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Optimization of Tilt Angle for Solar Collector to Receive Maximum Radiation

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Abstract: This article examines the theoretical aspects of choosing a tilt angle for the solar flat-plate collectors used at ten different stations in the world and makes recommendations on how the collected energy can be increased by varying the tilt angle. In this paper, the collector surface is assumed to be facing toward equator. For Indian stations, the calculations are based upon the measured values of monthly mean daily global and diffuse solar radiation on a horizontal surface. For other stations, the calculations are based upon the data of monthly mean daily global solar radiation and monthly average clearness index on a horizontal surface. It is shown that nearly optimal energy can be collected if the angle of tilt is varied seasonally, four times a year. Annual optimum tilt angle is found to be approximately equal to latitude of the location. It is found that the loss in the amount of collected energy is around 1 % if the angle of tilt is adjusted seasonally instead of using β_{opt} for each month of the year. The loss of energy when using the yearly average fixed angle is around 15 % compared with the monthly optimum tilt.

Key Words: Tilt angle, solar radiation, solar energy and solar collector.

INTRODUCTION

Solar energy is a very large, inexhaustible source of energy. Quantitative assessment of solar radiation incident on a tilt plane is very important to engineers designing solar energy collecting devices, to architects designing buildings, and to agronomists studying insolation on vegetation on mountain slopes. To meet all these requirements, one should know the intensity of radiation falling upon the sloping surface and its variation over a period of one year.

The most common form of solar energy utilization in the world is for water heating applications, mainly during the winter months. Generally conventional liquid flat-plate collectors are used in domestic hot water applications and the roofs of many houses and flats in cities are fitted with such collectors. These systems are manufactured and installed by ordinary ironmasters with little or no knowledge of solar or heat theory. These collectors are mounted with their surfaces facing towards the equator and the tilt angle is set approximately equal to latitude.

The optimum tilt (inclination) of solar collector with respect to user is an important subject from application of thermal / electrical energy point of view. By utilizing maximum solar energy through the optimum tilt, we are able to harness the energy needed without polluting our environment. It reduces the CO_2 emissions in the atmosphere which is a major culprit for Global warming. By reducing CO_2 emissions in the atmosphere, carbon credit can also be earned which is an international issue now a day.

Optimum tilt can be achieved by use of tracking systems. There are two types of tracking systems, Manual and Automatic. Following methods have been adopted in order to achieve optimum tilt. (1) Monthly based optimization: Both manual and automatic tracking system can be used for monthly based optimization. (2) Season based optimization: In this case also manual and automatic both tracking systems can be used. Automatic tracking system will be expensive as compared to manual one. (3) Annual based: In this case no need of automatic tracking system and hence only manual tracking system is used.

Solar radiation data is usually measured in the form of global and diffuse radiation on a horizontal surface at the latitude of interest. Flat-plate solar collectors are tilted so that they capture the maximum radiation and the problem of calculating solar radiation on a tilted surface is in determining the relative amount of beam and diffuse radiation contained in the measured horizontal global radiation. Since the fiat plate solar collectors are positioned at an angle to the horizontal, it is necessary to calculate the optimum tilt angle which maximizes the amount of collected energy. The best way to collect the maximum solar energy is by using solar tracking systems to follow the sun as it moves each day, and thus to maximize the collected beam radiation. It is possible to collect 40% more solar energy by using a two-axis tracking system [1] and it is estimated that in sunny climates, a flat-plate collector moved to face the sun twice a day can intercept nearly 95% of the energy collected using a fully automatic solar tracking system [1]. Tracking systems are expensive, need energy (usually solar energy is used) for their operation and they cannot easily be made applicable to solar collectors used for water heating purposes.

It is generally known that in the northern hemisphere, the optimum collector orientation is south facing ($\gamma = 0$) and the optimum tilt depends upon the latitude and the day of the year. In winter months, the optimum tilt is greater (usually latitude + 15^o), whilst in summer months, the optimum tilt is less (usually latitude-15°). There are many papers in the lit-

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1876-3871/09 2009 Bentham Open

erature which make different recommendations for the optimum tilt, based only on the latitude [2, 3]. In practice, the collector plate is usually oriented south facing and at a fixed tilt which is set to maximize the average energy collected over a year.

