Prediction on Deflection of Truss-Core Sandwich Panels in Weak Direction

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Abstract: A truss-core sandwich panel is consisted of two facing plates and a truss-core connecting the top and the bottom surface sheets together. To analyze the bending behavior of a truss-core sandwich panel under static load in its weak direction, the equation for predicting the deflection of a truss-core sandwich panel is deduced through assumptions and theoretical analysis. To verify the accuracy of the presented equation, a comparison between predicted theoretical results and finite element results has been conducted. A parametric study has also been carried out to determine the validity range of the proposed equation. Through the comparison between the parametric results and finite element results, the error variation has been investigated. It is found that the values obtained from the presented equation agree quite well with the results derived from the finite element analysis in a geometrical validity range of truss-core sandwich panels.

Keywords: Truss-core sandwich panels; deflection; weak direction; theoretical equation; validity range.

INTRODUCTION

A steel sandwich panel is composed of upper and lower surface plates with ultra-light inner cores. According to the various types of the cores, the cross sections of the steel sandwich panels can be designed into many kinds of forms, and some typical examples are shown in Fig. (1). The excellent structural forms make it a better choice for ship and offshore platform decking. Due to the flexibility of structural forms, the high flexural stiffness and the small relative density, sandwich panel structures have been widely used in many other practical fields, including civil engineering, navigation and aviation etc. Furthermore, the steel sandwich structures have excellent damping behavior and good energy absorption, which shows a high resistance to dynamic and blast loads.

The top and the bottom facing plates of the steel sandwich panel are connected together by the core stiffeners through mechanical means such as spot welds, rivets, self-tapping screws or other connection methods, making the possibilities in the two principal directions of the sandwich panel different. In the arrangement direction of the core stiffener, the shear rigidity, the flexural rigidity and the torsional rigidity are higher, and this direction is called the strong direction. Perpendicular to the direction of the core stiffener, however, the stiffness is relatively smaller because of its discontinuity in this direction. Hence, this direction is called the weak direction. This form of sandwich panel has been generally used in decking in offshore structures and in blast doors against shock waves, reflecting the superiority of the geometric structural form.

Fig. (1). Different types of cross sections of sandwich panel.

The study on the structural form of the sandwich panel is mainly focused on the physical parameters which influence the mechanical properties. The early representative study of sandwich plates was carried out by Libove et al. (1951) [1] on the elastic constants of corrugated sandwich plate. On this basis, in 1980, William [2] conducted further work to study the elastic constants of super-plastic forming corrugated plate. Later on, Nordstrand (1995) [3] discussed the elastic constants of the corrugated plate, and analyzed the post-buckling parameters of the sandwich-core. In 1999, Lok et al. [4, 5] deduced the equivalent constants of triangle-core sandwich panels. Fung, Tan and Lok (1994) [6] also investigated the equivalent elastic constants of Z-core sandwich panels. From the Refs. [7-9], it is found that constitutive models have contributed greatly to the improvement of efficiency in the simulations of deformation response of sandwich plates with various cores.

Many researchers have spent a lot of effort on investigating the dynamic response of sandwich structures with lattice truss-cores subjected to shock loadings. Xue et al. [10] used three-dimensional FE modeling of the geometry of the sandwich plates and compared the performance of sandwich plates with many other kinds of cores such as pyramidal truss, square honeycomb and folded plate cores. Fleck et al.
[11] developed an analytical method to investigate the dynamic response of metallic sandwich beams subjected to both air and water blasts. It is found that the presented analytical formula were in good agreement with the three-dimensional FE calculations by Xue and Hutchinson [10]. Hazian et al. (2003) [12] estimated the elastic and shear module of a core in a sandwich beam with an aluminum honeycomb using low-velocity impact tests and deflection theory with the sandwich beam without considering the core stiffness. In the contrast, the researches on the static performance of sandwich panels are relatively rare. In this paper, static performance bending behavior of Truss-core sandwich panel under bending in weak direction is studied.

