# Self-Consistent Hot Spot Tracing in Kinetic Simulations: A Method to Improve the Test Particles Method 

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#### Abstract

One of the most important advantages of particle simulation as compared to fluid simulation is the capacity for working with and tracing particles. In particle simulations, the test particle method is usually used to get some idea of the behavior of plasma or other substances. In this method, first, a small number of particles are injected into the frame of fixed electromagnetic fields. Then, movement of particles is investigated using the pattern of their electric fields. This method is useful; however, as we need to work with fixed fields in our system, it lacks precision. In this work, we adapted the particle simulations method, adding the flexibility of working with dynamic fields that come directly from the simulation.

Here we have tried to investigate particle entry from the solar wind with northward Interpanetary Magnetic Filed (IMF) to the magnetospheric cusp. As our initial results shows, self consistent path of particles does not follow the magnetic field lines going to the cusp that is slightly in contrast to the conventional non self consistent results from test particle method.


Keywords: Cusp particle entry, IMF, magnetosphere, particle in cell (PIC) simulation, self consistent hot spot tracing, solar wind, test particles method.

## INTRODUCTION

The test particle method involves first injecting a limited amount of particles randomly into the fixed frame of the magnetic field resulting from the simulation, and then examining how these particles move and react in response to the background electromagnetic fields [1]. In this method, as the number of injected particles is small, fields will not change due to the particles. This method gives a good idea of particle dynamics. However, for capturing the real physics of the simulation and achieving better insight into problems, we should include the field's dynamics. In this work, our aim was to see particle dynamics with live electromagnetic fields. Tracing particles in normal PIC simulation has many potential applications. For example, tracing particles could improve investigation of particle transportation via magnetic reconnection in plasma astrophysics and observing patterns of particle dynamics in different field topologies. Tracing one or several particles in PIC codes is common, but to our knowledge, there has been no particle tracing for hot spots in PIC codes.

Here we had only a simple hot spot tracing from Solar wind to the cusp and our initial results shows that particles mostly enters the cusp from the day-side that means re connection site in the tail ward side of the cusp is not injecting so many particles.

## Simulation Model

In our simulation, we use the same initial conditions to form the magnetosphere [2], radiating boundary conditions

[^0][3] and charge-conserving formulas [4, 5] as in the works of Nishikawa and Ohtani $[6,7]$. The grid size $\Delta \cong 0.5 R E$, and $\Delta t \cong 1$ is the time step $\left(\omega_{p e} \Delta t \cong 0.12\right)$. Here $\Delta=\Delta x=\Delta y=\Delta z$.

Initially, we use about $3 \times 10^{6}$ electron-ion pairs, which corresponds to a uniform particle density of $\tilde{n}=8.0$ pairs per cell across the simulation domain $(215 \Delta \times 145 \Delta \times 145 \Delta)$. All the simulation parameters are improved with respect to those used in previous works ( $\Delta \cong 1 R E, \quad \omega_{p e} \Delta t \cong 0.2, \quad \tilde{n}=4.0$ pair per cell in [8], or $\Delta \cong 1 R E, \quad \omega_{p e} \Delta t \cong 0.84, \quad \tilde{n}=0.8$ pair per cell in [Nishikawa, 1998]). Here " $\sim$ " denotes the normalized parameters defined as ('"e, and i"' denotes electron and ion, respectively): Thermal velocity $\tilde{v}_{t h e, i}=\frac{v_{t h e, i}}{\Delta / \Delta t}$ Debye length: $\tilde{\lambda}_{D e, i}=\frac{\tilde{v}_{t h e, i}}{\tilde{\omega}_{P e, i}}$, Larmorgyroradius: $\tilde{\rho}_{c e, i}=\frac{\tilde{v}_{t h e, i}}{\tilde{\omega}_{P e, i}}$, Inertia length: $\tilde{\lambda}_{c e, i}=\frac{\tilde{C}}{\tilde{\omega}_{P e, i}}$, Gyrofrequency: $\tilde{\omega}_{c e, i}=\frac{\omega_{c e, i}}{(\Delta t)^{-1}}=\frac{\tilde{B} \Delta m_{e}}{\Delta t m_{e, i}}$, Plasma frequency: $\quad \tilde{\omega}_{p e, i}=\frac{\omega_{p e, i}}{(\Delta t)^{-1}}=\sqrt{\frac{\tilde{q}_{e, i}^{2} \tilde{n}_{e, i}}{\varepsilon_{0} m_{e, i}}} \Delta t, \quad$ Gyroperiod: $\tilde{\tau}_{c e, i}=\frac{2 \pi}{\tilde{\omega}_{c e, i}}, \quad \beta$ ratio: $\tilde{\beta}_{e, i}=\frac{\tilde{T}_{e, i} \tilde{\omega}_{P e, i}^{2}}{\tilde{B}^{2}}$ values of normalized ambient plasma parameters used in our simulation are following: $\quad \tilde{v}_{t h e, i}=(0.09,0.045), \quad \tilde{\lambda}_{D e, i}=(0.75,1.5)$, $\tilde{\omega}_{p e, i}=(0.125,0.031), \tilde{\omega}_{c e, i}=(0.20,0.013), \tilde{\rho}_{c e, i}=(0.45,3.5)$, $\tilde{\lambda}_{c e, i}=(4.2,16.1), \quad \tilde{\boldsymbol{\tau}}_{c e, i}=(31.4,502), \quad \tilde{\beta}_{e, i}=(0.2,0.8)$,


