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More on Quantum Fast Fourier Transform by Linear Optics

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Abstract: It is shown by Barak and Ben-Aryeh that the quantum fast Fourier transform by linear optics is applicable for an n-qubit system. We point out that the quantum fast Fourier transform by linear optics is applicable not only for a qubit system but also for a qutrit system and a compound system of qubits and qutrits etc. We compute an upper bound for the number of the beam splitters, which gives a basis for planning an experiment.

Keywords: Beam splitter, KLM Scheme, quantum fast fourier transform.

1. INTRODUCTION

The quantum Fourier transform plays an important role in quantum algorithms such as Shor's prime factorization algorithm [1]. Moreover, the quantum Fourier transform by linear optics is an indispensable component of the KLM scheme [2]. In linear optics, we refer to the quantum Fourier transform with smaller number of beam splitters as the quantum fast Fourier transform. To carry out an experiment in optics, it is significant to diminish the number of beam splitters. Barak and Ben-Aryeh [3] have shown that for $N = 2^n$, which corresponds to an n-qubit system, an N-point quantum discrete fast Fourier transform is realizable. For $N = 2^n$, the classical Cooley-Tukey algorithm [4, 5] with radix 2 is applicable. The radix 2 algorithm consists of the so called butterfly operations. Any node connecting two butterfly operations has two inputs and two outputs. Since a beam splitter has two inputs and two outputs, Barak and Ben-Aryeh have placed a beam splitter on each node connecting two butterfly operations.

Apparently, for $N \neq 2^n$, it seems difficult to apply the Cooley-Tukey algorithm to the quantum discrete Fourier transform by linear optics. This is, however, not the case, because the quantum fast Fourier transform with radix r can be realized by combining $1/2r(r-1)$ beam splitters as a unit. The purpose of this letter is to point out that for $N \neq$ prime, the Cooley-Tukey algorithm is directly applicable to the quantum discrete Fourier transform by linear optics and that the number of beam splitters can be reduced. We also give an upper bound for the number of beam splitters to carry out an N-point quantum discrete Fourier transform.

2. FACTORIZATION OF FOURIER TRANSFORM

An N-point quantum discrete Fourier transform (F_N) in optics is described by

$$b_k^\dagger = 1/\sqrt{N}\sum_{j=0}^{N-1} a_j^\dagger W^{jk}, k = 0,1,\cdots,N-1, \tag{1}$$

where $W = e^{2\pi/Ni}$ and $a_j^\dagger$ is the j-th mode input photon creation operator and $b_k^\dagger$ is the k-th mode output photon creation operator. Fig. (**1**) is a schematic depiction of F_N. In the classical discrete Fourier transform $a_j^\dagger$ and $b_k^\dagger$ are replaced by the j-th component and the k-th component of two N-vectors, respectively. Since the quantum discrete Fourier transform F_N is an $N \times N$ unitary transformation, F_N can be realized with $1/2N(N-1)$ beam splitters and some phase shifters in linear optics [6].

The $N \times N$ unitary matrix in Eq. (1) is common to the classical and the quantum cases; therefore, the classical Cooley-Tukey algorithm is directly applicable to the quantum case. When N is factorized as $N = r_1 r_2$, according to the Cooley-Tukey algorithm, F_N can be factorized as a sequence of r_2 F_{r_1}'s and r_1 F_{r_2}'s. Therefore, the number of beam splitters to perform F_N is diminished to $1/2N(r_1 + r_2 - 2)$ in this case. When N is prime factorized as $N = r_1 r_2 \cdots r_s$, the number of beam splitters is diminished to $1/2N(r_1 + r_2 + \cdots + r_s - s)$. Especially when $N = 3^n$, which corresponds to an n-qutrit system, the number of beam splitters is diminished to Nn. The case $N = 2^n 3^m$ corresponds to a compound system of n-qubit and m-qutrit.

The above situation is illustrated in Fig. (**2**) for the case $N = 12 = 3 \times 4$, a one-qutrit and two-qubit system. In this case the number of beam splitters is diminished from 66 to 30. Each F_4 is factorized by four F_2's, and each F_4 is realized by four beam splitters [3]. Therefore, the number 30 diminishes further to 24.

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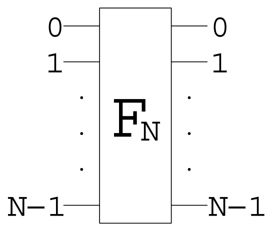

Fig. (1). An N -point quantum discrete Fourier transform. The left hand side indicates inputs $\{a_0^\dagger, a_1^\dagger, \cdots, a_{N-1}^\dagger\}$, and the right hand side indicates outputs $\{b_0^\dagger, b_1^\dagger, \cdots, b_{N-1}^\dagger\}$.

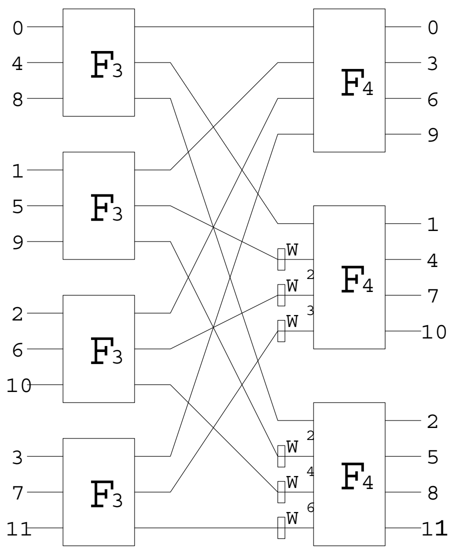

Fig. (2) A factorization of F_{12} by four F_3 's and three F_4 's. The small box W^m indicates corresponding phase shifter. The inputs and outputs are arranged according to the Cooley-Tukey algorithm.

CONCLUSION

We have pointed out that the classical Cooley-Tukey algorithm is applicable to the N -point quantum discrete Fourier transform in linear optics and that the number of beam splitters can be reduced when $N \neq$ prime; the cases $N = 3^n$ and $N = 2^n 3^m$ correspond to the n -qutrit system and the compound system of n -qubit and m -qutrit, respectively. The case of $N = 2^n$, which corresponds to the n -qubit system, studied by Barak and Ben-Aryeh is the most important and a special situation of our indication. We have given the upper bound for the number of beam splitters to carry out the N -point quantum discrete Fourier transform. To carry out the N -point quantum discrete Fourier transform we also need to know angles and phases of linear optics constituting F_r, r being a prime factor of N. Our result, however, gives a basis for planning an experiment.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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