THEORETICAL ANALYSIS OF DEFLECTION FOR A TRUSS-CORE SANDWICH PANEL

Typical Segment in a Truss-Core Sandwich Panel

A truss-core sandwich panel is consisted of two facing plates and many truss cores placed inclining to the surface sheets, as shown in Fig. (2), which are bonded together by a procedure of laser weld. Fig. (2) also illustrates the fact that the truss cores are arranged in only one direction namely x-axis, strengthening the stiffness of the structures in this direction. Perpendicular to the direction of the core stiffeners, i.e., in y-axis, the truss cores are not continuous. Due to this reason, the stiffnesses of the truss-core sandwich panel are quite different in x- and y-directions. A typical segment is isolated from the structure based on the assumption that the deformation of each segment of the truss-core sandwich panel is very similar to the one adjacent to it if the sandwich panel is subjected to uniform loading. Therefore, the typical segment, as shown in Fig. (2), can be studied representively for other segments of the sandwich panel. Based on the former assumption, the deflection of a truss-core sandwich panel can be calculated by multiplying the deflection of each typical segment with the segment numbers.

In order to describe a truss-core sandwich panel more clearly, some definitions are given in Fig. (2). Assuming the thicknesses of the top and the bottom facing plates are identical, it is expressed as $t$. The thickness of the inclined web is expressed as $T$. The height between the mid-plane of the two facing plates is denoted as $h$. The length of a typical segment is described as $2S$ while the distance between every two adjacent truss cores is same. $L$ and $B$ are used to represent the whole length and width of the truss-core sandwich panel respectively. The angle between the facing plate and the inclined truss-core web is expressed as $\alpha$.

Compatibility Conditions and Equations

If a truss-core sandwich panel is subjected to bending action, the total deflection can be divided into two parts: one is the deflection neglecting the rotation of the surface plate and only considering the shear action at the ends of the surface plate. The other is the deflection caused by the rotation of the truss-core web. In calculating the total deflection of the truss-core sandwich plate under the action of uniform loading, a typical segment can be isolated to analyze the deflection. For the typical segment, the inflection points are assumed in the two ends of the facing plates in a typical segment and the mid-point of the inclined truss-core webs, as shown in Fig. (3).

Considering the equilibrium of the internal forces of the typical segment, the shear forces and the bending moments at two ends of the facing plates must be the same. As shown in Fig. (4a), no axial forces exist in the typical segment while the sandwich panel is subjected to bending load. Therefore, only shear force and bending moment exist in the typical segment of the truss-core sandwich panel. $V$ and $M$ are expressed as the shear force and bending moment per unit width respectively. $V/2+\Delta V$ denotes the shear force in the top plate, while $V/2-\Delta V$ denotes the shear force in the bottom plate. For the bending moment at the two ends of the typical segment, $M$ is assigned at the left side while $M+\Delta M$ is assigned at the right side. For brevity in calculation, the bending moment $M$ and $M+\Delta M$ are replaced by a couple of forces $N$ and $N+\Delta N$ respectively, as shown in Fig. (4b). Due to the fact that the deformation caused by the couple of forces $N$ and $N+\Delta N$ is relatively very small, only the deformation caused by shear force is considered in present study. The final internal forces of the typical segment are shown in Fig. (4c), which also expresses that it is anti-symmetric with respect to shear forces. Therefore, a half segment with detailed definitions on some position is isolated from the truss-core sandwich panel hinging at both two ends, as shown in Fig. (5).
caused by rotation of the inclined truss-core of sandwich panel. In order to calculate the value of $\Delta_{\text{1DE}}$, the DE panel is chosen for analysis. The bending moment diagram ($M_p, M_i$) are drawn and shown in Fig. (7).

Fig. (3). Position of inflection points.

Fig. (4). Internal forces of the typical segment.

Fig. (5). Half segment of the typical segment.

Fig. (6). Internal forces and bending moment diagram of half typical segment.

Fig. (7). Bending moment diagram of DE panel.

Then $\Delta_{\text{1DE}}$ can be deduced as follow

$$
\Delta_{\text{1DE}} = \frac{1}{EI_1} \left[ \frac{1}{2} L_{\text{OE}} \times L_{\text{DE}} \times \left( \frac{V}{2} + \Delta V \right) \times \frac{2}{3} L_{\text{OE}} \right] = \frac{\left( \frac{V}{2} + \Delta V \right) L_1^2}{3EI_1}
$$

(2)

Where $I = Bt^3/12$ is the inertial moment of the facing plates about the mid-plane in thickness direction.

