SUPPLEMENTARY MATERIAL

Chinese Remainder Theorem

The Chinese Remainder Theorem (CRT) is based on modulo arithmetic which is also colloquially called clock arithmetic. In modular arithmetic, only the remainder left over after division of a number by the modulusis considered. The remainders form a cyclic group. Two integers m and n are said to be congruent modulo ***n*** (***n*** also an integer) if their difference is divisible by ***n***, as indicated by the ≡ symbol. For example, the number 20 would be represented by 8 using a 12 hour clock (i.e. 8 ≡ 20 mod 12) and 20 using military time (i.e. 20 ≡ 20 mod 24). The CRT states that any number can be uniquely represented by the remainders of different moduli under two conditions; first the number itself must be less than the product of the moduli and second, the moduli used are pairwise co-prime with each other i.e. their only common factor is 1. So 20 could also be represented by the factors of 24 as 2 ≡ 20 mod 3 and 4 ≡ 20mod 8. Thus if it was necessary to meet someone at 8 pm (or 20:00 hours) but others should not know the time, the time could be signaled publically as 2, 4 – since 3 and 8 are the only two factors of 24 that are co-prime, the knowledgeable recipient would then be able to compute exactly the time to meet using the CRT. The recipient would also know a 12 hour clock was not in use as 12 has co-prime factors 3 and 4 (and 0 ≡ 4mod 4).

Wiring of Hadamard Designs

The arrays discussed here are characterized by v ≡3 mod 4, where v is a prime, or the product of twin prime pairs, or primes that are of the form 2^n-1. The connection designated by “1” in the vector used to generate an array denote the quadratic residues for each modulus that are the residues of perfect squares mod v (i.e. a is a quadratic residue when $x^2 \equiv a \bmod v$, x∈{1,2,3..}. For example of 1^2, 2^2, 3^2 mod 7 are 1, 4 and 2). The perfect squares required to calculate the quadratic residues to construct a square array mod v can be represented by $x^2 = \sum_{k=1}^{\frac{v-1}{2}} (2v-1)$ i.e. by addition of odd numbers less than v. This permits determination of quadratic residues by the addition schemes using only odd numbers less than v. The sum mod v of each addition is a quadratic residue mod v, as well as the input for addition mod v to the next highest odd number. This approach does not require addition of any number greater than v,nor the use of any array of size greater than the v array. The first row is specified in (v-1)/2 steps and will have (v-1)/2 elements when v is prime. R code is available on request to show how this construction can be implemented.

The twin prime arrays can be derived using the quadratic residues for p and p+2. R code is available on request to show how this can be performed using a modified Jacobi sequence. The advantage of this approach is that giant arrays equal to the product of p and p+2 could be built in fewer steps than building them stepwise based on v =p x (p+2).

Supplementary Fig. (**1**).

(a)

7,4,2	0	1	2	3	4	5	6
1	1		1	1	1		
2		1		1	1	1	
3			1		1	1	1
4	1			1		1	1
5	1	1			1		1
6	1	1	1			1	
7		1	1	1			1

(b)

11,5,3		1	2	3	4	5	6	7	8	9	10
1	1			1		1	1	1			
2		1			1		1	1	1		
3			1			1		1	1	1	
4				1			1		1	1	1
5	1				1			1		1	1
6	1	1				1			1		1

7	1	1	1				1			1	
8		1	1	1				1			1
9	1		1	1	1				1		
10		1		1	1	1				1	
11			1		1	1	1				1

(c)

11,6,4		1	2	3	4	5	6	7	8	9	10
1	1		1	1		1	1	1			
2		1		1	1		1	1	1		
3			1		1	1		1	1	1	
4				1		1	1		1	1	1
5	1				1		1	1		1	1
6	1	1				1		1	1		1
7	1	1	1				1		1	1	
8		1	1	1				1		1	1
9	1		1	1	1				1		1
10	1	1		1	1	1				1	
11		1	1		1	1	1				1

(d)