Chiou and El-Naggar [4] gave a method to calculate the optimum tilt angle of an equator-facing collector in the heating seasons. Kern and Harris [2] calculated the optimum tilt angle for equator-facing collectors, based on only beam radiation.

E1-Sayed [5] has carried out an analysis to determine the optimum tilt angle by considering the effects of the latitude, number of glass covers, clearing index and the solar reflectivity.

This paper examines the theoretical aspects of choosing a tilt angle for the solar flat-plate collectors used at ten different locations in the world and makes recommendations on how the collected energy can be increased by varying the tilt angle seasonally four times a year. A computer program is developed to simulate the collected energy as the tilt angle is varied.

Estimation Techniques

As most published meteorological data give the total radiation on horizontal surfaces, correlation procedures are required to obtain insolation values on tilted surfaces from horizontal radiation. Monthly average daily total radiation on a tilted surface (H_T) is normally estimated by individually considering the direct beam (H_B), diffuse (H_S) and reflected components (H_R) of the radiation on a tilted surface. Thus for a surface tilted at a slope angle from the horizontal, the incident total radiation is given by

$$H_T = H_B + H_S + H_R \quad (1)$$

Several models have been proposed by various investigators [6-11] to calculate global radiation on tilted surfaces from the available data on a horizontal surface. The only difference among the models appears in the assessment of sky-diffuse component. Based on the assumptions made, the estimation models can be classified into isotropic [7] and anisotropic [6] ones. The daily beam radiation received on an inclined surface can be expressed as

$$H_B = (H - H_d)R_b \quad (2)$$

where H and H_d are the monthly mean daily global and diffuse radiation on a horizontal surface, and R_b is the ratio of the average daily beam radiation on a tilted surface to that on a horizontal surface. The daily ground reflected radiation can be written as

$$H_R = H\rho(1-\cos\beta)/2 \quad (3)$$

Liu and Jordan [7] have suggested that R_b can be estimated by assuming that it has the value which would be obtained if there were no atmosphere. For surfaces in the northern hemisphere, sloped towards the equator, the equation for R_b is given as below [7] and is used in the present study.

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_s' + \omega_s'\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \omega_s\sin\phi\sin\delta} \quad (4)$$

where

$$\omega_s' = \min\{\cos^{-1}(-\tan\phi\tan\delta), \cos^{-1}(-\tan(\phi-\beta)\tan\delta)\} \quad (5)$$

ω_s' is the sunset hour angle for the tilted surface for the mean day of the month. ''min'' means the smaller of the two terms in the bracket.

For surfaces in the southern hemisphere, sloped towards the equator, the equation for R_b is given as below [7].

$$R_b = \frac{\cos(\phi+\beta)\cos\delta\sin\omega_s' + \omega_s'\sin(\phi+\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \omega_s\sin\phi\sin\delta} \quad (4')$$

where

$$\omega_s' = \min\{\cos^{-1}(-\tan\phi\tan\delta), \cos^{-1}(-\tan(\phi+\beta)\tan\delta)\} \quad (5')$$

where ϕ is the latitude, δ is the declination angle and ω is the angle from the local solar noon. The declination angle is given as [12]

$$\delta = 23.45\sin[360(284+n)/365], \quad (6)$$

where n is the n th day of the year (1-365).

Assuming isotropic reflection, the daily ground reflected radiation can be written as

$$H_R = H\rho(1-\cos\beta)/2 \quad (7)$$

Diffuse Radiation Models

The methods to estimate the ratio of diffuse solar radiation on a tilted surface to that of a horizontal are classified as isotropic and anisotropic models. The isotropic models assume that the intensity of diffuse sky radiation is uniform over the sky dome. Hence, the diffuse radiation incident on a tilted surface depends on the fraction of the sky dome seen by it. The anisotropic models assume the anisotropy of the diffuse sky radiation in the circumsolar region (sky near the solar disc) plus and isotropically distributed diffuse component from the rest of the sky dome.

The sky-diffuse radiation can be expressed as

$$H_s = R_d H_d \quad (8)$$

where R_d is the ratio of the average daily diffuse radiation on a tilted surface to that on a horizontal surface.

