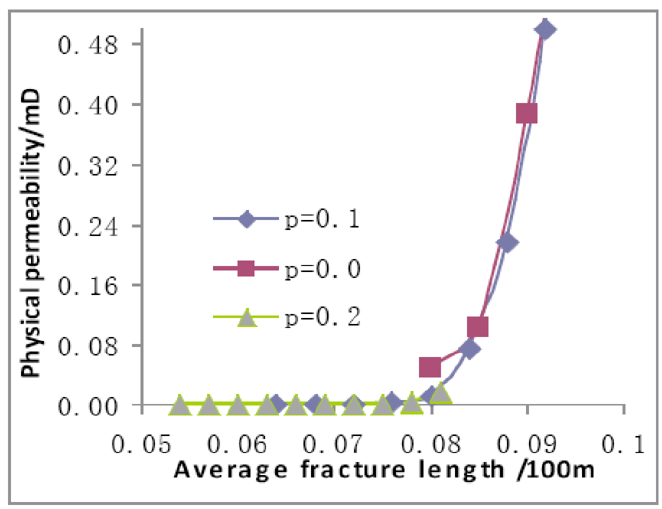

Fig. (8). Permeability versus average length.

The distributions of the seepage skeletons and pressures for pore-controlled percolation and fracture-controlled percolation are shown in Figs. (**9**~**12**).

(1) In Figs. (**9** and **10**), the conditions, $p = 0.2$, $\mu = 0.054$, are just satisfied the threshold. So it is pore-controlled percolation. The seepage penetrates into the area from the left side where the pressure is higher than that in other positions because no flow can form in the disconnected zone and so no pressure occurs. The distribution of the seepage is consistent with that of pressure. The zone with high pressure gradient is

located at the interface between the connected zone and the disconnected zone due to no seepage here . The catastrophic pressure is located at their interface. So the failure of the stratum will be first happen here. During the exploitation of oil and gas in the stratum of pore-controlled percolation, some measures such as hydraulic fracturing or blast-induced fracture techniques should be used to make the disconnected zone permeable. This kind of stratum is not suit for the geological sequestration of captured CO_2.

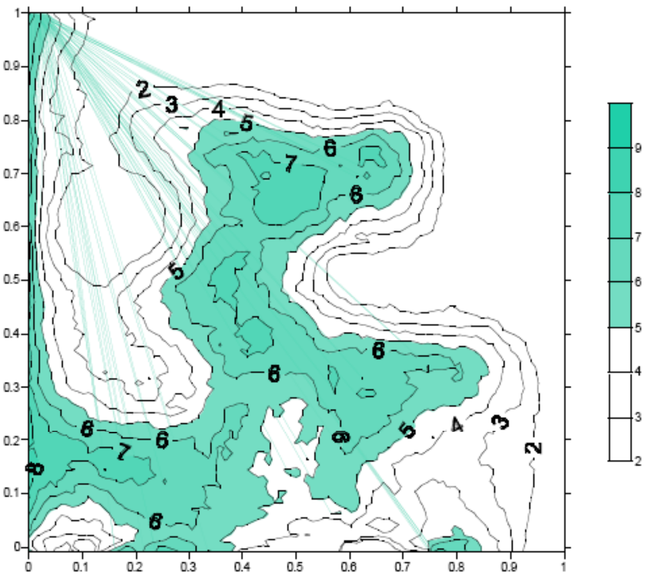

Fig. (9). Distribution of pressure at $P = 0.2, \mu = 0.054$.

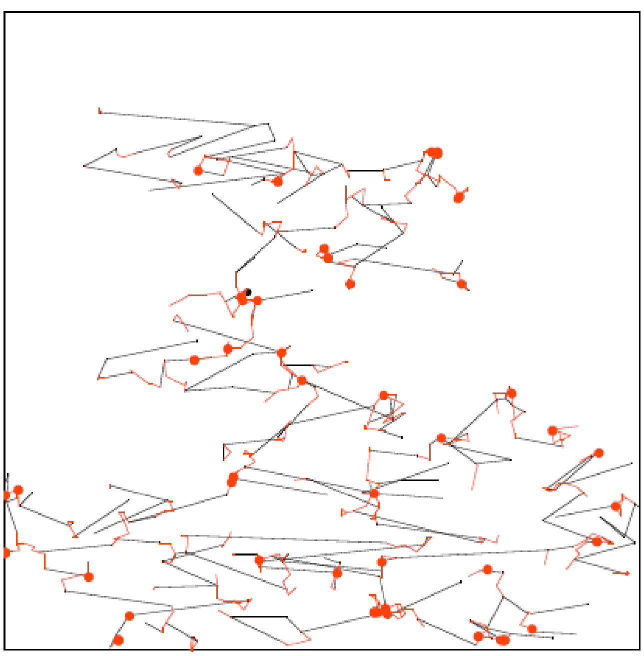

Fig. (10). Distribution of seepage at $P = 0.2, \mu = 0.054$.

(2) In Figs. (**11** and **12**), at the conditions of $p = 0.2$ and $\mu = 0.08$, the fracture network can form connected channel. So it is fracture-controlled percolation. In this case, seepage is fully distributed in the whole area, and the pressure decreases from the left side to the right side because the fracture networks are uniformly distributed in the whole zone according to the method described in the third section. The zone with high pressure gradient is located at the zone between two densely connected fracture clusters which is connected by pores since the flow in pores is smaller than that in fractures and so the pressure is higher at their interface marked by dashed line in Fig. (**12**). This kind of stratum is suit for the exploitation of oil and gas and the geological sequestration of captured CO_2.