To calculate the quantity $L_{\text{DE}}$, it is easy to find that $L_{\text{DA}} = L_{\text{FG}}, L_{\text{DE}} = L_{\text{KF}}, L_{\text{DE}} = S$, as shown in Fig. (8). Then the quantities $L_{\text{DE}}$ and $L_{\text{DA}}$ can be obtained by the following equations

$$
L_{\text{DE}} = \frac{1}{2} \left( S - h \cdot \cot \vartheta \right)
$$

(3)

$$
L_{\text{DA}} = \frac{1}{2} \left( S + h \cdot \cot \vartheta \right)
$$

(4)
Substituting Eq. (3) into Eq. (2), the following equation can be obtained

$$\Delta_{\text{DF}} = \left( \frac{V}{2} + \Delta V \right) \frac{h}{2} \left( S - h \cot \vartheta \right)$$

(5)

Before calculating \( \Delta_{2DE} \), the rotation angle at point D need to be obtained. Assuming the rotation angle is \( \beta \), the inclined DF panel is chosen for analysis. The forces at the top and the bottom facing plates can be equivalent to the force at point D, as shown in Fig. (9). The shear force at point D is denoted as \( 2\Delta V \) with its direction in vertical downward, and the horizontal force is denoted as \( \Delta N \) with its direction toward to the left. Therefore, the total bending moment at point D can be obtained from the following equation

$$M_d = \left( \frac{V}{2} + \Delta V \right) L_{\text{DF}} + \left( \frac{V}{2} - \Delta V \right) L_{\text{ba}}$$

$$= \left( \frac{V}{2} + \Delta V \right) \frac{h}{2} \left( S - h \cot \vartheta \right) + \left( \frac{V}{2} - \Delta V \right) \frac{h}{2} \left( S + h \cot \vartheta \right)$$

(6)

$$= \frac{VS}{2} - \Delta V h \cot \vartheta$$

Using similar method, values at point F can be obtained easily. The forces in inclined panel DF are expressed in Fig. (9a).

Displacement at Point G

The displacement at point G is also consisted of two parts: the deflection \( \Delta_{1FG} \) caused by shear force at the end of the bottom facing plate and \( \Delta_{2FG} \) caused by rotation of the inclined truss-core of sandwich panel. \( \Delta_{1FG} \) can be obtained from Fig. (10) using unit load method and expressed in Eq. (9).

\[
\Delta_{1FG} = \frac{1}{EI_{1}} \left[ \frac{1}{2} \times L_{\text{FG}} \times \left( \frac{V}{2} - \Delta V \right) \times \frac{2}{3} \times L_{\text{FG}} \right]
\]

(9)

Assuming the rotation angle at point F is \( \theta \), the inclined plate DF is isolated for analysis, and the bending moment diagram is shown in Fig. (11).
Then \( \theta \) can be calculated and shown as follows:

\[
\theta = \frac{1}{EI} \times \frac{1}{2} \times \frac{h}{\sin \alpha} \times \frac{1}{2} \times \left( \frac{VS}{2} - \Delta V h \cot \alpha \right) = \frac{\theta}{6EI} \sin \alpha \cdot (10)
\]

The deflection \( \Delta_{2FG} \) can be obtained in the following equation:

\[
\Delta_{2FG} = \theta \cdot L_{FG} = \frac{\left( \frac{VS}{2} - \Delta V h \cot \alpha \right)(S + h \cot \alpha)h}{12EI \sin \alpha} (11)
\]

**Overall Deflection of a Truss-Core Sandwich Panel**

As can be known from the compatibility condition and continuous consistent recurrence, the deflection at point E is equal to that at point G, which is shown as follows:

\[
\Delta_{1DE} + \Delta_{2DE} = \Delta_{1FG} + \Delta_{2FG} (12)
\]

Substituting Eqs. (5), (8), (9), (11) into Eq. (12), the following equation can be obtained:

\[
\left( \frac{V}{2} + \Delta V \right) \left( \frac{1}{2}(S - h \cot \alpha) \right) \frac{(S - h \cot \alpha)}{3EI} + \frac{1}{3EI} = \frac{(S - h \cot \alpha) \times h \times \left( \frac{VS}{2} - \Delta V h \cot \alpha \right)}{12EI \sin \alpha} (13)
\]

Eq. (13) can be simplified into the Eq. (14), then \( \Delta V \) is obtained in the following:

\[
\Delta V = \frac{m}{n} \frac{V}{n} (14)
\]

Where

\[
m = \frac{3S^2 h \cot \alpha + h^3 \cot^3 \alpha}{6EI} + \frac{Sh^2 \cot \alpha}{3EI \sin \alpha} (15)
\]

\[
n = \frac{S^3 + 3Sh^2 \cot^2 \alpha}{6EI} + \frac{2h^3 \cot^2 \alpha}{3EI \sin \alpha} (16)
\]