The diffuse radiation models chosen for study were as follows [13].

Isotropic Models

Badescu model (2002)

$$R_d = [3+\cos(2\beta)]/4 \quad (9)$$

Tian *et al.* model (2001)

$$R_d = 1-\beta/180 \quad (10)$$

Koronakis model (1986)

$$R_d = 1/3[2 + \cos\beta] \tag{11}$$

Liu and Jordan model (1962)

$$R_d = [1 + \cos\beta]/2 \tag{12}$$

Anisotropic Models

Reindl *et al.* model (1990)

$$R_d = \frac{H_b}{H_o}R_b + (1 - \frac{H_b}{H_o})[(1+\cos\beta)/2][1 + \sqrt{H_b/H}\sin^3(\beta/2)] \tag{13}$$

Skartveit and Olseth model (1986)

$$R_d = \frac{H_b}{H_o}R_b + \Omega\cos\beta + (1 - \frac{H_b}{H_o} - \Omega)(1+\cos\beta)/2, \tag{14}$$

where

$$\Omega = \{\max[0, (0.3 - 2\frac{H_b}{H_o})]\} \tag{15}$$

Steven and Unsworth model (1980)

$$R_d = 0.51R_b + \left\{\left[1+\cos\beta\right]/2\right\} - \frac{1.74}{1.26\pi} \left[\sin\beta - (\beta \times \frac{\pi}{180})\cos\beta - \pi\sin^2(\beta/2)\right] \tag{16}$$

Hay model (1979)

$$R_d = \frac{H_b}{H_o}R_b + (1 - \frac{H_b}{H_o})[(1+\cos\beta)/2] \tag{17}$$

Total Radiation on a Tilted Surface

Total radiation on a tilted surface, can thus be expressed as

$$H_T = (H - H_d)R_b + H\rho(1-\cos\beta)/2 + H_d R_d \tag{18}$$

As no information is available on ground albedo, ρ values are assumed to be 0.2. According to Equation (18), we need the direct and diffuse components of global radiation for estimating global solar radiation on tilted surfaces. For this study these components were not available separately, so we used Miguel *et al.* [14] model to estimate the daily direct and diffuse components from measured daily global irradiance (Appendix A.1 and A.2). Computer programmes were written at various stages of the work. Data of monthly mean daily global radiation and monthly average clearness index (except for Indian stations) on horizontal surfaces needed for the study was taken from Duffie and Beckman [12].

Experimental Data

For Indian stations, long term monthly-mean hourly global radiation data for a measuring site are obtained from hourly global radiation by averaging individual hourly values for each month over a period of one to eleven years. The long term monthly-mean daily global irradiation is obtained as the sum of each individual hourly irradiation for that day. The solar radiation data have been collected for the period of 1991-2001 from India Meteorology Department (IMD) Pune, India.

The present results are based on a large number of different instrument types, calibration methods, and climates, so that the overall error should remain small. Most other errors are random, and they tend to decrease rapidly as the averaging period increases. Therefore, they should be negligible here.

These data have been obtained using a thermoelectric pyranometer. The pyranometer used are supposed to be calibrated once a year with reference to the World Radiometric Reference (WRR). Critical information such as calibration history, instrument changes, data quality control process, and shading due to obstructions in the horizon, is simply not available for these stations. Therefore it is to be expected that some data sites have larger uncertainties, with possibly more incorrect or missing data than others. But this situation is also confronted by any investigator using this kind of radiation data directly to design solar energy systems.

Methodology

Using Equations (1) to (18), total solar radiation falling on tilted surface was computed for tilt angle 0, 10, 20, 30, 40, 50, 60, 70, 80 and 90 degrees for each month of the year and for each station. Using MSEXCEL graphics software package, graphs were plotted between the total insolation versus tilt angle for each month and each station. Second order polynomial equations were developed to fit the curves. These polynomial equations were differentiated with respect to tilt angle and then equated to zero to obtain the optimum tilt angle corresponding to maximum insolation. Thus optimum tilt angle was computed for each month and for each station.

Computer programmes in Matlab were developed using the above formulae to calculate the monthly average daily total radiation on a surface facing towards equator as the tilt angle is changed from 0 to 90^o. The solar reflectivity (ρ) was assumed to be 0.2.