Fig. (1). Ion density, global 3D PIC simulation with northward IMF direction at the time step 3000. The marked circle is the maximum density in the high altitude cusp.
$\tilde{T}_{e, i}=(0.008,0.032)$. The center of the current loop that generates the dipolar terrestrial magnetic field is located at $(80 \Delta, 72.5 \Delta, 73 \Delta)$. Within the time range $0<\tilde{t}<1000 \Delta \tilde{t}$, a drift velocity $\tilde{v}_{\text {Sol }}=-0.5 \tilde{C}$ representing the solar wind, is applied along the x-direction without an IMF. Here $\tilde{C}=0.5$ is the speed of light. The injected solar wind density also has $\tilde{n}=8.0$ electron-ion pairs per cell, the mass ratio is $m_{i} / m_{e}=16$, and the electron and ion thermal velocities are $\tilde{v}_{t h e}=\left(\tilde{T}_{e} / v_{e}\right)^{1 / 2}=0.18 \tilde{C}, \quad$ and $\quad \tilde{v}_{t h i}=\left(\tilde{T}_{i} / v_{i}\right)^{1 / 2}=0.09 \tilde{C}$, respectively. The electron plasma frequency is $\tilde{\omega}_{p e, i}=0.125$.

## METHOD

In the tracing particles method, we first determine the region of interest then selecting particles including ions or electrons in this region. In the next step, we re-run the same code with the same amount of particles. All parts of the code initialization, including particle number and fields, were the same; the only change was that this time the code was run with tracing of marked particles. We can also turn off other diagnostics from a number of time steps before the present time and then see how particles moved to the area of interest, or how they established a favorite pattern.

In this method, first we find any hot spots in our simulation outputs $[2,9,10]$ as can be seen from the following figure (Fig. 1):

In the red dashed circle area (Fig. 1), we have the maximum high altitude cusp density. Researchers are
interested to see how this hot spot forms during solar wind interaction with the earth's magnetic field and to observe how particles are transferred via the reconnection region to the cusp.

Secondly, we mark the particles (ions or electrons) in this area, and then go back a number of time steps in the simulation (for example 1000 or 2000 time steps). Next, as in particle simulation, we work directly with particles and trace them. Tracing these marked particles, we see how they are pushed by the Lorentz force in the system. We can directly see how this hot spot formed. By tracing about 40 particles in the dashed circle from 500 time steps before, we have the following results (Fig. 2a, b):

Above are the results in the $\mathrm{X}-\mathrm{Z}$ plane. For clarity, results for $\mathrm{X}-\mathrm{Y}$ and $\mathrm{Y}-\mathrm{Z}$ plots are not shown.