The deflection of a typical segment \( \Delta_1 \) is generalized as the following:

\[
\Delta_1 = 2(\Delta_{1DE} + \Delta_{2DE} + \Delta_{1FG} + \Delta_{2FG}) (17)
\]

Considering that beam is a problem of plane stress while panel is in plane strain state, elastic modulus \( E \) in the equations should be replaced by \( E/(1-\nu^2) \). Assuming that \( n \) is the segment number, the deflection of a truss-core sandwich panel can be obtained by multiplying \( k/2 \) with \( \Delta_1 \). Therefore, the overall deflection of the sandwich panel \( \Delta_0 \) is depicted as the following equation.

\[
\Delta_0 = k \cdot \frac{\left( \frac{S - h \cot \alpha}{2} \right) \left( \frac{V}{2} + \Delta V \right)}{3EI} + k \cdot \frac{(S - h \cot \alpha) \times h \times \left( \frac{VS}{2} - \Delta V h \cot \alpha \right)}{12EI \sin \alpha} (18)
\]

Finally, substituting Eq. (14) and \( V = F/2 \) into Eq. (18), the following equation can be finally obtained:

\[
\Delta_0 = kF(1 - \nu^2) \left[ \frac{(S - h \cot \alpha) \times h \times \left( \frac{VS}{2} - \Delta V h \cot \alpha \right)}{2EI} \right] (19)
\]

**CASE STUDY**

A simply supported sandwich beam with 7 truss-cores is selected for analysis, as shown in Fig. (12). Same steel materials with elastic modulus \( E = 206 \text{ GPa} \) and Poisson’s ratio \( \nu = 0.3 \) are given to the truss-core sandwich panel. At the mid-span of the sandwich beam in weak direction, a vertical line load with a value of \( F = 75 \text{ N/m} \) is applied to the tip.

In order to study the validity range of Eq. (19), several parameters have been analyzed. The parameters include the facing plate thickness \( t \), the inclined web thickness \( T \), the angle \( \alpha \) between facing plates and inclined web plates, the height \( h \) between the mid-plane of the two facing plates, and the half segment length \( S \).

**Fig. (12).** A truss-core sandwich beam.

**Influence of the Thickness Ratio \( T/t \)**

To verify the influence of the thickness ratio \( T/t \), 8 truss-core sandwich beam models have been analyzed. For the analyzed sandwich beam, the length and the width are selected to be 1350 mm and 50 mm respectively. Other dimensions are listed in Table 1.

**Table 1. Dimensions of Truss-Core Sandwich Beams**

<table>
<thead>
<tr>
<th>Model</th>
<th>( \nu/mm )</th>
<th>( T/mm )</th>
<th>( \alpha/\text{degree} )</th>
<th>( h/mm )</th>
<th>( S/mm )</th>
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</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.5</td>
<td>2</td>
<td>45(^\circ)</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>B1</td>
<td>0.67</td>
<td>2</td>
<td>45(^\circ)</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>C1</td>
<td>0.8</td>
<td>2</td>
<td>45(^\circ)</td>
<td>50</td>
<td>100</td>
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</table>
Table 1. contd…

<table>
<thead>
<tr>
<th>Model</th>
<th>t/mm</th>
<th>T/mm</th>
<th>α/degree</th>
<th>h/mm</th>
<th>S/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>D₁</td>
<td>1</td>
<td>2</td>
<td>45°</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>E₁</td>
<td>1.33</td>
<td>2</td>
<td>45°</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>F₁</td>
<td>2</td>
<td>2</td>
<td>45°</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>G₁</td>
<td>5</td>
<td>2</td>
<td>45°</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>H₁</td>
<td>5</td>
<td>1</td>
<td>45°</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Substituting these parameters into the Eq. (19), the theoretical values can be obtained. Using the software ABAQUS, finite element values of the truss-core sandwich beam models have been obtained. Both the theoretical values and the finite element values are tabulated in Table 2. Meanwhile, errors can be assessed by comparing theoretical values with finite element values. For a clear comparison, both the two kinds of results are plotted together in Fig. (13).