15,7,4	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1		1			1	1		1	1	1				
2		1		1			1	1		1	1	1			
3			1		1			1	1		1	1	1		
4				1		1			1	1		1	1	1	
5					1		1			1	1		1	1	1
6	1					1		1			1	1		1	1
7	1	1					1		1			1	1		1
8	1	1	1					1		1			1	1	
9		1	1	1					1		1			1	1
10	1		1	1	1					1		1			1
11	1	1		1	1	1					1		1		
12		1	1		1	1	1					1		1	
13			1	1		1	1	1					1		1
14	1			1	1		1	1	1					1	
15		1			1	1		1	1	1					1

(e)

15,8,5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1				1			1	1		1		1	1	1
2	1	1				1			1	1		1		1	1

3	1	1	1				1			1	1		1		1
4	1	1	1	1				1			1	1		1	
5		1	1	1	1				1			1	1		1
6	1		1	1	1	1				1			1	1	
7		1		1	1	1	1				1			1	1
8	1		1		1	1	1	1				1			1
9	1	1		1		1	1	1	1				1		
10		1	1		1		1	1	1	1				1	
11			1	1		1		1	1	1	1				1
12	1			1	1		1		1	1	1	1			
13		1			1	1		1		1	1	1	1		
14			1			1	1		1		1	1	1	1	
15				1			1	1		1		1	1	1	1

(f)

19,9,5		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1		1		1					1	1			1			1	1	1
2	1	1		1		1					1	1			1			1	1
3	1	1	1		1		1					1	1			1			1
4	1	1	1	1		1		1					1	1			1		
5		1	1	1	1		1		1					1	1			1	
6			1	1	1	1		1		1					1	1			1
7	1			1	1	1	1		1		1					1	1		
8		1			1	1	1	1		1		1					1	1	
9			1			1	1	1	1		1		1					1	1
10	1			1			1	1	1	1		1		1					1
11	1	1			1			1	1	1	1		1		1				
12		1	1			1			1	1	1	1		1		1			
13			1	1			1			1	1	1	1		1		1		
14				1	1			1			1	1	1	1		1		1	
15					1	1			1			1	1	1	1		1		1
16	1					1	1			1			1	1	1	1		1	
17		1					1	1			1			1	1	1	1		1
18	1		1					1	1			1			1	1	1	1	
19		1		1					1	1			1			1	1	1	1

(g)

19,10,6	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1		1		1					1	1		1	1			1	1	1
2	1	1		1		1					1	1		1	1			1	1
3	1	1	1		1		1					1	1		1	1			1
4	1	1	1	1		1		1					1	1		1	1		

5		1	1	1	1		1		1					1	1		1	1	
6			1	1	1	1		1		1					1	1		1	1
7	1			1	1	1	1		1		1					1	1		1
8	1	1			1	1	1	1		1		1					1	1	
9		1	1			1	1	1	1		1		1					1	1
10	1		1	1			1	1	1	1		1		1					1
11	1	1		1	1			1	1	1	1		1		1				
12		1	1		1	1			1	1	1	1		1		1			
13			1	1		1	1			1	1	1	1		1		1		
14				1	1		1	1			1	1	1	1		1		1	
15					1	1		1	1			1	1	1	1		1		1
16	1					1	1		1	1			1	1	1	1		1	
17		1					1	1		1	1			1	1	1	1		1
18	1		1					1	1		1	1			1	1	1	1	
19		1		1					1	1		1	1			1	1	1	1

(h)