We investigated the results in comparison to the Interplanetary Magnetic Field (IMF) rotation. In this simulation, IMF is built in 1500 time step with the northward direction and will rotate smoothly to Dawn-Dusk from 2900 to 3200 time steps (for 300 time steps) [10]. As can be seen from the results, particles are moving normally before rotation and immediately after IMF rotation (Time Step: 2900), particles start to gather in the high altitude cusp. These results are is in good agreement with experimental observations that show magnetospheric cusp dynamics depend strongly on IMF rotations [11]. This tracing shows the relation of IMF rotation and particle entry to the cusp. Further, it shows that particles enter the cusp not from the reconnection region but from magnetosheath, as observed in experiments by Lavraud experimental [11].


Fig. (2). results of particle tracing in 3D PIC simulation a) from 2500 to 2780 , b) from 2800 to 3100 . Results are particle movement in dynamic fields, but for simplicity we plotted particles in the fixed field of 2900 time step.

Particle entry also can be investigated using plasma currents or the test particle method. However, using the tracing method gives much more direct and exact vision as dynamic fields using original simulation with the same number of particles has been used.

A fixed magnetic field for every 100 time steps was used for the plots in Fig. (3), but a dynamic magnetic field has been used in the tracing particles.

The following diagram illustrates the concise algorithm of the tracing particle method:

## Particle Index

In the PIC simulation, in sorting subroutines particles may eliminate or exchange information with other particles. In this simulation we investigated how many particles, out of 44 particles selected, would be reliable for tracing. For this purpose we measured the history of displacement for each particle. If the displacement was small within boundaries, we determined that this particle was reliable for our analysis. We found that we can rely on 28 particles out of 44 particles (measurement data not shown). Notably, almost 63 percent of particles are reliable for our tracing work.

## Comparing with Test Particle Method Results

We compared the above results with the results of the test particle method. Both cases with static Electro-Magnetic


Fig. (3). Diagram of particle tracing method in PIC simulation.
(EM) fields and dynamic fields were considered. The same number of particles used in the tracing method was used for
(a)


Fig. (4). Results of Test particle method with the initial EM fields of 2500 time step and about $3 \times 10^{6}$ electron-ion pairs for cases of a) static EM fields, b) dynamic EM fields.
the test particle method. Particles were injected evenly into the box from 2500 time step; simulation continued until 3000 time step. Fig. (4a, b) shows the results for static and dynamic EM fields respectively:

As it can be seen from Figs. (1, 4a) the test particle method also shows the hot spot in the cusp region. However, the spot is located at lower altitude as compared to the PIC. The comparison in Figs. $(\mathbf{1}, \mathbf{4 b})$ shows that using a dynamic field with the test particle method yields poor results. Furthermore, when the fixed frame of EM fields is used, it is impossible to catch the physics relating to IMF rotation, which is very important in solar-terrestrial simulation.

The relative time required for the tracing particles method, as compared with the test particle method, depends on the number of particles used in the simulation. However, in the case of the proposed tracing particle method, we need only trace particles about 200 time steps back. As shown in Fig. (3), tracing from 2800 time step would be enough and gives the required physics information.

## CONCLUSION

To understand results in PIC simulation, scientists usually use the test particles method, which uses fixed magnetic field and a limited number of particles. In this work, we decided to see if we can do the same thing with dynamics magnetic field and full particle interactions. Then we saw that we can mark particles in the considered region and go backward in the simulation, tracing those particles to visualize particle dynamics in the hot spot. Compared to the test particle method, this method may have a higher cost in computer time required for simulation, but is worthwhile as it gives more exact dynamics of the particles and fields. Here, our initial results, Fig. (2) challenges the common accepted physics of particle entry to the cusp from tailward reconnection site that has been done by non self consistent test particle method. However to make this idea clear, it needs more investigation using this method that we will try to do in future works.

## Future Work

As our simulation is a full particle simulation, it is difficult to see particle rotations and movements using normal 3D plots. In future studies we will set up a 3D visualization screen (Virtual Reality) to see more exact 3D
results and better trace particles with the background 3D magnetic field. We hope to achieve even better results and derive greater understanding of particle entry to the cusp from reconnection region using tracing PIC simulation particles and 3D visualization.

## CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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