RESULTS

Fig. (**1**) shows the monthly average daily global solar radiation H and the monthly average extra-terrestrial daily radiation H_o on a horizontal surface in the city of New Delhi in India. The average winter value of H is 14.82 MJ/m^{2}day and its average summer value is 22.97 MJ/m^{2}day. The beam and diffuse components of the monthly average daily global radiation on a horizontal surface are shown in Fig. (**2**). In winter months, the beam and diffuse components are nearly equal, and thus both components make the same contribution to the global radiation. In summer months, the beam component is more than diffuse component and thus the main contribution comes from the beam component. In monsoon season, the diffuse component is more than beam component.

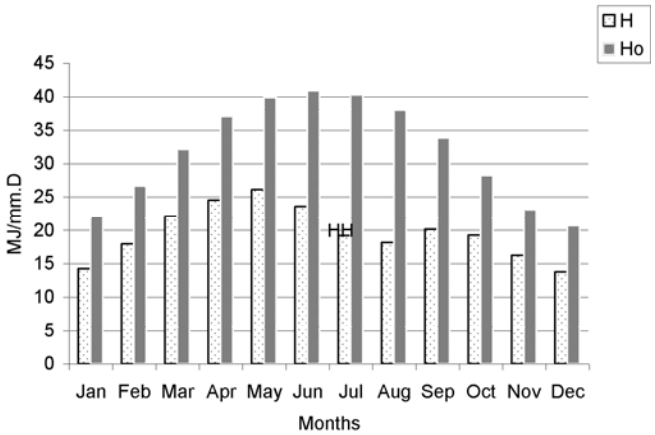

Fig. (1). Monthly average daily global radiation (H) and monthly averaged extra-terrestrial daily radiation (Ho) on a horizontal surface in New Delhi.

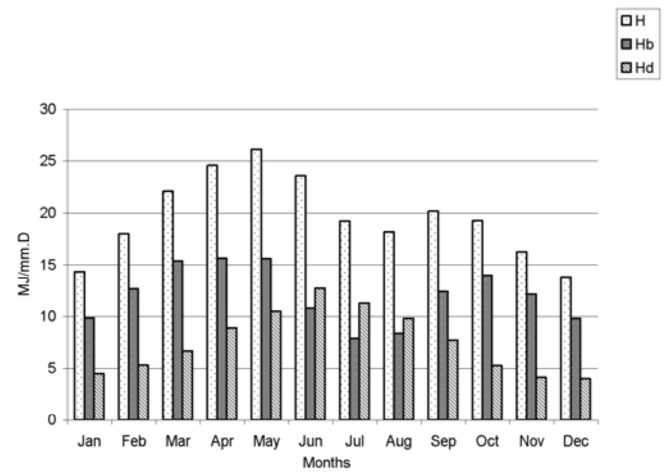

Fig. (2). Monthly average daily global radiation (H), beam radiation (Hb) and diffuse radiation (Hd) on a horizontal surface in New Dehli.

Figs. (**3(a)** and **3(b)**) show the average daily total solar radiation at New Delhi on a south facing surface as the angle of tilt is varied from 0 to 90^{o} in steps of 10^{o}. It is clear from these graphs that a unique β_{opt} exists for each month of the year for which the solar radiation is at a peak for the given month. Similar trend has been observed for all other stations selected under present study. Graphs for other stations have not been shown in order to avoid repeatation. Table **1** gives a summary list of β_{opt} for each month of the year at New Delhi. The optimum angle of tilt of a flat-plate collector in January is 56^{o} and the total monthly solar radiation falling on the collector surface at this tilt is 870 MJ/m^2. The optimum tilt angle in June goes to a minimum of zero degree and the total monthly solar radiation at this angle is 796 MJ/m^2. The optimum tilt angle then increases during the winter months and reaches a maximum of 58^{o} in December which collects 759 MJ/m^2 of solar energy monthly.