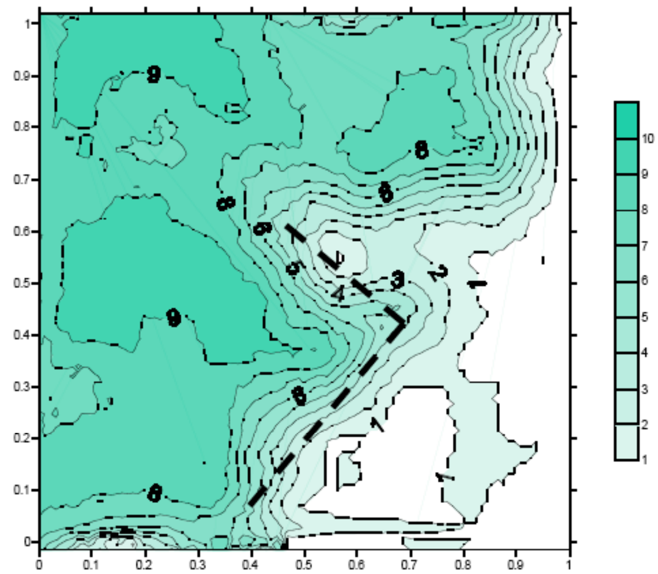

Fig. (11). Distribution of pressure at $P = 0.2, \mu = 0.08$.

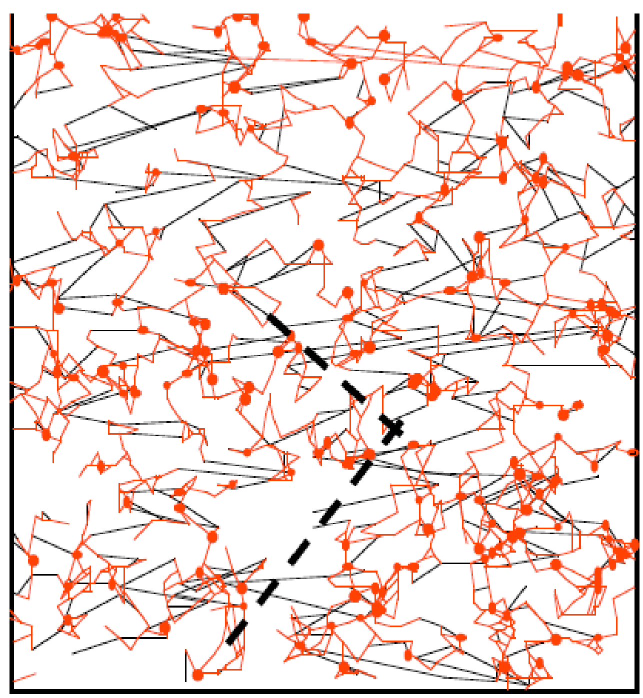

Fig. (12). Distribution of seepage at $P = 0.2, \mu = 0.08$.

Generally, it can be seen that except for the simulation of fracture network in large scale fast and simply, the model presented in this paper can distinguish and eliminate the disconnected fractures, which can lead to a more accuarate permeability than the fracture tensor model.

4.1. Compared with the Theory of Seepage Tensor Model Used in Fractural Rocks

The comparison between the computing results by the double percolation model and that of the fracture tensor model is shown in Fig. (**13**). The tendency of the permeability with the average fracture length computed by these two methods are similar though the values are about 40-50% different which may be caused by the different of the connective area.

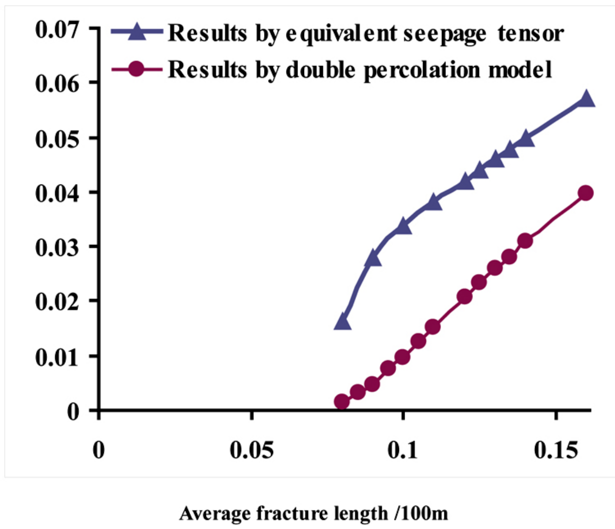

Fig. (13). Comparison of computing and theoretical results.

5. CONCLUSIONS

By introducing the cubic law used in fracture seepage, the double percolation model can be used to compute the permeability of fractural porous media.

The model presented in this paper can consider the low permeability while the fractural seepage tensor model cannot. However, the results are similar computed by these two models when the pore percolation can be neglected.

The seepage can be divided into two types: "pore-controlled" and "fracture-controlled" seepage according to the fracture length. The critical value is the average length of fractures when the seepage can form by only the fracture network.

In the condition of "pore-controlled" seepage, the percolation area and permeability are both small. The pressure gradient is located at the interface between the seepage and non-seepage zones. In the condition of "fracture-controlled" seepage, the permeability is several orders higher than that of "pore-controlled" seepage. The percolation area is large. The pressure gradient is located between connected clusters.

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