Table 2. Comparison of the Results

<table>
<thead>
<tr>
<th>Model</th>
<th>T/t</th>
<th>FE value (mm)</th>
<th>Theoretical value(mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>4</td>
<td>-1.1010</td>
<td>-1.1340</td>
<td>2.9</td>
</tr>
<tr>
<td>B₁</td>
<td>3</td>
<td>-0.4868</td>
<td>-0.4876</td>
<td>1.68</td>
</tr>
<tr>
<td>C₁</td>
<td>2.5</td>
<td>-0.3048</td>
<td>-0.2985</td>
<td>2</td>
</tr>
<tr>
<td>D₁</td>
<td>2</td>
<td>-0.1766</td>
<td>-0.1665</td>
<td>5.7</td>
</tr>
<tr>
<td>E₁</td>
<td>1.5</td>
<td>-0.09692</td>
<td>-0.08617</td>
<td>12.5</td>
</tr>
<tr>
<td>F₁</td>
<td>1</td>
<td>-0.0505</td>
<td>-0.0416</td>
<td>17.6</td>
</tr>
<tr>
<td>G₁</td>
<td>0.4</td>
<td>-0.01297</td>
<td>-0.00819</td>
<td>36.7</td>
</tr>
<tr>
<td>H₁</td>
<td>0.2</td>
<td>-0.0165</td>
<td>-0.0099</td>
<td>40</td>
</tr>
</tbody>
</table>

From the comparison, it can be concluded that both the theoretical values and finite element values grow rapidly with thickness ratio $T/t$ increasing regularly, while errors between theoretical results and FE results become more and more small. As can be seen from Fig. (13b), when $T/t$ is larger than 2.0, the error is acceptable with the range of 5%. On the contrary, when the ratio $T/t$ is less than 2.0, the differences between theoretical values and FE results become larger, causing the presented equation to be not suitable. In this case, the models deform in a way not satisfying the above assumption—the inflection points in the typical segment are located in the mid-point of the facing plates and the truss-core webs. Accordingly, the accuracy and reliability of the derived equation should be assessed carefully when it is used for design purpose.

**Influence of Angle α**

In order to verify the influence of angle $α$, 7 models have been analyzed. In the models, the thicknesses of facing plates and inclined web cores are all selected as 2 mm, while other dimensions are: $h$=50 mm, $S$=100 mm. Parameter $α$ is changing among these values: 30°, 40°, 45°, 60°, 70°, 80°, 90°. Theoretical values and FE results are plotted together in Fig. (14a), while the error-angle relationship is described in Fig. (14b).

It is depicted clearly that the trend of theoretical values agrees with that of FE values. The derived equation is reasonably accurate when the angle $α$ is not less than 60°, in which the error is less than 5%.

**Influence of Half Segment Length $S$**

Similarly, to find the influence of half segment length $S$, 8 FE models have been analyzed. The parameters $T$, $t$, $h$, and $α$ are kept as 2 mm, 2 mm, 50 mm, and 45° respectively. In Fig. (15), the deflection comparison and errors between theoretical values and FE results are described clearly with the half segment length $S$ changing among the following values: 75, 80, 90, 100, 125, 150, 200, 300.

It can be concluded from Fig. (15a) that FE results and theoretical values increase rapidly as the half segment length...
$S$ increases. Fig. (15b) shows that the error between theoretical and FE results is less than 4.5% when half segment length $S$ is larger than 150mm. Accordingly, $S > 150$ mm is regarded as the reasonable range in the presented equation.

(a) Deflection of theoretical and FE results
(b) Error between theoretical and FE results

Fig. (14). Comparison of the deflection and errors of truss-core sandwich beams.

Influence of Height $h$

In order to verify the influence of the height $h$, 8 models have been analyzed with the height $h$ changing from 20 mm to 90 mm by increasing each step of 10 mm. In the models, $T$, $t$, $S$ and $\alpha$ are assigned to be 2 mm, 0.5 mm, 100 mm and 45° respectively. The relationship of deflection comparison between theoretical and FE results to height $h$ is plotted together in Fig. (16a), while error-$h$ relationship is drawn in Fig. (16b).

CONCLUSIONS

Based on recurrence conditions and compatibility conditions, the bending behavior of truss-core sandwich panels in weak direction is analyzed and an equation for calculating the deformation of truss-core sandwich panel is derived. In this study, several FE models have been simulated to verify the accuracy of the presented equation. The presented
weak direction when a geometrical validity range of is assured.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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Declared none.

REFERENCES