Figs. (**4**(**a**), **4**(**b**) and **4**(**c**)) show the tilt angles for each month of the year when the collector panel is tilted at the optimum angle at different stations in India, Northern hemi-

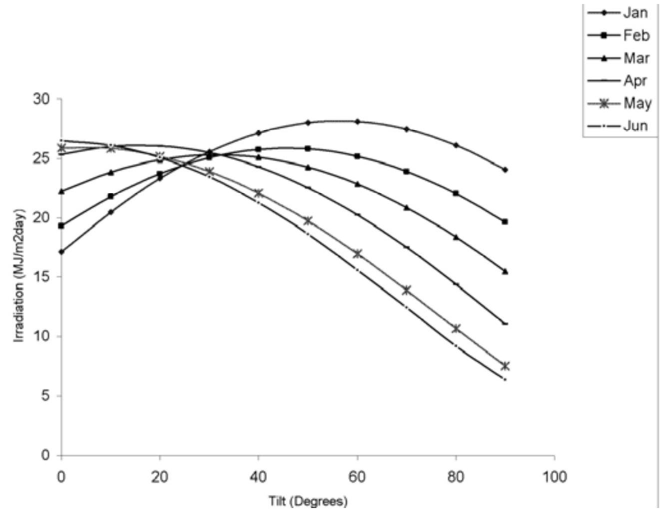

Fig. 3(a). Monthly average daily total solar radiation on a south facing panel in New Delhi for the months of January-June.

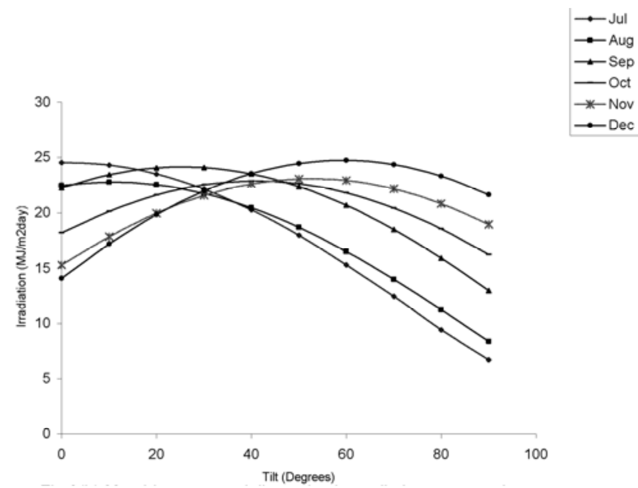

Fig. 3(b). Monthly average daily total solar radiation on a south facing panel in New Delhi for the months of July-December.

Table 1. Optimum Tilt Angle (β_{opt}) for Each Month of the Year for a South Facing Solar Collector at New Delhi

Month	β_{opt}(degrees)	Monthly Radiation(MJ/m^2.month)
Jan	56	870.2940
Feb	45	721.6720
Mar	32	783.8598
Apr	14	780.8160
May	0	803.3030
Jun	0	795.8400
Jul	0	760.8950
Aug	6	705.4670
Sep	25.5	721.9230
Oct	40	704.8780
Nov	53	694.2570
Dec	58	758.8614

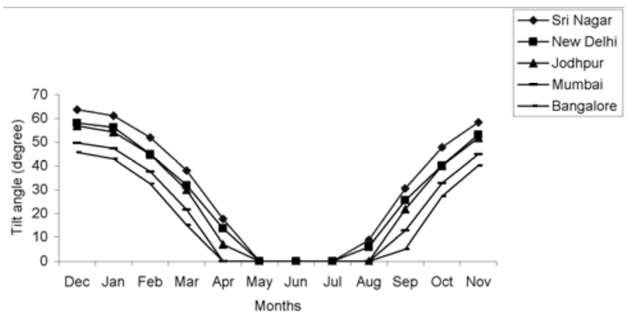

Fig. 4(a). Optimum average tilt angle for each month of the year at five different Indian locations.

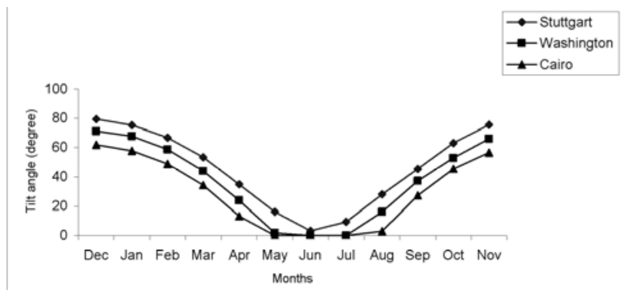

Fig. 4(b). Optimum average tilt angle for each month of the year three different locations of northern hemisphere.