23,11,6	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1						1		1			1	1			1	1		1		1	1	1
2	1	1						1		1			1	1			1	1		1		1	1
3	1	1	1						1		1			1	1			1	1		1		1
4	1	1	1	1						1		1			1	1			1	1		1	
5		1	1	1	1						1		1			1	1			1	1		1
6	1		1	1	1	1						1		1			1	1			1	1	
7		1		1	1	1	1						1		1			1	1			1	1
8	1		1		1	1	1	1						1		1			1	1			1
9	1	1		1		1	1	1	1						1		1			1	1		
10		1	1		1		1	1	1	1						1		1			1	1	
11			1	1		1		1	1	1	1						1		1			1	1
12	1			1	1		1		1	1	1	1						1		1			1
13	1	1			1	1		1		1	1	1	1						1		1		
14		1	1			1	1		1		1	1	1	1						1		1	
15			1	1			1	1		1		1	1	1	1						1		1
16	1			1	1			1	1		1		1	1	1	1						1	
17		1			1	1			1	1		1		1	1	1	1						1
18	1		1			1	1			1	1		1		1	1	1	1					
19		1		1			1	1			1	1		1		1	1	1	1				
20			1		1			1	1			1	1		1		1	1	1	1			
21				1		1			1	1			1	1		1		1	1	1	1		
22					1		1			1	1			1	1		1		1	1	1	1	
23						1		1			1	1			1	1		1		1	1	1	1

(h)c(31,15,8)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1		1	1		1	1	1		1						1			1	1				1	1	1	1			1	
2		1		1	1		1	1	1		1						1			1	1				1	1	1	1			1
3	1		1		1	1		1	1	1		1						1			1	1				1	1	1	1		
4		1		1		1	1		1	1	1		1						1			1	1				1	1	1	1	
5			1		1		1	1		1	1	1		1						1			1	1				1	1	1	1
6	1			1		1		1	1		1	1	1		1						1			1	1				1	1	1
7	1	1			1		1		1	1		1	1	1		1						1			1	1				1	1
8	1	1	1			1		1		1	1		1	1	1		1						1			1	1				1
9	1	1	1	1			1		1		1	1		1	1	1		1						1			1	1			
10		1	1	1	1			1		1		1	1		1	1	1		1						1			1	1		
11			1	1	1	1			1		1		1	1		1	1	1		1						1			1	1	
12				1	1	1	1			1		1		1	1		1	1	1		1						1			1	1
13	1				1	1	1	1			1		1		1	1		1	1	1		1						1			1
14	1	1				1	1	1	1			1		1		1	1		1	1	1		1						1		
15		1	1				1	1	1	1			1		1		1	1		1	1	1		1						1	
16			1	1				1	1	1	1			1		1		1	1		1	1	1		1						1
17	1			1	1				1	1	1	1			1		1		1	1		1	1	1		1					
18		1			1	1				1	1	1	1			1		1		1	1		1	1	1		1				
19			1			1	1				1	1	1	1			1		1		1	1		1	1	1		1			
20				1			1	1				1	1	1	1			1		1		1	1		1	1	1		1		
21					1			1	1				1	1	1	1			1		1		1	1		1	1	1		1	
22						1			1	1				1	1	1	1			1		1		1	1		1	1	1		1
23	1						1			1	1				1	1	1	1			1		1		1	1		1	1	1	
24		1						1			1	1				1	1	1	1			1		1		1	1		1	1	1
25	1		1						1			1	1				1	1	1	1			1		1		1	1		1	1

26	1	1		1						1			1	1				1	1	1	1			1		1		1	1		1
27	1	1	1		1						1			1	1				1	1	1	1			1		1		1	1	
28		1	1	1		1						1			1	1				1	1	1	1			1		1		1	1
29	1		1	1	1		1						1			1	1				1	1	1	1			1		1		1
30	1	1		1	1	1		1						1			1	1				1	1	1	1			1		1	
31		1	1		1	1	1		1						1			1	1				1	1	1	1			1		1

Supplementary Fig. (1). Square sets belonging to the series (t, 2t-1, 4t-1 or t+1,2t, 4t-1) are designated by the label in the top right box. Inputs are represented by columns and outputs by rows. Connections between inputs and outputs are indicated by the number 1. Biologically these sets can be generated during neuronal development by reiterated the simple pattern in the first row by cyclic permutation. The first row of each set is highlighted to allow comparison of the elements. Evolutionary, besides the scheme discussed in the paper, it is possible for one set to develop from another. The steps from the v=7 to the v=23 arrays involves addition of a new connection and a change in spacing between existing connections. The new connections are highlighted with the light gray box. For v=31, the array may have arisen from using the elements from v=1 and v=19 that are outlined with the dotted box.