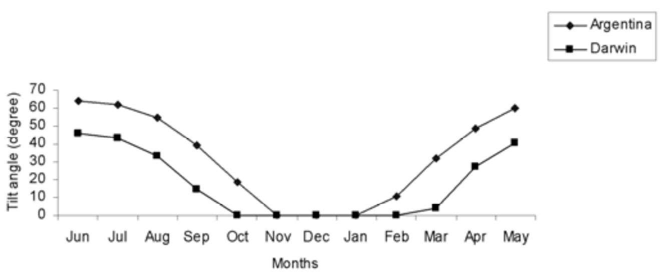

Fig. 4(c). Optimum average tilt angle for each month of the year two different locations of northern hemisphere.

sphere and southern hemisphere respectively. Table **2** shows the seasonal average, and the yearly average tilt angles at New Delhi. The seasonal average was calculated by finding the average value of the tilt angle for each season and the implementation of this requires the collector tilt to be changed four times a year. In spring the tilt should be 24^o, in summer zero degree, in autumn 30^o and in winter 56^o. The yearly average tilt was calculated by finding the average value of the tilt angles for all months of the year. The yearly average tilt was found to be 30^o and this results in a fixed tilt

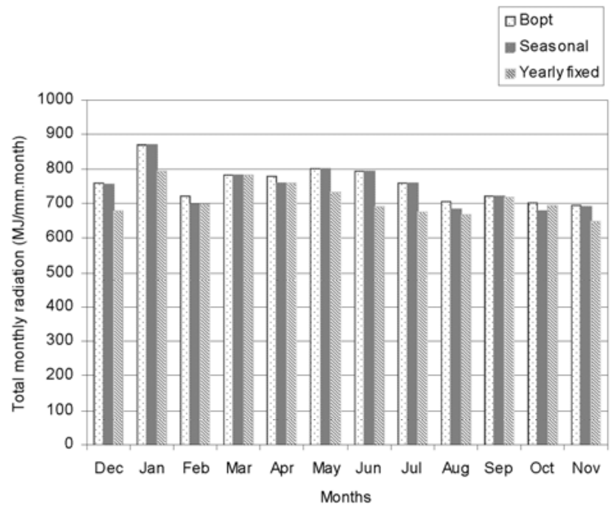

Fig. (5). Total monthly solar radiation for optimum, seasonally adjusted and yearly fixed titl angles in New Delhi.

throughout the year. Fig. (**5**) shows the monthly solar energy collected when the angle of tilt is optimum, when the seasonal average angles are used, and when the yearly average angle is used throughout the year. The collected energy is tabulated in detail in Table **2**. When the monthly optimum tilt angle was used, the yearly collected solar energy was 9103 MJ/m^2. With the seasonally adjusted tilt angles, the yearly collected solar energy was 9015 M J/m^2. Finally, with the yearly average tilt angle, the yearly collected solar energy was 7879 MJ/m^2. Fig. (**6**) shows the yearly average optimum tilt angles for all the ten selected stations.

CONCLUSIONS

Monthly based optimum tilt is different for different stations (Fig. **4**). Season based optimum tilt is also different for different stations (Fig. **4**, Table **2**). Annual based optimum tilt is approximately equal to latitude of the location. All the eight diffuse radiation models yield the same optimum tilt.

The results show that the average optimum tilt angle at New Delhi for the winter months is 47.5^o (latitude + 19°) and for the summer months 13 ^{o} (latitude -16°). This, in general, is in agreement with the results of many other researchers [15, 16].

It is clear from Table **2** that the loss in the amount of collected energy is less than 1% (0.97) if the angle of tilt is adjusted seasonally instead of using β_{opt} for each month of the year at New Delhi. The loss of energy when using the yearly average fixed angle is around 13.4% compared with the op-

Table 2. Optimum Tilt, Seasonally Adjusted Tilt and Yearly Average Tilt and Monthly Solar Radiation on a Tilted South Facing Plane at New Delhi. The Numbers Inside Brackets are the Tilt Angles

Tilt	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Yearly
β_{opt}	870.29 (56)	721.67 (45)	783.86 (32)	780.82 (14)	803.30 (0)	795.84 (0)	760.89 (0)	705.47 (6)	721.92 (25.5)	704.88 (40)	694.26 (53)	758.86 (58)	9102.1
Seasonally adjusted β	870.29 (56)	701.43 (30)	783.52 (30)	760.08 (30)	803.30 (0)	795.84 (0)	760.89 (0)	684.04 (24)	721.75 (24)	681.91 (24)	693.58 (56)	758.38 (56)	9015.0
Yearly average β	795.34 (30)	701.43 (30)	783.52 (30)	760.08 (30)	732.22 (30)	691.05 (30)	678.16 (30)	667.52 (30)	720.33 (30)	695.73 (30)	649.20 (30)	682.22 (30)	7878.6

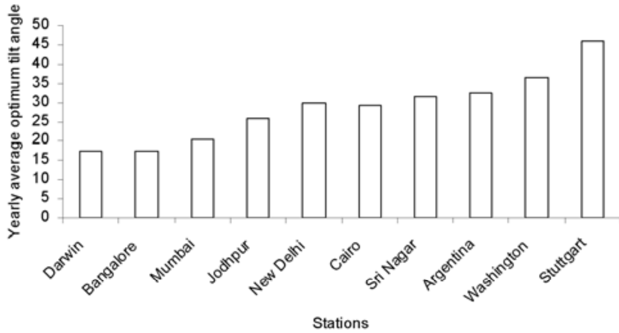

Fig. (6). Yearly average optimum tilt angle for different stations.

timum tilt at New Delhi. It can be concluded that a yearly average fixed tilt can be used in many general applications (e.g. domestic water heating) in order to keep the manufacturing and installation costs of collectors low. For higher efficiency, the collector should be designed such that the angle of tilt can easily be changed at least on a seasonal basis, if not monthly. Alternatively, solar tracking systems can be used in industrial installations where higher efficiency is required.

ACKNOWLEDGEMENTS

The authors are grateful to the Indian Meteorological Department, Pune, India for providing the hourly global and diffuse radiation data for the period of 11 years from 1991 to 2001.

APPENDIX A

Computing daily diffuse and direct components from daily global irradiance [14]. A simple physical based method proposed by Miguel *et al*. (2001) was used for estimating daily diffuse and direct components from daily global irradiance. For three different ranges of atmospheric transmissivity ($k=H/H_o$), the resulting correlations are given by the following expression:

$Hd/H = 0.952$ if $k \le 0.13$, $0.868 + 1.335\,k - 5.782\,k^2 + 3.721\,k^3$ if $0.13 < k \le 0.80$, 0.141 if $k > 0.80$ (A.1)

And H_b can be calculated as follows:

$H_b = H - H_d$ (A.2)

NOMENCLATURE

H = daily global radiation incident on a horizontal surface, MJ $m^{-2}day^{-1}$

H_d = daily diffuse radiation incident on a horizontal surface, MJ $m^{-2}day^{-1}$

H_B = daily beam radiation incident on an inclined surface, MJ $m^{-2}day^{-1}$

H_R = daily ground- reflected radiation incident on an inclined surface, MJ $m^{-2}day^{-1}$

H_S = daily sky-diffuse radiation incident on an inclined surface, MJ $m^{-2}day^{-1}$

H_o = extraterrestrial daily radiation incident on a horizontal surface, MJ $m^{-2}day^{-1}$

H_T = daily global radiation on a tilted surface, MJ $m^{-2}day^{-1}$

R_b = ratio of average daily beam radiation incident on an inclined surface to that on a horizontal surface

R_d = ratio of average daily diffuse radiation incident on an inclined surface to that on a horizontal surface

β = surface slope from the horizontal, degrees

δ = declination, degrees

ω_s = sunrise hour angle, degrees

ω_s' = sunrise hour angle for a tilted surface, degrees

ϕ = latitude, degrees

ρ = ground albedo

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Received: October 14, 2008 Revised: November 17, 2008 Accepted: November 19, 2008